

# Thermodynamics of hot and magnetized deconfined QCD matter in one-loop HTL approximation

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26 November, 2019

CETHENP 2019, VECC

# Outline

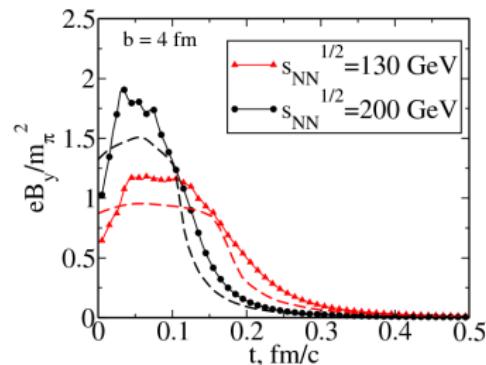
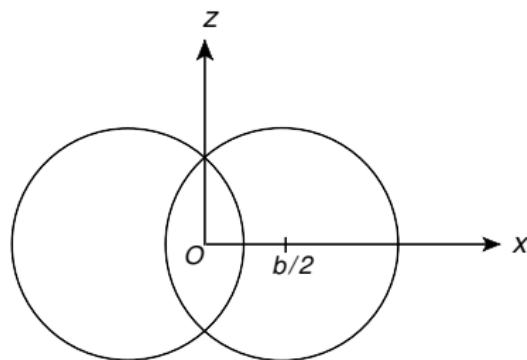
1 Motivation

2 Formalism

3 Results

4 Summary

# Magnetic field in non-central HIC



arXiv:0907.1396

- Magnetic field strength decreases with time .
- In a direction perpendicular to the reaction plane.

# Fermion propagator in magnetic field

- Weak magnetic field limit  $eB < m_{th}^2 < T^2$ .
- In this limit the sum over Landau levels in Schwinger propagator can be rearranged as sum over the magnetic field strength.
- Strong magnetic field limit  $m_{th}^2 < T^2 < eB$ .
- Only lowest Landau level contributes.

# General structure of gluon self energy in presence of magnetic field

- ① Four velocity  $u_\mu$  of heat bath is introduced because of presence of medium. We choose rest frame of heat bath  $u_\mu = (1, 0, 0, 0)$ .
- ② We consider magnetic field along z direction  $n_\mu = (0, 0, 0, 1)$ .
- ③ Rotational symmetry is broken due to the presence of magnetic field.
- ④ Constituent tensors constructed out of  $\eta_{\mu\nu}, P_\mu P_\nu, u_\mu u_\nu, n_\mu n_\nu, P_\mu u_\nu + u_\mu P_\nu, P_\mu n_\nu + n_\mu P_\nu, u_\mu n_\nu + n_\mu u_\nu$ .
- ⑤ Photon self energy  $\Pi^{\mu\nu} = bB^{\mu\nu} + cR^{\mu\nu} + dQ^{\mu\nu} + aN^{\mu\nu}$ .
- ⑥  $b = B^{\mu\nu}\Pi_{\mu\nu}, c = R^{\mu\nu}\Pi_{\mu\nu}, d = Q^{\mu\nu}\Pi_{\mu\nu}, a = \frac{1}{2}N^{\mu\nu}\Pi_{\mu\nu}$ .

# General structure of gluon effective propagator

## Effective gluon propagator

$$\begin{aligned}\mathcal{D}_{\mu\nu} = & \frac{\xi P_\mu P_\nu}{P^4} + \frac{(P^2 - d)B_{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2} + \frac{R_{\mu\nu}}{P^2 - c} \\ & + \frac{(P^2 - b)Q_{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2} + \frac{aN_{\mu\nu}}{(P^2 - b)(P^2 - d) - a^2}\end{aligned}$$

- Form factors are calculated upto  $\mathcal{O}[(eB)^2]$  in weak magnetic field approximation (EPJC 79 (2019) no.8, 658).
- Leads to three dispersive modes of gluon.

# General structure of quark propagator in presence of magnetic field

Fermion self energy

$$\Sigma(K) = -a \not{K} - b \not{\psi} - b' \gamma_5 \not{\psi} - c' \gamma_5 \not{\psi}.$$

Effective fermion propagator

$$S^*(K) = P_- \frac{\not{L}(K)}{L^2} P_+ + P_+ \frac{\not{R}(K)}{R^2} P_-$$

$$\begin{aligned} L^\mu &= (1+a)K^\mu + (b+b')u^\mu + c'n^\mu, \\ R^\mu &= (1+a)K^\mu + (b-b')u^\mu - c'n^\mu. \end{aligned}$$

- Form factors in weak and strong magnetic field are calculated in PRD 97, 034024 (2018) and PRD 99, 094002 (2019).

# Pressure of QGP in magnetized medium

- Equation of state is needed for hydrodynamic evolution of the system.
- Free energy is calculated using the effective propagator of quarks and gluons in magnetized medium.

## Free energy of quarks

$$F_q = -N_c \sum_f \sum_{\{P\}} \ln (\det[S_{eff}^{-1}])$$

- where  $S_{eff}^{-1} = P - \Sigma$ .

## Free energy of gluons

$$F_g = (N_c^2 - 1) \left[ \frac{1}{2} \sum_P \ln [\det(D_{\mu\nu,eff}^{-1})] - \sum_P \ln(-P^2) \right]$$

- For hard quasiparticles ( $P \sim T$ ) we make a high temperature expansion of the ‘Log’ term.
- We keep terms upto  $\mathcal{O}[g^4]$ .
- For soft gluons ( $P \sim gT$ ), we put  $p_0 = 0$  because this is the lowest Matsubara mode.
- $g^3$  term appears from soft part.

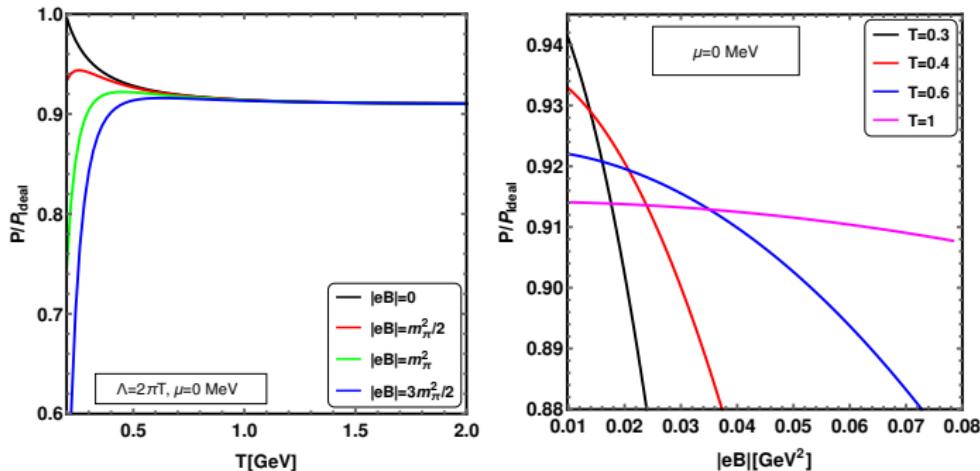
## Weak magnetic field

- Total pressure of the weakly magnetized quark-gluon plasma is given by  $P = -(F_q + F_g)$ .
- We can consider the pressure as **isotropic** because the magnetic field strength is very small.
- We calculate the ratio of total QGP pressure in presence of weak magnetic field to that of free quarks and gluons.
- Pressure of free quarks and gluons is given by

$$P_{\text{Ideal}}(T) = N_c N_f \frac{7\pi^2 T^4}{180} + (N_c^2 - 1) \frac{\pi^2 T^4}{45}$$

# Weak magnetic field

PRD 100, 034031 (2019)



**Figure:** Variation of the scaled one-loop pressure with temperature and magnetic field for  $N_f = 3$ .

# Anisotropic pressure in presence of strong magnetic field

- $F = \epsilon - Ts - eB \cdot M.$
- Longitudinal pressure  $P_z = -F.$
- Transverse pressure  $P_{\perp} = -F - eB \cdot M = P_z - eB \cdot M.$
- Magnetization  $M = -\frac{\partial F}{\partial(eB)}.$
- Longitudinal pressure of free quarks and gluons in presence of strong magnetic field

$$P_z^i = N_c N_F \sum_f (q_f B) \frac{T^2}{12} + (N_c^2 - 1) \frac{\pi^2 T^4}{45}$$

- Transverse pressure

$$P_{\perp}^i = (N_c^2 - 1) \frac{\pi^2 T^4}{45}$$

# Strong magnetic field

- Longitudinal pressure

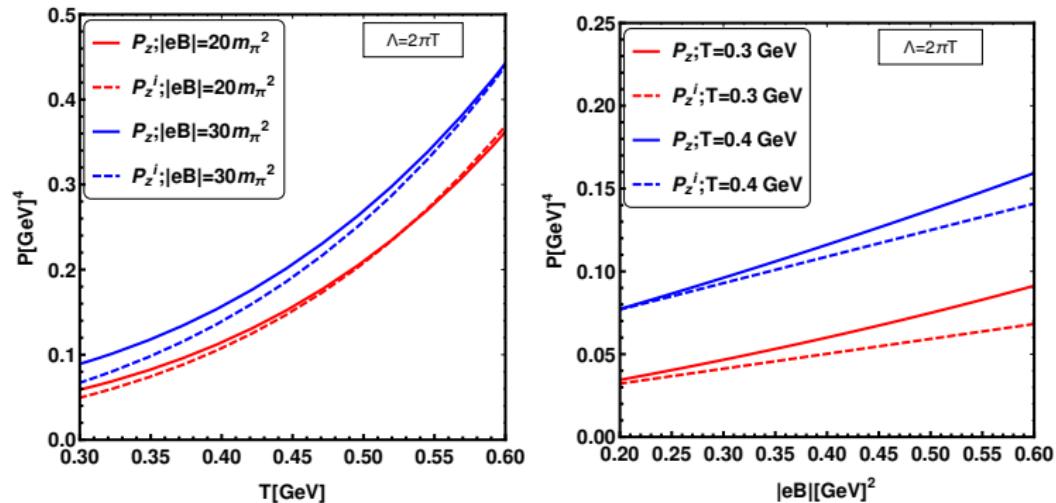


Figure: Variation of the one-loop longitudinal pressure with temperature and magnetic field for  $N_f = 3$ .

# Strong magnetic field

- Transverse pressure

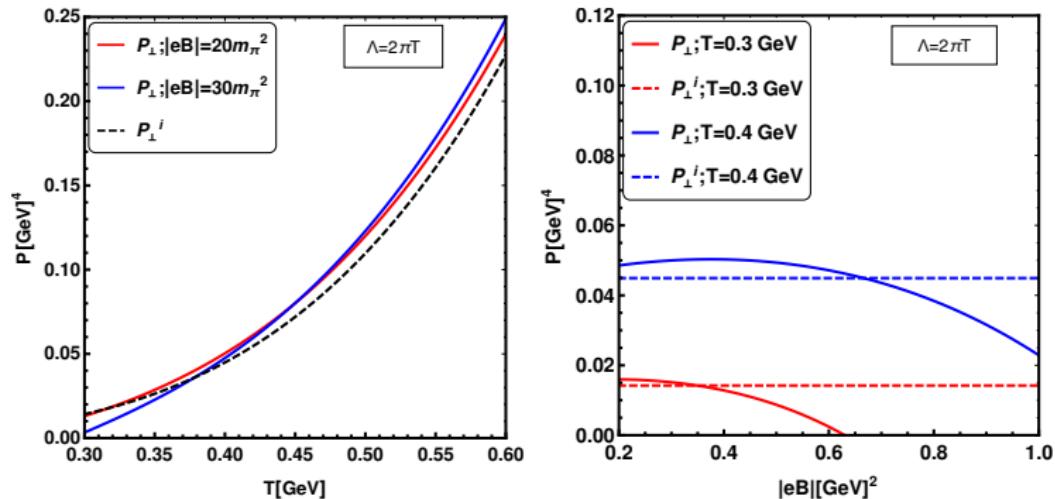
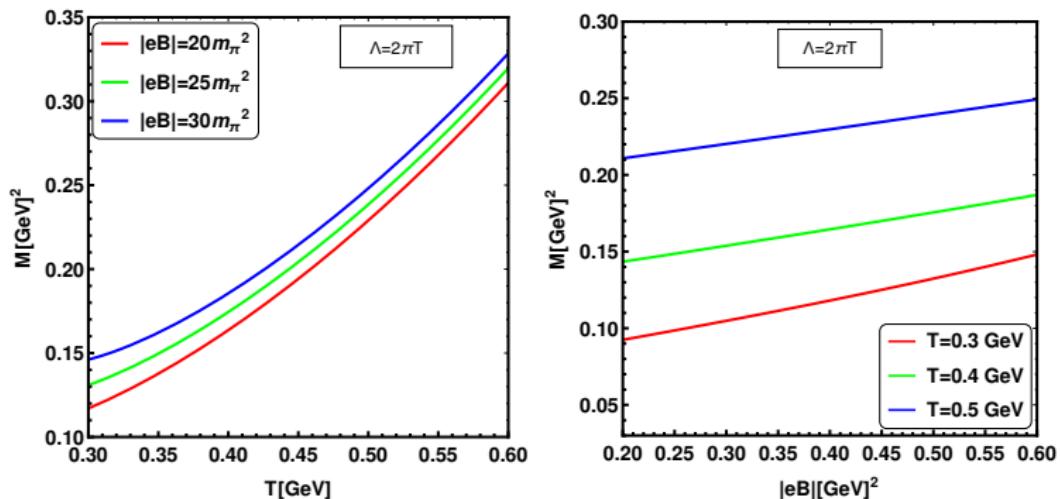


Figure: Variation of the one-loop transverse pressure with temperature and magnetic field for  $N_f = 3$ .

# Magnetization

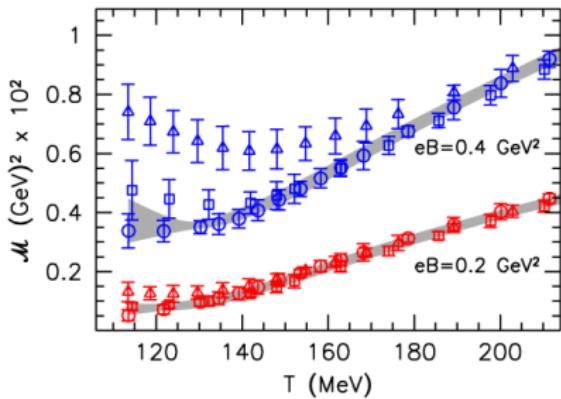
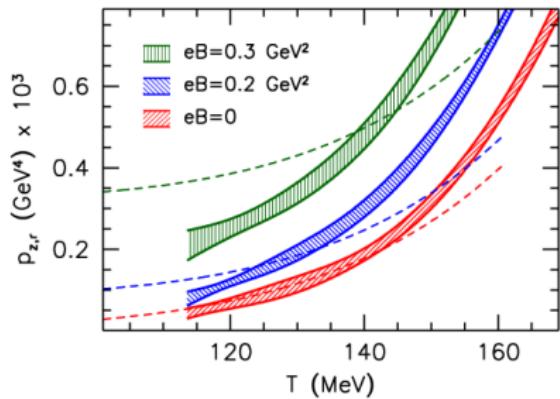
PRD 99, 094002 (2019)



**Figure:** Variation of the magnetization with temperature and magnetic field for  $N_f = 3$ .

# Lattice results

JHEP 1408 (2014) 177



## Summary

- ① There is negligible change in pressure of hot and dense QGP in presence of weak magnetic field.
- ② This pressure can be considered as isotropic.
- ③ We calculated anisotropic pressure in presence of strong magnetic field.
- ④ Longitudinal pressure is greater than the transverse pressure.
- ⑤ The magnetized medium shows paramagnetic nature.
- ⑥ Results can be improved by performing higher loop order calculations.

# Thank You

Gluon thermal mass

$$m_{th}^2 = \frac{1}{6}g^2T^2(C_A + \frac{1}{2}N_f)$$

Fermion thermal mass

$$m^2 = \frac{1}{8}g^2T^2C_F$$

For gluons

$$e^2T^2 \rightarrow g^2T^2(C_A + \frac{1}{2}N_f)$$

For quarks

$$e^2T^2 \rightarrow g^2T^2C_F$$

Debye mass correction for weak field limit

$$\delta m_D^2 = \frac{e^2}{6\pi^2 T^2} (eB)^2 \sum_{l=1}^{\infty} (-1)^{l+1} l^2 K_0 \left( \frac{m_f l}{T} \right)$$

$$b_0(p_0, p) = \Pi_L = \frac{m_D^2}{\bar{u}^2} \left( 1 - \frac{p_0}{2p} \log \frac{p_0 + p}{p_0 - p} \right)$$

$$c_0(p_0, p) = d_0(p_0, p) = \frac{1}{2} \Pi_T = \frac{m_D^2}{2p^2} \left[ p_0^2 - (p_0^2 - p^2) \frac{p_0}{2p} \log \frac{p_0 + p}{p_0 - p} \right]$$

$$\begin{aligned}
b_2 &= \frac{\delta m_D^2}{\bar{u}^2} + \frac{2(e^2 B)^2}{\bar{u}^2 \pi^2} \left[ \left( g_k + \frac{\pi m_f - 4T}{32m_f^2 T} \right) (A_0 - A_2) \right. \\
&\quad \left. + \left( f_k + \frac{8T - \pi m_f}{128m_f^2 T} \right) \left( \frac{5A_0}{3} - A_2 \right) \right]
\end{aligned}$$

$$\begin{aligned}
c_2 &= -\frac{8(e^2 B)^2}{3\pi^2} g_k + \frac{(e^2 B)^2}{\pi^2} \left( g_k + \frac{\pi m_f - 4T}{32m_f^2 T} \right) \times \\
&\quad \left[ -\frac{7}{3} \frac{p_0^2}{p_1^2} + \left( 2 + \frac{3}{2} \frac{p_0^2}{p_1^2} \right) A_0 + \left( \frac{3}{2} + \frac{5}{2} \frac{p_0^2}{p_1^2} + \frac{3}{2} \frac{p_3^2}{p_1^2} \right) A_2 \right. \\
&\quad \left. - \frac{3p_0 p_3}{p_1^2} A_1 - \frac{5}{2} \left( 1 - \frac{p_3^2}{p_1^2} \right) A_4 - \frac{5p_0 p_3}{p_1^2} A_3 \right]
\end{aligned}$$

$$d_2 = F_1 + F_2$$

where

$$\begin{aligned} F_1 &= -\frac{2(e^2B)^2p^2}{\pi^2p_\perp^2} \left[ -g_k \left\{ -\frac{p_0^2p_3^2}{3p^4} - \frac{A_0}{4} + \left( \frac{3}{2} + \frac{p_0^2p_3^2}{p^4} \right) A_2 - \frac{5}{4}A_4 \right\} \right. \\ &\quad + \left( \frac{\pi}{32m_fT} - \frac{1}{8m_f^2} \right) \left\{ \frac{A_0}{4} - \left( \frac{3}{2} + \frac{p_0^2p_3^2}{p^4} \right) A_2 + \frac{5}{4}A_4 \right\} \\ &\quad \left. - f_k \frac{p_0^2p_3^2}{p^4} \left( \frac{14}{3} - 5A_0 + A_2 \right) + \frac{p_0^2p_3^2}{p^4} \frac{8T - \pi m_f}{128Tm_f^2} (5A_0 - A_2) \right] \end{aligned}$$

$$F_2 = -\frac{(e^2B)^2}{3\pi^2m_fT} \frac{p^0p^3}{p_\perp^2} \frac{1}{1 + \cosh \frac{m_f}{T}} \left( \frac{3A_1}{2} - A_3 \right)$$

$$A_n = \int \frac{d\Omega}{4\pi} \frac{p_0 c^n}{P \cdot \hat{K}}$$

$$\begin{aligned}
F_q^r &= N_c N_f \left[ -\frac{7\pi^2 T^4}{180} \left( 1 + \frac{120\hat{\mu}^2}{7} + \frac{240\hat{\mu}^4}{7} \right) + \frac{g^2 C_F T^4}{48} (1 + 4\hat{\mu}^2) (1 + 12\hat{\mu}^2) \right. \\
&\quad + \frac{g^4 C_F^2 T^4}{768\pi^2} (1 + 4\hat{\mu}^2)^2 (\pi^2 - 6) + \frac{g^4 C_F^2}{27N_f} M_B^4 \left( 12 \ln \frac{\hat{\Lambda}}{2} - 6\aleph(z) + \frac{36\zeta(3)}{\pi^2} \right. \\
&\quad \left. \left. - 2 - \frac{72}{\pi^2} \right) \right].
\end{aligned}$$