Photon Generation by Laser-Compton Scattering

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Contents:
0. Laser-Compton Scattering (LCS)
1. Short history on KEK LCS experiment
2. Results at KEK-ATF-EXT
3. 2-mirror optical cavity for laser power accumulation
4. Experiments at the KEK ATF using 4-mirror optical cavity
5. Results at STF and cERL
6. Results at LUCX
0. Laser-Compton Scattering (LCS)

Theoretical Description of Compton Scattering

At the initial electron rest frame, Four-momentums of electrons and photons;

\[ p_{\gamma 1}^\mu = \frac{E_1^*}{c} \left( \frac{c}{\cos \theta^*}, \frac{\sin \theta^*}{\sin \phi^*}, 0 \right), \]
\[ p_{e1}^\mu = \frac{mc^2}{c} \left( 0, 0, 0 \right), \]
\[ p_{\gamma 2}^\mu = \frac{E_2^*}{c} \left( \frac{c}{\cos \phi^*}, \frac{\sin \phi^*}{\sin \phi^*}, 0 \right), \]
\[ p_{e2}^\mu = \frac{mc^2 \gamma^*}{c} \left( \frac{c}{\beta^* \cos \psi^*}, \frac{\beta^* \sin \psi^*}{\beta^* \sin \psi^*}, 0 \right) \]

Due to the energy momentum conservation;

\[ p_{\gamma 1}^\mu + p_{e1}^\mu = p_{\gamma 2}^\mu + p_{e2}^\mu, \]

\[ |p_{e2}^\mu|^2 = p_{\gamma 1}^\mu + p_{e1}^\mu - p_{\gamma 2}^\mu|^2 \]

\[ = p_{\gamma 1}^\mu p_{\gamma 1\mu} + p_{e1}^\mu p_{e1\mu} + p_{\gamma 2}^\mu p_{\gamma 2\mu} + 2p_{\gamma 1}^\mu p_{e1\mu} - 2p_{\gamma 2}^\mu p_{e1\mu} - 2p_{\gamma 1}^\mu p_{\gamma 2\mu}, \]

From above equations;

\[ E_2^* = \frac{mc^2 E_1^*}{mc^2 + E_1^* [1 - \cos(\theta^* - \phi^*)]} \]
At the laboratory frame by Lorentz transformation as;

\[ E_1^* = \gamma E_1 (1 - \beta \cos \theta), \]
\[ E_2^* = \gamma E_2 (1 - \beta \cos \phi), \]

\[ \theta^* = \sin^{-1} \left[ \frac{\sin \theta}{\gamma (1 - \beta \cos \theta)} \right] = \cos^{-1} \left( \frac{\beta - \cos \theta}{1 - \beta \cos \theta} \right), \]
\[ \phi^* = \sin^{-1} \left[ \frac{\sin \phi}{\gamma (1 - \beta \cos \phi)} \right] = \cos^{-1} \left( \frac{\beta - \cos \phi}{1 - \beta \cos \phi} \right), \]

\[ E_2 = \frac{mc^2 \gamma E_1 (1 - \beta \cos \theta)}{mc^2 \gamma (1 - \beta \cos \phi) + E_1 \{1 - \cos (\theta - \phi)\}}. \]

\[ \theta = 180^\circ, \phi = 0 \]
\[ E_{2\text{max}} = \frac{\gamma E_1 (1 + \beta)^2}{1 + \frac{2\gamma E_1}{mc^2} (1 + \beta)} \sim 4\gamma^2 E_1 \]

\[ \theta = 90^\circ, \phi = 0 \]
\[ E_{2-90^\circ} = \frac{\gamma E_1 (1 + \beta)}{1 + \frac{\gamma E_1}{mc^2} (1 + \beta)} \sim 2\gamma^2 E_1 \]
The differential cross section of Compton scattering in the electron is given by Klein-Nishina equation.

\[
\frac{d\sigma}{d\Omega^*} = \frac{r_0^2}{2} \left( \frac{E_2^*}{E_1^*} \right)^2 \left[ \frac{E_1^*}{E_2^*} + \frac{E_2^*}{E_1^*} - \sin^2(\phi^* - \theta^*) \right].
\]

Above equation can be converted to the laboratory frame as;

\[
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega^*} \frac{d\Omega^*}{d\Omega} = \frac{d\sigma}{d\Omega^*} \frac{\sin\phi^*}{\sin\phi} \frac{d\phi^*}{d\phi} = \frac{d\sigma}{d\Omega^*} \frac{-1}{\sin\phi} \frac{d\cos\phi^*}{d\phi}
\]

\[
= \frac{d\sigma}{d\Omega^*} \frac{1}{\gamma^2 (1 - \beta \cos\phi)^2}
\]

Energy distribution of the scattered photon in the laboratory frame as;

\[
\frac{d\sigma}{dE_2} = \frac{d\sigma}{d\Omega^*} \frac{d\Omega^*}{dE_2}
\]
1. Short history on KEK LCS experiment
**ATF Status**

**Energy** = 1.28 GeV  
**Intensity** = $1.0 \times 10^{10}$ e-/bunch

**Vertical emittance measurement in damping ring**

**1. Laser wire scanner result**

$\sigma_y = 9.8 \pm 1.1_{\text{stat}} \pm 0.4_{\text{sys}} \, \mu\text{m}$

$\varepsilon_y = (1.8 \pm 0.4_{\text{stat}} \pm 0.1_{\text{sys}}) \times 10^{-11} \, \text{m} \cdot \text{rad}$

Intensity = $5 \times 10^9$

Preliminary

![Graph showing signal vs laser wire vertical position]

*Measured by Kyoto Univ. with ATF collaboration*

New data obtained in Dec. are under analysis.

**2. SR Interference monitor result**

<table>
<thead>
<tr>
<th>$\sigma_x = 50.0 , \mu\text{m}$</th>
<th>$\varepsilon_x = 1.30 \times 10^{-9} , \text{m} \cdot \text{rad}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y = 6.96 , \mu\text{m}$</td>
<td>$\varepsilon_y = 1.86 \times 10^{-11} , \text{m} \cdot \text{rad}$</td>
</tr>
</tbody>
</table>

Intensity = $(1.0 - 1.2) \times 10^{10}$ e-/bunch (4mA)
JFY2003

Optical cavity development

New Laser wire  X & Y scan  May 2003

Two wire chamber
TEM\(_{00}\) Mode

Normalized beam emittance in Linear Colliders

X profile  Y profile
Experimental Area is at the KEK-ATF extraction line

**JFY 2003-2004**

1.28 GeV S-band Linac

1.28 GeV Damping Ring

Accelerator Test Facility

Laser-electron collision point

2. Results at KEK-ATF EXT

**Expected Value**

- Low background in the experiment
- Consistent with expected value

**Number of Gamma-rays**

Entry [Counts]

For on-axis collision and (electron beam size) \ll (laser spot size), luminosity is defined only by $M^2$ of laser.
3. Two mirror optical cavity for laser power accumulation

3-1. Cavity stability

\[ R(z_1) = z_1 + \frac{z_0^2}{z_1} \]

\[ R(z_2) = z_2 + \frac{z_0^2}{z_2} \]

The condition that the beam in Cavity is stable;

\[ D = z_2 - z_1 \]

\[ -R_1 = z_1 + \frac{z_0^2}{z_1} \]

\[ R_2 = z_2 + \frac{z_0^2}{z_2} \]
Conversely, solving

\[ z_1 = \frac{-D (R_2 - D)}{R_1 + R_2 - 2D} \]

\[ z_2 = \frac{D (R_1 - D)}{R_1 + R_2 - 2D} \]

\[ z_0^2 = \frac{D (R_1 - D)(R_2 - D)(R_1 + R_2 - D)}{(R_1 + R_2 - 2D)^2} \]

\[ w_0^2 = \frac{\lambda}{\pi} \cdot \sqrt{\frac{D (R_1 - D)(R_2 - D)(R_1 + R_2 - D)}{|R_1 + R_2 - 2D|}} \]

\[ z_0 = \pi w_0^2 / \lambda \]

Spot size \( w_1, w_2 \) on the mirror

\[ w_0^2 = \frac{\lambda D}{\pi} \cdot \sqrt{\frac{g_1 g_2 (1 - g_1 g_2)}{|g_1 + g_2 - 2g_1 g_2|}} \]

\[ w_1 = \frac{\lambda D}{\pi} \cdot \sqrt{\frac{g_2}{g_1 (1 - g_1 g_2)}} \]

\[ w_2 = \frac{\lambda D}{\pi} \cdot \sqrt{\frac{g_1}{g_1 (1 - g_1 g_2)}} \]

\[ g_{1,2} \equiv 1 - D/R_{1,2} \]
Stability conditions

Since the spot size is a real number,

\[ g_1 g_2 (1 - g_1 g_2) \geq 0 \]

\[ 0 \leq (1 - D / R_1)(1 - D / R_2) \leq 1 \]

Optical Cavity example (symmetric concentric type)
Wavelength: 532nm
Mirror radius of curvature: 0.5m
Laser waist size: 100\(\mu\)m

\[ w_0^2 = \frac{\lambda}{\pi} \sqrt{D \left( 2 \frac{R}{2} - D \right)} \]

\[ w \left( \frac{D}{2} \right) = \frac{\lambda}{\pi w_0} \sqrt{\frac{RD}{2}} \]

Laser size on mirror: 840.7 \(\mu\)m
Mirror spacing: 0.985851018m
3-2. Derivation of enhancement factor

We don't consider polarization, think of it as a Scalar function.

\[ E'_{rj} = E'_{ri} \tau_{ij} + E_{lj} \rho_{ji} \]
\[ E'_{li} = E_{lj} \tau_{ji} + E'_{ri} \rho_{ij} \]

\[ E'_{li} = E_{lj} \left( \frac{\tau_{ji} \tau_{ij} - \rho_{ji} \rho_{ij}}{\tau_{ij}} \right) + E_{rj} \frac{\rho_{ij}}{\tau_{ij}} \]

Fresnel equation: \( \rho_{ij} = -\rho_{ji}, \tau_{ji} \tau_{ij} + (\rho_{ij})^2 = 1 \)

\[ E'_{li} = \frac{1}{\tau_{ij}} E_{lj} + \frac{\rho_{ij}}{\tau_{ij}} E_{rj} \]
Matrix Formalism

\[ \vec{E}_j \equiv \begin{pmatrix} E_{lj} \\ E_{rj} \end{pmatrix} , \vec{E}_j' \equiv \begin{pmatrix} E'_{ij} \\ E'_{rj} \end{pmatrix} , H_{ij} \equiv \frac{1}{\tau_{ij}} \begin{pmatrix} 1 & \rho_{ij} \\ \rho_{ij} & 1 \end{pmatrix} \]

\[ \vec{E}_i = H_{ij} \vec{E}_j \]

\( H_{ij} \) is the interface transition matrix.

The phase change when propagating from end to end in one layer is \( e^{-i\beta_j} \) \((\beta_j = kn_j d_j / \cos \theta_j, k \) is wave number, \( n_j \) is refractive index of layer \( j, d_j \) is thickness of layer \( j, \theta_j \) is the angle of light with respect to the normal to the interface).
Then,

\[
\vec{E}_j = L_j \vec{E}_j
\]

Thus, \(L_j\) is called the layer propagation matrix.

If the incident wave is to the right and the \(N\)th layer is the last layer,

You obtain,

\[
\vec{E}_N = \begin{pmatrix} 0 \\ \vec{E}_{rN} \end{pmatrix}
\]

From the above,

\[
\vec{E}_1 = H_{12} \vec{E}_2 = H_{12}L_2 \vec{E}_2 = \cdots = H_{12}L_2H_{23}L_3 \cdots L_{N-1}H_{N-1,N} \vec{E}_N
\]

\[
S_{1N} \equiv H_{12}L_2H_{23}L_3 \cdots L_{N-1}H_{N-1,N}
\]

\[
\vec{E}_1 = S_{1N} \vec{E}_N
\]
In other words, all multiple effects in all layers from the input to the output including reflection are represented by this Stack matrix $S_{1N}$.

Single Slab

In this case, the stack matrix $S$ is

$$S = H_{12}L_2H_{23}$$

$$H_{12} = \frac{1}{\tau_{12}} \begin{pmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{pmatrix}, L_2 = \begin{pmatrix} e^{-i\beta_2} & 0 \\ 0 & e^{i\beta_2} \end{pmatrix}, H_{23} = \frac{1}{\tau_{23}} \begin{pmatrix} 1 & \rho_{23} \\ \rho_{23} & 1 \end{pmatrix} = \frac{1}{\tau_{21}} \begin{pmatrix} 1 & -\rho_{12} \\ -\rho_{12} & 1 \end{pmatrix}$$

$$\beta_2 = kn_2d / \cos \theta_2$$

When calculating,

$$S = \frac{1}{\tau_{12}\tau_{21}} \begin{pmatrix} e^{-i\beta_2} - \frac{\rho_{12}^2 e^{i\beta_2}}{\rho_{12} e^{-i\beta_2} - \rho_{12} e^{i\beta_2}} & \rho_{12} e^{i\beta_2} - \rho_{12} e^{-i\beta_2} \\ \rho_{12} e^{-i\beta_2} - \rho_{12} e^{i\beta_2} & e^{i\beta_2} - \frac{\rho_{12}^2 e^{-i\beta_2}}{\rho_{12} e^{i\beta_2} - \rho_{12} e^{-i\beta_2}} \end{pmatrix} \equiv \begin{pmatrix} S_{11} & S_{12} \\ -S_{12} & S_{22} \end{pmatrix}$$
From Single slab, the reflection and transmission coefficients $\rho$ and $\tau$ are:

$$
\begin{align*}
\begin{pmatrix}
E'_{l1} \\
E'_{r1}
\end{pmatrix} &=
\begin{pmatrix}
S_{11} & S_{12} \\
-S_{12} & S_{22}
\end{pmatrix}
\begin{pmatrix}
0 \\
E_{r3}
\end{pmatrix}
\end{align*}
$$

$$
\rho \equiv \frac{E'_{l1}}{E'_{r1}} = \frac{S_{12}E_{r3}}{S_{22}E_{r3}} = \frac{S_{12}}{S_{22}} = \frac{\rho_{12}(e^{i\beta_2} - e^{-i\beta_2})}{e^{i\beta_2} - \rho_{12}e^{-i\beta_2}} = \frac{\rho_{12}(e^{i\beta_2} - e^{-i\beta_2})e^{-i\beta_2}}{1 - \rho_{12}e^{-2i\beta_2}}
$$

$$
\tau \equiv \frac{E'_{r3}}{E'_{r1}} = \frac{E_{r3}}{S_{22}E_{r3}} = \frac{1}{S_{22}} = \frac{\tau_{12}\tau_{21}}{e^{i\beta_2} - \rho_{12}e^{-i\beta_2}} = \frac{\tau_{12}\tau_{21}e^{-i\beta_2}}{1 - \rho_{12}e^{-2i\beta_2}}
$$

Therefore, the reflectance, transmittance $R_m$ and $T_m$ are

$$
R_m = |\rho|^2 = \frac{4|\rho_{12}|^2 \sin^2 \beta_2}{|1 - \rho_{12}e^{-2i\beta_2}|^2}, \quad T_m = |\tau|^2 = \frac{|\tau_{12}\tau_{21}|^2}{|1 - \rho_{12}e^{-2i\beta_2}|^2}
$$

Here, the absorption due to the influence of the Slab (dielectric multilayer film) material and the like is neglected.

In fact, we has to adopt phase shift by absorption but we consider $\rho_{12} > 0$ for simplicity.
Here, as reflection and transmission at the interface between layer 1 (vacuum layer) and 2 (dielectric film), we introduce the rates $R_1$ and $T_1$.

\[
R_1 = \rho_{12}^2, \quad T_1 = \tau_{12} \tau_{21}
\]

\[
R_1 + T_1 = 1
\]

\[
\left|1 - \rho_{12}^2 e^{-2i\beta_2}\right|^2 = 1 + R_1^2 - 2R_1 \cos 2\beta_2
\]

Then,

\[
R_m = \frac{4R_1 \sin^2 \beta_2}{1 + R_1^2 - 2R_1 \cos 2\beta_2}
\]

\[
T_m = \frac{T_1^2}{1 + R_1^2 - 2R_1 \cos 2\beta_2}
\]

**Cavity transmittance**

Ignoring absorption, the layers 1, 3, 5 are evacuated. The mirrors of layers 2 and 4 have a thickness $d$ and a refractive index $n$. The layer 3 is in between the layers 2 and 4. The width is $D$. 
Stack Matrix S is;

\[ S = H_{12}L_2H_{23}L_3H_{34}L_4H_{45} \]

\[ H_{12} = H_{34} = \frac{1}{\tau_{12}} \begin{pmatrix} 1 & \rho_{12} \\ \rho_{12} & 1 \end{pmatrix}, \quad L_2 = L_4 = \begin{pmatrix} e^{-i\beta_2} & 0 \\ 0 & e^{i\beta_2} \end{pmatrix}, \]

\[ H_{23} = H_{45} = \frac{1}{\tau_{21}} \begin{pmatrix} 1 & -\rho_{12} \\ -\rho_{12} & 1 \end{pmatrix}, \quad L_3 = \begin{pmatrix} e^{-i\beta_1} & 0 \\ 0 & e^{i\beta_1} \end{pmatrix}, \]

\[ \beta_1 = kD, \beta_2 = knd \]

Introducing the stack matrix for the mirror and calculating the transmission coefficient,

\[ \tau = \frac{e^{i\beta_1} (\tau_{12} \tau_{21})^2}{4 \rho_{12}^2 \sin^2 \beta_2 + e^{2i(\beta_1 + \beta_2)} (1 - \rho_{12}^2 e^{-2i\beta_2})^2} \]

So, the transmittance is

\[ T = |d|^2 = \frac{|\tau_{12} \tau_{21}|^4}{4 \rho_{12}^2 \sin^2 \beta_2 + e^{2i(\beta_1 + \beta_2)} (1 - \rho_{12}^2 e^{-2i\beta_2})^2} \]
The denominator can be modified as follows.

\[ 4 \rho_{12}^2 \sin^2 \beta_2 + e^{2i(\beta_1 + \beta_2)} (1 - \rho_{12}^2 e^{-2i\beta_2})^2 \]

\[ = \left| 1 - R_1 e^{-2i\beta_2} \right|^4 \left\{ \frac{4 R_1 \sin^2 \beta_2}{\left(1 - R_1 e^{-2i\beta_2} \right)^2} + 1 + 2 \cdot \frac{4 R_1 \sin^2 \beta_2}{\left| 1 - R_1 e^{-2i\beta_2} \right|^2} \cos 2(\beta_1 - \delta) \right\} \]

\[ -\delta \equiv \arg(e^{i\beta_2} - R_1 e^{-i\beta_2}) \]

The above equation is calculated using the mirror's reflectance, refractive index \( R_m \) and \( T_m \).

Rewriting,

\[ T = \frac{T_m^2}{1 + R_m^2 + 2R_m \cos 2(\beta_1 - \delta)} = \left( \frac{T_m}{1 - R_m} \right)^2 \cdot \frac{1}{1 + \frac{4R_m}{(1 - R_m)^2} \cos^2(\beta_1 - \delta)} \]

Using the definition of \( 1-R_m = T_m \) and Finesse here,

\[ T = \frac{1}{1 + \frac{4 F^2}{\pi^2} \cos^2(\beta_1 - \delta)} \]

\[ \therefore F \equiv \frac{\pi \sqrt{R_m}}{1 - R_m} \]
Since $\beta_1 = kD$, when the Cavity interval $D$ is changed, $T$ obeys following equation with integer of $m$.

$$D = \frac{1}{k} \left\{ \left( m + \frac{1}{2} \right) \pi + \delta \right\} = \frac{\lambda}{2} \left\{ \left( m + \frac{1}{2} \right) + \frac{\delta}{\pi} \right\}$$

The maximum value is 1 at this time.

$$D = \frac{1}{k} \{ m\pi + \delta \} = \frac{\lambda}{2} \left( m + \frac{\delta}{\pi} \right)$$

Minimum at this time.

$$\left( \frac{1 - R_m}{1 + R_m} \right)^2$$

The cycle of this variation is $\lambda / 2$. This $T$ is called Airy function, if this peak is sharp enough, finesse $F$ is the distance between the peaks (free spectral range: FSR) $\lambda / 2 = \text{ratio of } \Delta D \text{ to peak FWHM} = \delta D$, $F = \Delta D / \delta D$ become.

**Derivation of enhancement factor**

Consider a rightward x-axis with the origin on the right side of layer 2 as the coordinate origin. Find the light intensity in the mirror.
\[
\begin{pmatrix}
E'_{l3} \\
E'_{r3}
\end{pmatrix} = H_{34} L_4 H_{45} \begin{pmatrix} 0 \\ E_{r5} \end{pmatrix} = \frac{1}{\tau_m} \begin{pmatrix} e^{2i\delta} & \rho_m \\ -\rho_m & 1 \end{pmatrix} \begin{pmatrix} 0 \\ E_{r5} \end{pmatrix} = \frac{1}{\tau_m} \begin{pmatrix} \rho_m E_{r5} \\ E_{r5} \end{pmatrix}
\]

\[
T_m = \left| \tau_m \right|^2, R_m = \left| \rho_m \right|^2
\]

\[
\begin{pmatrix}
E'_{l1} \\
E'_{r1}
\end{pmatrix} = S \begin{pmatrix} 0 \\ E_{r5} \end{pmatrix} = \begin{pmatrix} S_{12} E_{r5} \\ S_{22} E_{r5} \end{pmatrix}
\]

Then, \( E_{r5} = \frac{1}{S_{22}} E'_{r1} \)

So,

\[
\begin{pmatrix}
E'_{l3} \\
E'_{r3}
\end{pmatrix} = \frac{1}{\tau_m S_{22}} \begin{pmatrix} \rho_m E'_{r1} \\ E'_{r1} \end{pmatrix}
\]

Now, the electric field at position \( x \) is,

\[
\begin{pmatrix}
E^x_{l} \\
E^x_{r}
\end{pmatrix} = \begin{pmatrix} e^{-i\beta_x} & 0 \\ 0 & e^{i\beta_x} \end{pmatrix} \begin{pmatrix} E'_{l3} \\ E'_{r3} \end{pmatrix}, \beta_x \equiv k(D - x)
\]
\[
\left( \begin{array}{c}
E_l^x \\
E_r^x
\end{array} \right) = \left( \begin{array}{cc}
e^{-i\beta_x} & 0 \\
0 & e^{i\beta_x}
\end{array} \right) \frac{1}{\tau_m S_{22}} \left( \begin{array}{c}
\rho_m E'_{r1} \\
E'_{r1}
\end{array} \right)
\]

Then,

\[
E^x \equiv E_l^x + E_r^x = \frac{E'_{r1}}{\tau_m S_{22}} (\rho_m e^{-i\beta_x} + e^{i\beta_x})
\]

\[
\frac{E^x}{E'_{r1}} = \frac{\tau}{\tau_m} (\rho_m e^{-i\beta_x} + e^{i\beta_x}), \therefore \frac{1}{S_{22}} = \tau
\]

Therefore, the required amplification factor \( P \) is

\[
P \equiv \left| \frac{E^x}{E'_{r1}} \right|^2 = \frac{\left| \tau \right|^2}{\left| \tau_m \right|^2} \left| \rho_m e^{-i\beta_x} + e^{i\beta_x} \right|^2
\]

\[
\left| \rho_m e^{-i\beta_x} + e^{i\beta_x} \right|^2 = 1 + R_m + 2 \sqrt{R_m} \sin(2\beta_x - \delta), \therefore \rho_m = \sqrt{R_m} e^{i\delta}
\]
Then,

\[ P = \frac{T}{T_m} \{1 + R_m - 2\sqrt{R_m} \sin(2kx - 2\beta_1 + \delta)\} \]

Therefore, \( P \) changes sinusoidally with a period \( \lambda / 2 \) with respect to \( x \). The average value is

\[ \frac{1 + R_m}{1 - R_m} T \]

and the Cavity transmittance \( T \) is the maximum value of near 1.

Maximum value at that time is

\[ \frac{2}{1 - R_m} \approx \frac{2}{3} F \quad (R_m \approx 1). \]
Actually there is a power loss, so when I make a Cavity with two mirrors, as Effective Reflectance is $R_{\text{eff}} \equiv r_2 r_1 x$, Transmitted light in On Resonance is shown by below equation. The formula of the power of the reflected light and the accumulated light is shown below. Derivation of these expressions should do when you have time. The transmission coefficient, reflection coefficient and power loss coefficient of the mirror 1 are represented by $r_1, t_1, x$ and the transmission coefficient, reflection coefficient, and power loss coefficient of the mirror 2 be $r_2, t_2, x$.

\[
\begin{align*}
R_1 &= |r_1|^2, \quad R_2 = |r_2|^2, \quad T_1 = |t_1|^2, \quad T_2 = |t_2|^2, \quad X = |x|^2 \\
T_{\text{cav}} &= \frac{T_1 T_2 X}{(1 - R_{\text{eff}})^2} \\
R_{\text{cav}} &= \left( R_1 + T_1 \right) - \frac{T_1 R_{\text{eff}} \left[ \frac{1}{R_{\text{eff}}} - R_{\text{eff}} - T_1 \frac{R_{\text{eff}}}{R_1} \right]}{A \left( R_{\text{eff}} \right)} \\
S_{\text{cav}} &= \frac{T_1 \sqrt{X} \left( 1 + R_2 X \right)}{(1 - R_{\text{eff}})^2} \\
A \left( R \right) &= (1 - R)^2 + 4 R \sin^2 \frac{\theta}{2}, \quad (\theta = 0) \\
\text{Finesse} : F \left( R_{\text{eff}} \right) &= \frac{2 \pi}{2 \theta_1 / 2} = \frac{\pi \sqrt{R}}{1 - R}
\end{align*}
\]
The following is almost realized and operated.

\[ R_{\text{eff}} = 99.87\%, X = 99.97\%, T = 0.07\%, R = 99.9\% \]

\[ r_1 = r_2 = 0.9995, x = 0.99985, t_1 = t_2 = 0.026 \]

\[ T_{\text{cav}} = 0.29, R_{\text{cav}} = 0.213, S_{\text{cav}} = 828, F = 3140 \]

The ultimate goal is to develop the following Super Cavity within 5 years.

\[ R_{\text{eff}} = 99.9987\%, X = 99.9997\%, \]
\[ T = 0.0007\%, R = 99.999\% \]

\[ r_1 = r_2 = 0.999995, x = 0.9999985, \]
\[ t_1 = t_2 = 0.002646 \]

\[ T_{\text{cav}} = 0.29, R_{\text{cav}} = 0.213, S_{\text{cav}} = 82839, F = 241659 \]

Assuming the above amplification factor, head-on collision on Laser Compton scattering

The generated X-ray dose is estimated below;
4. Experiments at KEK ATF using 4-mirror optical cavity

**Optical cavity**

**ATF parameter**
1.3 GeV
1 $\times 10^{10}$ electron/bunch
Up to 10 bunch/train
2.16 $\times 10^6$ turns/s
4 mirror 3D cavities were at the ATF

KEK-Hiroshima
installed 2011.

LAL-Orsay-KEK
installed summer 2010.
\(\gamma\)-ray Generation

5 bunches/train

5.6ns

5 bunch/train (7.7mA)

\[2970 \pm 20\text{ MeV}\]

\[\Rightarrow \sim 120\gamma \text{s} \text{ / train}\]

ATF 2.16MHz

\[\sim 2.6 \times 10^8 /\text{sec}\]

Gamma energy [GeV]

Gamma yield [A.U.]
New feedback control using polarization resonance characteristics.

Different slope in left and right pol.
Laser power = 2.6kW
Timing jitter = 8ps
Enhancement 1230
due to mirror contamination and injection coupling.

1.4% fluctuation ➔ 16pm control
5. Results at STF and cERL

2D four-mirror cavity to generate X-ray with two cylindrical lenses. STF project

Development for Next Generation Compact High Brightness X-ray Source using Super Conducting RF Acceleration Technique

Quantum Beam Technology Program: Beam commissioning started from mid. of February 2012 to March 2013.
2D four mirror cavity to generate X-ray with two cylindrical lenses.
Change to head collision scheme to get another enhancement of 5 and to increase laser pulse duration ~20ps.

Quantum project at STF

Beam size 10μm
Achieved beam at STF for QBT project

1ms beam from L-band RF Gun

Min. emittance so far

Min. beam size @ WM-PRM-05

Min. beam size @ WM-PRM-07

Measurement of beam emittance by wire scanner 06.13.2012

Dispersion measurement

@ WM-PRM -05, -07, dispersion measurements

SC phase 240deg 40.58 MeV/c (B1 150.20A)

SC phase 250deg 40.02MeV/c (B1 148.10A)

Min. beam size so far

WM-PRM-05

WM-PRM-07

Laser beam size : about 80μm

Target : less than 20μm

Background level is acceptable.
Detected Signal by MCP (22\textsuperscript{nd} Mar. 2013)

Raw X-ray spectrum detected by SOI detector

Blue line: laser on
Red line: laser off

Detected flux $10^6$ photons/sec of 28keV X-ray.

Success of 28keV X-ray detection
Two sets of 2D four mirror optical cavity to generate X-ray. cERL project

Electron beam
Energy: 20 MeV
CW beam with Energy Recovery
Repetition: 162.5 MHz
~ 100 μA

(Plan) → 1 mA (2015) → 10 mA (2016)
Beam Optics for the LCS

- Low-beta insertion for small beam sizes at IP
- Transport beams to the dump with small beam losses

Beam optics was established

Design optics (example: “70% middle” optics)

\[ \sigma_x^* = 21 \, \mu m, \quad \sigma_y^* = 33 \, \mu m \text{ at IP} \]

Beam sizes at IP were estimated from Q-scan data

\[ \sigma_x^* \sim 13 \, \mu m, \quad \sigma_y^* \sim 25 \, \mu m \text{ (example)} \]

Bunch charge: 0.5 pC/bunch,
Normalized emittances: \((\epsilon_{nx}, \epsilon_{ny}) = (0.47, 0.39) \text{ mm-mrad}\)
Electron: 20 MeV, $\sigma_{x,y} \approx 130,30\mu m$

Laser: 1064 nm, $\sigma_{x,y} \approx 30\mu m$

Crossing angle: $18^\circ$

$E = 6.91$ keV, $\Delta E = 173$ eV (FWHM)

Ave. beam intensity: 57.7 uA

Laser power in cavity: 10kW

1200 cps by $\phi 4.66$ mm (SDD) (expected 3000 cps)

$\rightarrow$ Total photon flux

$4.3 \times 10^7 /\text{sec} @\text{IP}$
Evaluation of present LCS performance

SR X-ray (PF-AR)
30 msec exposure for angiography

LCS X-ray (LUCX)
10 sec exposure at $10^7$ photons/sec
still improving...

LCS X-ray (cERL)
100 sec exposure at $10^7$ photons/sec

Assume the angiography by LCS X-ray, need **100** improvement

- intense electron beam
- intense laser pulse
- smaller beam size
- increase repetition rate
- ...
- small crossing angle
- higher e- beam energy
- ...

integrate challenges!
6. Results at LUCX

Four mirror 2D optical cavity to generate X-ray. LUCX Project

- To downsize the accelerator, we have installed a 3.6cell rf-gun and a 12cell booster.
  - 3.6cell rf-gun
    - Beam test has been started from Jan 2012.
  - 12cell booster
    - This booster was installed in last June.

Microwave resonator cavity for soft X-ray generation

New optical cavity for hard X-ray generation

3m accelerating tube

1.6cell Rf-gun

12cell booster

3.6cell RF-gun
Development of multi-bunch e\textsuperscript{-} beam at LUCX

2012: 150 bunches, 90 nC
2013: 300 bunches, 380 nC
2014: 1000 bunches, 600 nC, 24 MeV

Energy compensation by RF amplitude modulation

LUCX beam target:
30 MeV, 357 MHz
bunch spacing 2.8 ns
more than 1000 bunches/pulse,
beam size $\sim 100$ um

Electron beam stability: 0.2\%(in $\sigma$)

Number of bunches (1000)

Electron Bunch Charge

3.6cell RF-gun
LCS Collision point
RF cavity
X-ray detector
X-ray

LUCX beam target:
30 MeV, 357 MHz
bunch spacing 2.8 ns
more than 1000 bunches/pulse,
beam size $\sim 100$ um

Electron beam stability: 0.2\%(in $\sigma$)

Number of bunches (1000)

Electron Bunch Charge
Burst mode optical enhancement cavity

Use PDH method

Main laser (CW)

CCW circulate

Piezo
cw trans.
ccw trans.

Split
2

1000 times higher peak power can be achieved on electron beam timing, which stabilized by reverse path in the optical cavity.

AFAD 2015 26-27Jan2015@NSRRC, Taiwan

K. Sakaue (Waseda univ.)
We installed the Pockels cell in order to collect the amplification power at the electron beam timing.
Burst Storage with PC

Histogram of peak power stored in the optical cavity

Power jitter is also small, about 5% in rms

1MW peak power storage was successfully achieved, but the cavity mirror was broken.
Laser power stability in the optical cavity

Stability of stored pulse intensity: 15% rms
About 0.2nm accuracy of cavity length adjustment was not achieved two years ago.

Stability of stored pulse intensity: 3.5% rms
About 0.2nm accuracy of cavity length adjustment is achieved now.

Optical cavity is not stable due to the vibration and laser injection optics drift.
## Summary of Laser-Electron Beam Parameters at the photon-electron collision point

### LASER

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wavelength</td>
<td>1064nm</td>
</tr>
<tr>
<td>Repetition</td>
<td>357MHz</td>
</tr>
<tr>
<td>Power</td>
<td>1MW</td>
</tr>
<tr>
<td>Pulse energy</td>
<td>2.8mJ</td>
</tr>
<tr>
<td>Size (H)</td>
<td>60μm</td>
</tr>
<tr>
<td>Size (V)</td>
<td>25μm</td>
</tr>
<tr>
<td>Pulse duration</td>
<td>7ps</td>
</tr>
<tr>
<td>Col. angle</td>
<td>7.5°</td>
</tr>
</tbody>
</table>

### ELECTRON

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy</td>
<td>22MeV</td>
</tr>
<tr>
<td>Repetition</td>
<td>357MHz</td>
</tr>
<tr>
<td>Charge</td>
<td>0.6nC</td>
</tr>
<tr>
<td>N. bunch</td>
<td>1000</td>
</tr>
<tr>
<td>Size (H)</td>
<td>40μm</td>
</tr>
<tr>
<td>Size (V)</td>
<td>70μm</td>
</tr>
<tr>
<td>Bunch length</td>
<td>15ps</td>
</tr>
<tr>
<td>Emi (H)</td>
<td>5πmmrad</td>
</tr>
<tr>
<td>Emi (V)</td>
<td>6πmmrad</td>
</tr>
</tbody>
</table>

### Laser pulse Energy in the Optical cavity

### Total charge per train

9keV LCS X-ray Energy
We found the present multi-pixel detector was saturated. This problem will be solved using SOI detector because of the pixel size of 17µm soon.

Measured X-ray intensity distribution and the comparison with CAIN simulation

CAIN Simulation assumed following measured values.
Number of X-ray: $3.6 \times 10^7$ Photons/sec
X-ray Energy: 9keV
Source size: 60um x 25um

X-ray energy distribution at the HyPix-3000

- Energy band width (FWHM): ~4%
- X-ray flux at the HyPix-3000: 11.83 photons/sec/pixel
- This pixel size is 100um x 100um.

Calculated by CAIN
To get clear X-ray imaging every second, we have to increase the X-ray flux by factor $\sim 10$.

**High brightness X-ray facility based on LCS**

Normal conducting accelerator system for compact high brightness X-ray under design

Thank you for your attention!

Downsizing to 6m x 8m by new technologies