Thermodynamics of strongly interacting matter in background magnetic field

Nilanjan Chaudhuri

Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700064, India

CETHENP

November 25-27, 2019

Outline

Introduction

Formalism

- NJL model at finite temperature and density
- Effective potential in presence of magnetic field
- Inclusion of AMM of quarks

Results

Summary

Outline

Introduction

Formalism

NJL model at finite temperature and density

Effective potential in presence of magnetic field

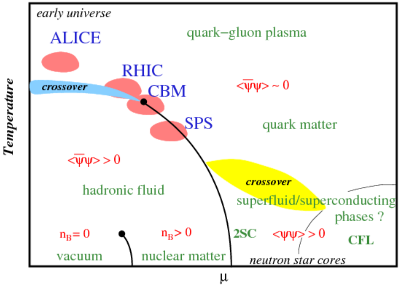
Inclusion of AMM of quarks

Results

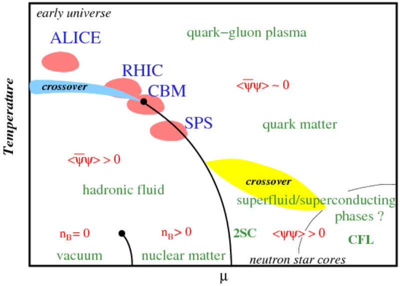
Summary

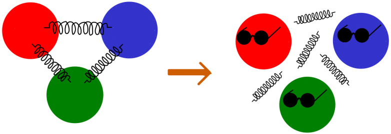
Introduction

Phases of QCD Matter



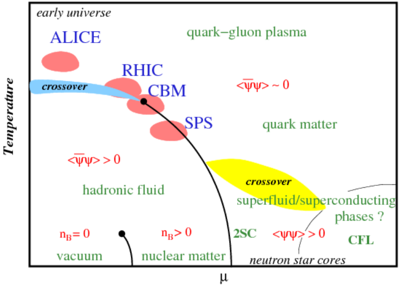
Phases of QCD Matter

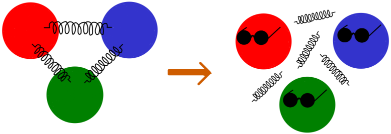




Introduction

Phases of QCD Matter





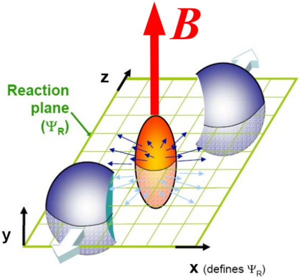
- Hadronic phase :
  Quarks and gluons are bound into hadrons. We can observe it experimentally.
- QGP phase :
  Quarks and gluons are free in the medium. Only indirect observation possible.

Introduction

Magnetic field

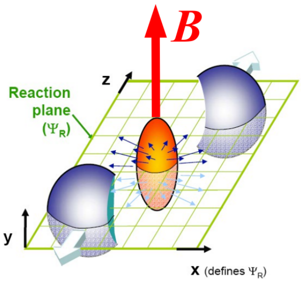
Introduction

Magnetic field



Introduction

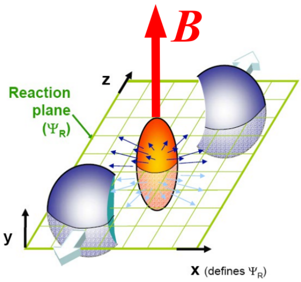
Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.

Introduction

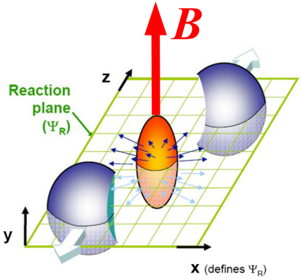
Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.

Introduction

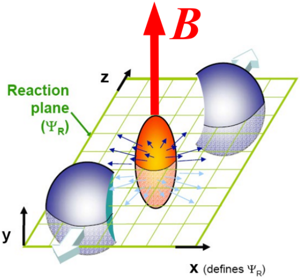
Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

Introduction

Magnetic field

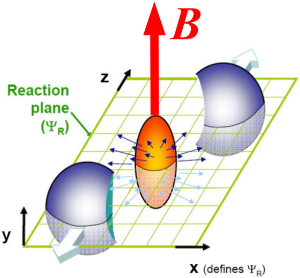


- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

LHC: $eB \approx 15\ m_\pi^2$ **RHIC:** $eB \approx 5\ m_\pi^2$

Introduction

Magnetic field



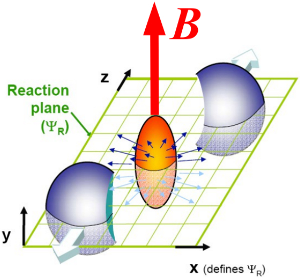
- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

LHC: $eB \approx 15\ m_\pi^2$ **RHIC:** $eB \approx 5\ m_\pi^2$

Strong magnetic field in other physical systems:

Introduction

Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

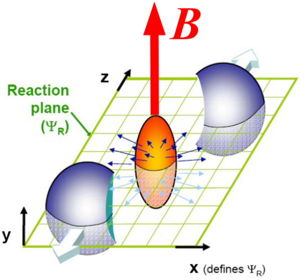
LHC: $eB \approx 15\ m_\pi^2$ **RHIC**: $eB \approx 5\ m_\pi^2$

Strong magnetic field in other physical systems:

Neutron star: $eB \approx 10^{-7}\ m_\pi^2$

Introduction

Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

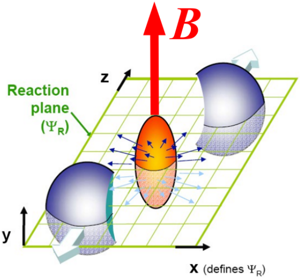
LHC: $eB \approx 15\ m_\pi^2$ **RHIC**: $eB \approx 5\ m_\pi^2$

Strong magnetic field in other physical systems:

Neutron star: $eB \approx 10^{-7}\ m_\pi^2$ **Magnetar**: $eB \approx 10^{-6}$-$10^{-3}\ m_\pi^2$

Introduction

Magnetic field



- Noncentral heavy-ion collisions should produce magnetic field.
- The magnetic field is directed out of the collision plane.
- The presence of a conducting medium with substantially delays the decay of B_y. [**Kharzeev** PRC 89 054905, **Tuchin** PRC 93 014905]

LHC: $eB \approx 15\ m_\pi^2$ **RHIC**: $eB \approx 5\ m_\pi^2$

Strong magnetic field in other physical systems:

Neutron star: $eB \approx 10^{-7}\ m_\pi^2$ **Magnetar**: $eB \approx 10^{-6}\text{-}10^{-3}\ m_\pi^2$

Conversion unit : $m_\pi^2 \approx 0.02\ \text{GeV}^2$ and $1\ \text{GeV}^2 \approx 10^{15}$ Tesla

Introduction

Introduction

Life is tough with QCD!!

Introduction

Life is tough with QCD!!

- **pQCD**

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.
  - Sign problem restricts the use at high chemical potential.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.
  - Sign problem restricts the use at high chemical potential.
- **Effective models**

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.
  - Sign problem restricts the use at high chemical potential.
- **Effective models**
  - QCD interactions are replaced by effective interactions.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.
  - Sign problem restricts the use at high chemical potential.
- **Effective models**
  - QCD interactions are replaced by effective interactions.
  - Works only in a special domain of energy.

Introduction

Life is tough with QCD!!

- **pQCD**
  - Need a small coupling constant, works at high energy / high T, μ.
  - Not justified in regions close to the phase boundary and below.
- **LQCD**
  - Non-perturbative study.
  - Sign problem restricts the use at high chemical potential.
- **Effective models**
  - QCD interactions are replaced by effective interactions.
  - Works only in a special domain of energy.
  - Allows finite chemical potential studies.

Introduction

NJL Model

Introduction

NJL Model

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\psi\bar{(}x)\psi(x)\right)^2 + \left(\psi\bar{(}x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**

Introduction

NJL Model

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**
  - It shares the global symmetries of the QCD action, most essentially the chiral symmetry (when $m = 0$)

$$\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\gamma_5\right)\psi$$

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**
  - It shares the global symmetries of the QCD action, most essentially the chiral symmetry (when $m = 0$)

$$\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\gamma_5\right)\psi$$

  - Also symmetric under $\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\right)\psi$.

Introduction

NJL Model

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\psi(\bar{x})\psi(x)\right)^2 + \left(\psi(\bar{x})i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**
  - It shares the global symmetries of the QCD action, most essentially the chiral symmetry (when $m = 0$)

$$\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\gamma_5\right)\psi$$

  - Also symmetric under $\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\right)\psi$.
- **Shortcomigs**

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**
  - It shares the global symmetries of the QCD action, most essentially the chiral symmetry (when $m = 0$)

    $$\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\gamma_5\right)\psi$$

  - Also symmetric under $\psi \longrightarrow \exp\left(-\frac{i}{2}\tau\cdot\theta\right)\psi$.
- **Shortcomigs**
  - Since gluons are frozen, lacks confinement.

The Lagrangian of 2-flavour **Nambu–Jona-Lasinio** model

$$\mathscr{L} = \bar{\psi}(x)\left(i\partial\!\!\!/ - m\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

- **Advantages:**
  - It shares the global symmetries of the QCD action, most essentially the chiral symmetry (when $m = 0$)

$$\psi \longrightarrow \exp\left(-\frac{i}{2}\tau \cdot \theta\gamma_5\right)\psi$$

  - Also symmetric under $\psi \longrightarrow \exp\left(-\frac{i}{2}\tau \cdot \theta\right)\psi$.
- **Shortcomigs**
  - Since gluons are frozen, lacks confinement.
  - NJL is non-renormalizable $\implies$ cannot remove regularization parameter.

Outline

Outline

NJL model at finite temperature and density

NJL model at finite temperature and density

Using mean field approximation i.e. $(\bar{\psi}\psi)^2 \approx 2\langle\bar{\psi}\psi\rangle(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle^2$, the thermodynamic potential become

$$\Omega(T,\mu;M) = \frac{(M-m)^2}{4G} - \frac{2N_cN_f}{\beta}\int\frac{d^3\vec{p}}{(2\pi)^3}\left[\beta E_{\vec{p}} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right)\right]$$

NJL model at finite temperature and density

Using mean field approximation i.e. $(\bar{\psi}\psi)^2 \approx 2\langle\bar{\psi}\psi\rangle(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle^2$, the thermodynamic potential become

$$\Omega(T,\mu;M) = \frac{(M-m)^2}{4G} - \frac{2N_cN_f}{\beta}\int\frac{d^3\vec{p}}{(2\pi)^3}\left[\beta E_{\vec{p}} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right)\right]$$

Now the constituent quark mass M can be obtained self-consistently from the stationarity condition i.e. $\partial\Omega/\partial M = 0$, which implies

$$M = m + 4N_cN_fG\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{M}{E_{\vec{p}}}\left[1-n^+-n^-\right]$$

NJL model at finite temperature and density

Using mean field approximation i.e. $(\bar{\psi}\psi)^2 \approx 2\langle\bar{\psi}\psi\rangle(\bar{\psi}\psi) - \langle\bar{\psi}\psi\rangle^2$, the thermodynamic potential become

$$\Omega(T,\mu;M) = \frac{(M-m)^2}{4G} - \frac{2N_cN_f}{\beta}\int\frac{d^3\vec{p}}{(2\pi)^3}\left[\beta E_{\vec{p}} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right)\right]$$

Now the constituent quark mass M can be obtained self-consistently from the stationarity condition i.e. $\partial\Omega/\partial M = 0$, which implies

$$M = m + 4N_cN_fG\int\frac{d^3\vec{p}}{(2\pi)^3}\frac{M}{E_{\vec{p}}}\left[1-n^+-n^-\right]$$

Note : The medium independent momentum integral is UV divergent and a 3-momentum cut-off parameter, Λ is introduced to regularize the vacuum term.

Outline

Effective potential in presence of magnetic field

Effective potential in presence of magnetic field

- Dispersion relation of quarks in presence of a uniform magnetic field is given by

$$\omega_{nfs}(\vec{p}, n, s) = p_z^2 + M^2 + (2n + 1 - s)\,|q_f|\,B$$

where the magnetic field along z-direction i.e. $\vec{B} = B\hat{z}$.

Effective potential in presence of magnetic field

- Dispersion relation of quarks in presence of a uniform magnetic field is given by

$$\omega_{nfs}(\vec{p}, n, s) = p_z^2 + M^2 + (2n + 1 - s)\,|q_f|\,B$$

where the magnetic field along z-direction i.e. $\vec{B} = B\hat{z}$.

- In presence of a magnetic field one should do the following modification

$$N_f \int \frac{d^3p}{(2\pi)^3} \;\rightarrow\; \sum_{n,f,s} \frac{|q_f|\,B}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

Effective potential in presence of magnetic field

- Dispersion relation of quarks in presence of a uniform magnetic field is given by

$$\omega_{nfs}(\vec{p}, n, s) = p_z^2 + M^2 + (2n + 1 - s)\,|q_f|\,B$$

where the magnetic field along z-direction i.e. $\vec{B} = B\hat{z}$.

- In presence of a magnetic field one should do the following modification

$$N_f \int \frac{d^3p}{(2\pi)^3} \;\rightarrow\; \sum_{n,f,s} \frac{|q_f|\,B}{2\pi} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi}$$

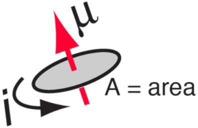
- Thus we get the thermodynamic potential in presence of magnetic field as

$$\Omega(T, \mu; M) = \frac{(M - m)^2}{4G} - \frac{2N_c}{\beta} \sum_f \sum_s \sum_{n=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \frac{|q_f|\,B}{2\pi} \Big[\beta\omega_{nfs}(\vec{p}, n, s) - \ln\left(1 - n_f^+\right) - \ln\left(1 - n_f^-\right)\Big]$$

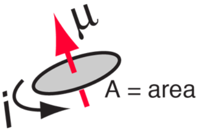
Outline

Magnetic moment

Magnetic moment



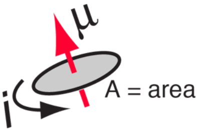
Magnetic moment



Classical

$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

Magnetic moment



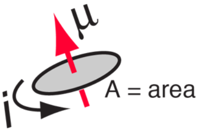
Classical

$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

Quantum mechanical

$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

Magnetic moment



Classical

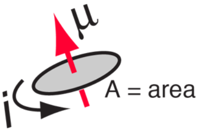
$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

Quantum mechanical

$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

- Dirac equation predicts that any charged fermion must have a magnetic moment.

Magnetic moment



Classical

$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

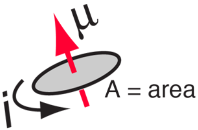
Quantum mechanical

$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

- Dirac equation predicts that any charged fermion must have a magnetic moment.

$$\not{D}^2 = D^2 + \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}$$

Magnetic moment



Classical

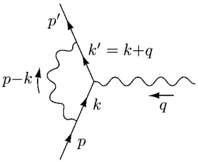
$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

Quantum mechanical

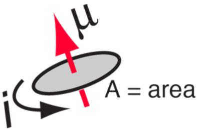
$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

■ Dirac equation predicts that any charged fermion must have a magnetic moment.

$$\not{D}^2 = D^2 + \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}$$



Magnetic moment



Classical

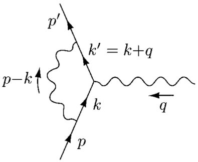
$$\vec{\mu}_L = \frac{e}{2m}\vec{L}$$

Quantum mechanical

$$\vec{\mu}_S = g\frac{e}{2m}\vec{S}$$

- Dirac equation predicts that any charged fermion must have a magnetic moment.

$$\not{D}^2 = D^2 + \frac{e}{2}F_{\mu\nu}\sigma^{\mu\nu}$$



- **correction up to order α^2**

$$\mathbf{g = 2 + \alpha/\pi = 2.00232}$$

AMM of quarks

AMM of quarks

- The spin magnetic moment of a system of quarks in presence of uniform magnetic field along $\hat{z}$

$$\mu_f = \frac{q_f e}{2M_f}(1 + a_f)$$

where a_f is the anomalous part.

AMM of quarks

- The spin magnetic moment of a system of quarks in presence of uniform magnetic field along $\hat{z}$

$$\mu_f = \frac{q_f e}{2M_f}(1 + a_f)$$

where a_f is the anomalous part. From here one can define a positive quantity

$$I_f = \frac{M_f}{1 + a_f} = \frac{\mu_N}{\mu_f} q_f m_p$$

AMM of quarks

- The spin magnetic moment of a system of quarks in presence of uniform magnetic field along $\hat{z}$

$$\mu_f = \frac{q_f e}{2M_f}(1 + a_f)$$

where a_f is the anomalous part. From here one can define a positive quantity

$$I_f = \frac{M_f}{1 + a_f} = \frac{\mu_N}{\mu_f} q_f m_p$$

- From constituent quark model

$$\mu_p = \frac{1}{3}\left(4\mu_u - \mu_d\right) \qquad \mu_n = \frac{1}{3}\left(4\mu_d - \mu_u\right)$$

AMM of quarks

- The spin magnetic moment of a system of quarks in presence of uniform magnetic field along $\hat{z}$

$$\mu_f = \frac{q_f e}{2M_f}(1 + a_f)$$

where a_f is the anomalous part. From here one can define a positive quantity

$$I_f = \frac{M_f}{1 + a_f} = \frac{\mu_N}{\mu_f} q_f m_p$$

- From constituent quark model

$$\mu_p = \frac{1}{3}(4\mu_u - \mu_d) \qquad \mu_n = \frac{1}{3}(4\mu_d - \mu_u)$$

which implies ($\mu_p \approx 2.79\mu_N$ and $\mu_n \approx -1.91\mu_N$)

$$\boldsymbol{\mu_{\mathbf{u}} \approx 1.852\ \mu_{\mathbf{N}}} \qquad \boldsymbol{\mu_{\mathbf{d}} \approx -0.972\ \mu_{\mathbf{N}}}$$

AMM of quarks

AMM of quarks

- What if we consider finite values of AMM of quarks ?

AMM of quarks

- What if we consider finite values of AMM of quarks ?
- The immediate change is in the form of energy. Thus we can write

$$\Omega = \frac{(M-m)^2}{4G} - \frac{2N_c}{\beta} \sum_f \frac{|q_f B|}{2\pi} \sum_{n=0}^{\infty} \sum_s \int_{-\infty}^{\infty} \frac{dp_z}{2\pi} \Big\{ \beta\omega_{nfs} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right) \Big\}$$

AMM of quarks

- What if we consider finite values of AMM of quarks ?
- The immediate change is in the form of energy. Thus we can write

$$
\Omega = \frac{(M-m)^2}{4G} - \frac{2N_c}{\beta}\sum_f \frac{|q_f B|}{2\pi}\sum_{n=0}^{\infty}\sum_s \int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\Big\{\beta\omega_{nfs} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right)\Big\}
$$

where

$$
\omega_{nfs} = \left[p_z^2 + \left\{\left(\sqrt{|q_f B|\,(2n+1-s) + M^2} - s\kappa_f q_f B\right)^2\right\}\right]^{1/2}
$$

AMM of quarks

- What if we consider finite values of AMM of quarks ?
- The immediate change is in the form of energy. Thus we can write

$$\Omega = \frac{(M-m)^2}{4G} - \frac{2N_c}{\beta}\sum_f \frac{|q_f B|}{2\pi}\sum_{n=0}^{\infty}\sum_s \int_{-\infty}^{\infty}\frac{dp_z}{2\pi}\Big\{\beta\omega_{nfs} - \ln\left(1-n^+\right) - \ln\left(1-n^-\right)\Big\}$$

where

$$\omega_{nfs} = \left[p_z^2 + \left\{\left(\sqrt{|q_f B|\,(2n+1-s) + M^2} - s\kappa_f q_f B\right)^2\right\}\right]^{1/2}$$

- One can find out the constituent quark mass by minimizing the thermodynamic potential

$$M = m + 2GN_c\sum_f |q_f B|\sum_{n=0}^{\infty}\sum_{\{s\}}\int_{-\infty}^{\infty}\frac{dp_z}{4\pi^2}\frac{M}{\omega_{nfs}}\left(1-\frac{s\kappa_f q_f B}{M_{nfs}}\right)\left(1-n^+-n^-\right)$$

Regularization with AMM

Regularization with AMM

- This regularization will be valid iff $\Lambda^2 - \vec{p}_\perp^2 \geq 0$ and $\vec{p}_\perp^2 \geq 0$ as $p_z, \vec{p}_\perp$ are real quantities. First condition will always be there for finite values of eB but the second condition is only due to non-zero values of AMM of quarks.

Regularization with AMM

- This regularization will be valid iff $\Lambda^2 - \vec{p}_\perp^2 \geq 0$ and $\vec{p}_\perp^2 \geq 0$ as $p_z, \vec{p}_\perp$ are real quantities. First condition will always be there for finite values of eB but the second condition is only due to non-zero values of AMM of quarks.
- Putting these condition back we finally get

$$
\begin{aligned}
M &= m + 4GN_c \sum_{n,f,s} |q_f B| \int_0^{\Lambda_z} \frac{dp_z}{4\pi^2} \Theta\left(\Lambda^2 - \vec{p}_\perp^2\right) \Theta\left(\vec{p}_\perp^2\right) \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right) \\
&\quad -4GN_c \sum_f |q_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\infty} \frac{dp_z}{4\pi^2} \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right) \left(n^+ + n^-\right)
\end{aligned}
$$

Regularization with AMM

- This regularization will be valid iff $\Lambda^2 - \vec{p}_\perp^2 \geq 0$ and $\vec{p}_\perp^2 \geq 0$ as $p_z, \vec{p}_\perp$ are real quantities. First condition will always be there for finite values of eB but the second condition is only due to non-zero values of AMM of quarks.
- Putting these condition back we finally get

$$
\begin{aligned}
M \quad = \quad & m + 4GN_c \sum_{n,f,s} |q_f B| \int_0^{\Lambda_z} \frac{dp_z}{4\pi^2} \Theta\left(\Lambda^2 - \vec{p}_\perp^2\right) \Theta\left(\vec{p}_\perp^2\right) \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right) \\
& -4GN_c \sum_f |q_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\infty} \frac{dp_z}{4\pi^2} \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right) \left(n^+ + n^-\right)
\end{aligned}
$$

- If we take the limits $\kappa_f \to 0$ and $q_f B \to 0$ in the above equation, it go back to it's vacuum expression.

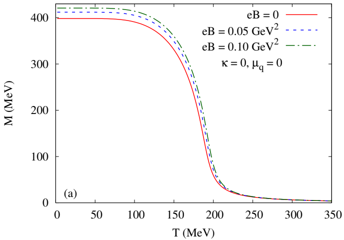
Outline

Variation of constituent quark mass (M) and $\partial M/\partial T$ with temperature T

Parameters: $m_0 = 5.6$ MeV, $\Lambda = 587.9$ MeV, $G = 2.44/\Lambda^2$.

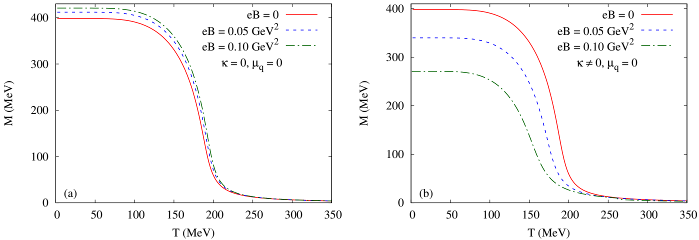
Variation of constituent quark mass (M) and $\partial M/\partial T$ with temperature T

Parameters: $m_0 = 5.6$ MeV, $\Lambda = 587.9$ MeV, $G = 2.44/\Lambda^2$.



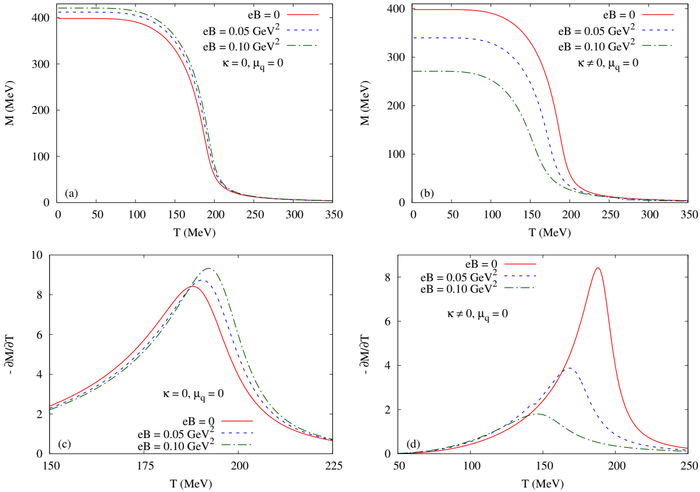
Variation of constituent quark mass (M) and $\partial M/\partial T$ with temperature T

Parameters: $m_0 = 5.6$ MeV, $\Lambda = 587.9$ MeV, $G = 2.44/\Lambda^2$.



Variation of constituent quark mass (M) and $\partial M/\partial T$ with temperature T

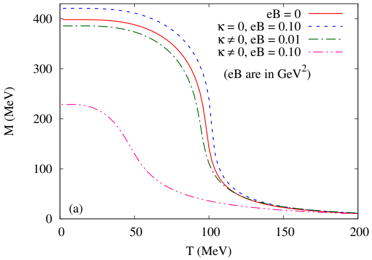
Parameters: $m_0 = 5.6$ MeV, $\Lambda = 587.9$ MeV, $G = 2.44/\Lambda^2$.



PRD 99 116025

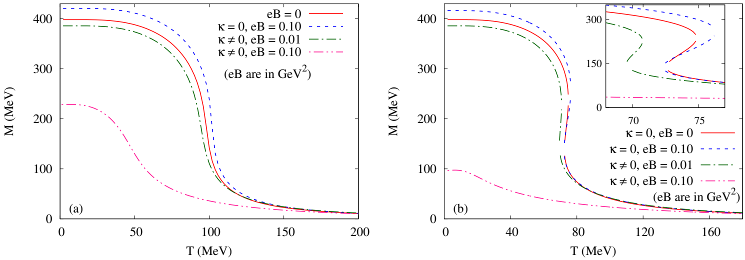
M vs T/μ_q

M vs T/μ_q



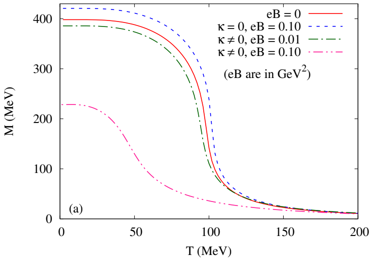
eB = 0
κ = 0, eB = 0.10
κ ≠ 0, eB = 0.01
κ ≠ 0, eB = 0.10
(eB are in GeV²)
(a)
M (MeV)
T (MeV)
0
100
200
300
400
0
50
100
150
200

M vs T/μ_q

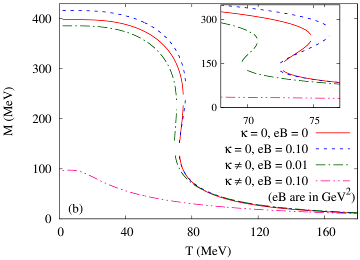


eB = 0
κ = 0, eB = 0.10
κ ≠ 0, eB = 0.01
κ ≠ 0, eB = 0.10
(eB are in GeV^2)
M (MeV)
T (MeV)
(a)
κ = 0, eB = 0
κ = 0, eB = 0.10
κ ≠ 0, eB = 0.01
κ ≠ 0, eB = 0.10
(eB are in GeV^2)
M (MeV)
T (MeV)
(b)

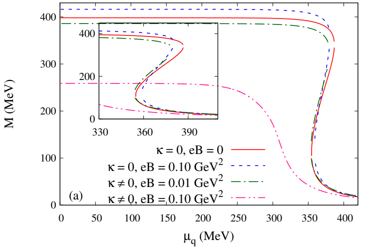
M vs T/μ_q



eB = 0
κ = 0, eB = 0.10
κ ≠ 0, eB = 0.01
κ ≠ 0, eB = 0.10
(eB are in GeV^2)
(a)
M (MeV)
T (MeV)

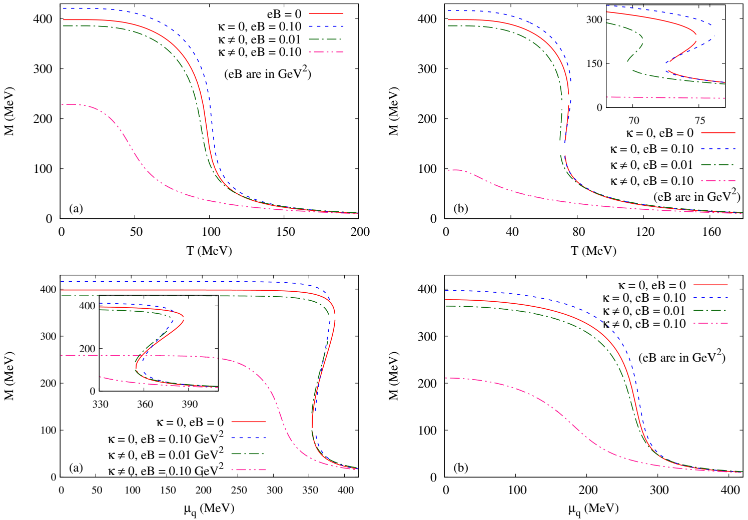


κ = 0, eB = 0
κ = 0, eB = 0.10
κ ≠ 0, eB = 0.01
κ ≠ 0, eB = 0.10
(eB are in GeV^2)
(b)
M (MeV)
T (MeV)



κ = 0, eB = 0
κ = 0, eB = 0.10 GeV^2
κ ≠ 0, eB = 0.01 GeV^2
κ ≠ 0, eB = 0.10 GeV^2
(a)
M (MeV)
μ_q (MeV)

M vs T/μ_q

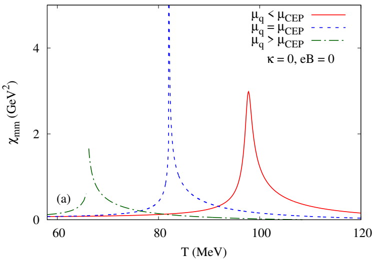


PRD 99 116025

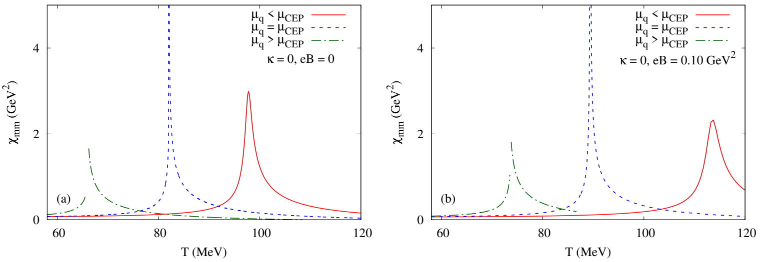
Variation of χ_{mm} with temperature at different values of quark chemical potential

Variation of χ_{mm} with temperature at different values of quark chemical potential

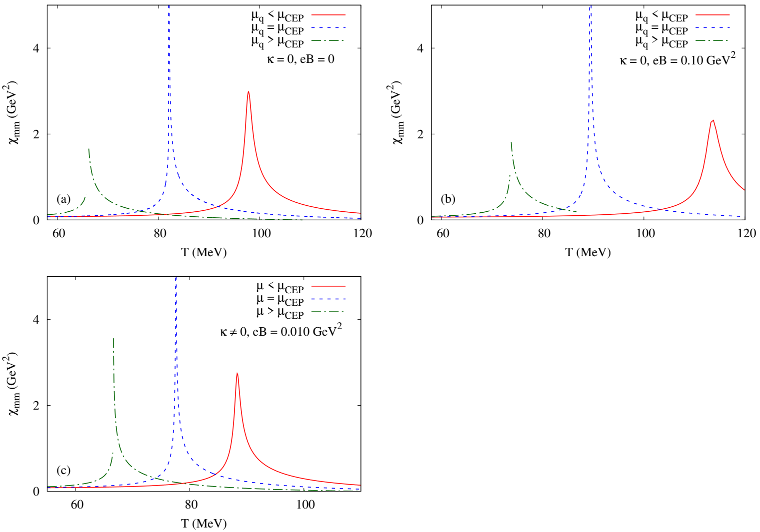
Variation of χ_{mm} with temperature at different values of quark chemical potential



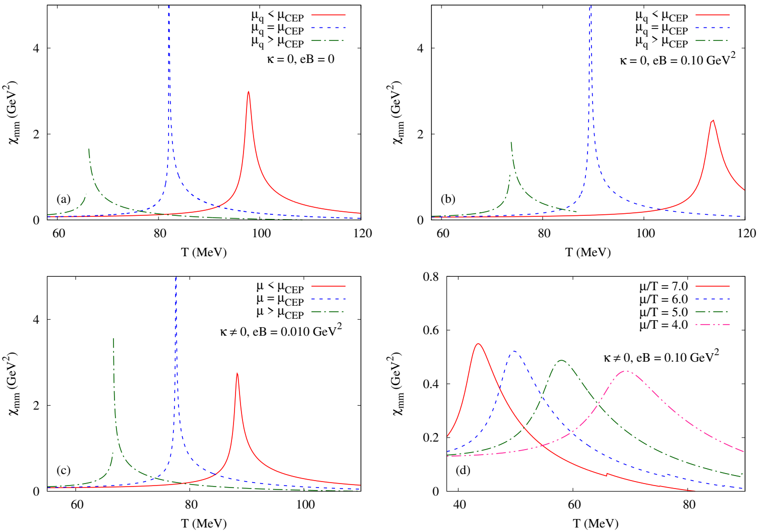
Variation of χ_{mm} with temperature at different values of quark chemical potential



Variation of χ_{mm} with temperature at different values of quark chemical potential



Variation of χ_{mm} with temperature at different values of quark chemical potential



PRD 99 116025

Phase diagram

PRD 99 116025

Phase diagram

PRD 99 116025

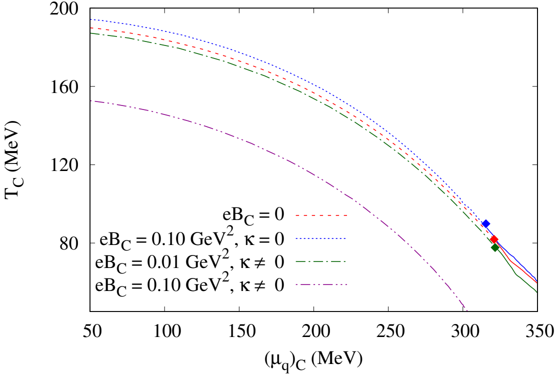


Figure: T_C-$(\mu_q)_C$ phase diagram in NJL model for three different conditions.

Phase diagram

PRD 99 116025

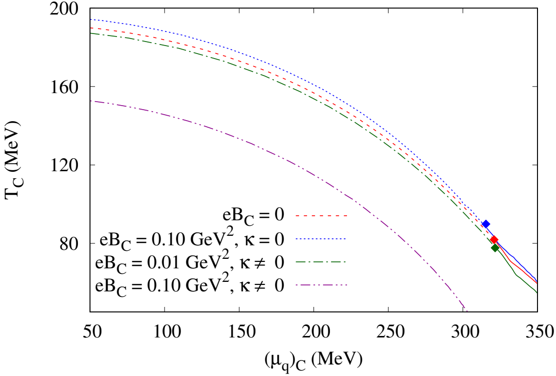


Figure: T_C-$(\mu_q)_C$ phase diagram in NJL model for three different conditions.

- The red, green and blue square points represent CEPs

Outline

Summary

- In this work, we found that the transition temperature from symmetry broken to restored phase increases with external magnetic field showing the enhancement of the quark anti-quark condensate, which can be identified as magnetic catalysis.
- The opposite behaviour is observed when AMM of quarks is taken into consideration, indicating inverse magnetic catalysis.
- Critical behaviour of chiral susceptibility (χ_{mm}) has been examined in the vicinity of the phase transition.
- The phase diagram of hot and dense magnetized quark matter is obtained and for finite values of eB, the CEP is found to shift towards lower values of temperature and follows an opposite trend when we exclude AMM of quarks.
- Interestingly at high eB for finite values of AMM, the transition remains crossover for larger range of T_C and $(\mu_q)_C$.

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

collaborators

- Dr. Snigdha Ghosh, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Pradip Roy, Saha Institute of Nuclear Physics, 1/AF Bidhannagar, Kolkata 700064, India
- Dr. Sourav Sarkar, Variable Energy Cyclotron Centre, 1/AF Bidhannagar, Kolkata 700 064, India

back ups

back ups

Expression for D_T

$$D_T = 1 - \frac{GN_c}{\pi^2} \sum_f |e_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\Lambda_z} dp_z \left[\frac{s\kappa_f e_f B M^2}{E_{nfs} M_{nf}^3} + \frac{1}{E_{nfs}} \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right) - \frac{M^2}{E_{nfs}^3} \right.$$
$$\left. \times \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right)^2 \right] - \frac{GN_c}{\pi^2} \sum_f |e_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \frac{M^2}{(\Lambda^2 + M^2)^{1/2}} \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right) \frac{s\kappa_f e_f B}{\Lambda_z M_{nf}}$$
$$+ \frac{GN_c}{\pi^2} \sum_f |e_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\infty} dp_z \left[\frac{s\kappa_f e_f B M^2}{E_{nfs} M_{nf}^3} + \frac{1}{E_{nfs}} \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right) \right.$$
$$\left. - \frac{M^2}{E_{nfs}^3} \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right)^2 \right] \left(n^+ + n^-\right)$$
$$- \frac{GN_c}{\pi^2} \sum_f |e_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\infty} dp_z \beta \left(\frac{M}{E_{nfs}}\right)^2 \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right)^2 \left[n^+(1 - n^+) + n^-(1 + n^-)\right.$$

back ups

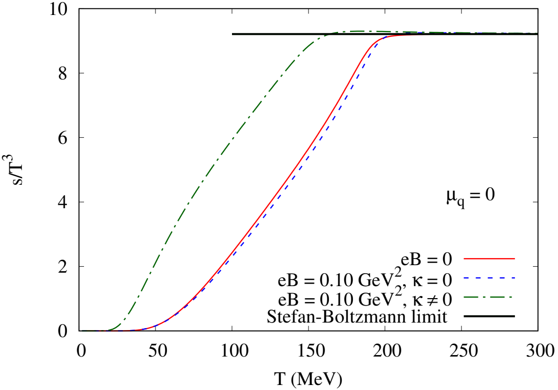


Figure: Variation of scaled entropy density as function of temperature at $\mu_q = 0$.

back ups

$$M = m + 4GN_c \sum_f |e_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\Lambda_z} \frac{dp_z}{4\pi^2} \Theta(\Lambda^2 - \vec{p}_\perp^{\,2})\ \Theta(\vec{p}_\perp^{\,2}) \frac{M}{E_{nfs}} \left(1 - \frac{s\kappa_f e_f B}{M_{nf}}\right)$$

First put all the terms containing the $\kappa_f e_f B$ equals to zero.

$$\begin{aligned} M &= m + 4GN_c \lim_{B\to 0} \sum_f |e_f B| \sum_{n=0}^{\infty} (2 - \delta_{n0}) \\ &\quad \int_0^{\sqrt{\Lambda^2 - 2n|e_f B|}} \frac{dp_z}{4\pi^2} \Theta(\Lambda^2 - 2n\,|e_f B|)\ \frac{M}{\sqrt{p_z^2 + 2n\,|e_f B| + M^2}} \\ &= m + \frac{MGN_c}{\pi^2} \lim_{B\to 0} \sum_f |e_f B| \sum_{n=0}^{\infty} (2 - \delta_{n0}) \Theta(\Lambda^2 - 2n\,|e_f B|) \tanh^{-1} \sqrt{\frac{\Lambda^2 - 2n\,|e_f B|}{\Lambda^2 + M^2}} \end{aligned}$$

Separating out the contribution of the LLL from the above equation($\tau_f = 2n\,|e_f B|$)

$$M = m + \frac{MGN_c}{\pi^2} \lim_{B\to 0} \sum_f |e_f B| \left[\tanh^{-1} \sqrt{\frac{\Lambda^2}{\Lambda^2 + M^2}} + 2 \sum_{\tau_f = 2|e_f B|}^{\infty}{}' \Theta(\Lambda^2 - \tau_f) \tanh^{-1} \sqrt{\frac{\Lambda^2 - \tau_f}{\Lambda^2 + M^2}}\right.$$

$\sum'$ denotes an increment of $2e_f B$ of its index rather than 1. Now as $e_f B \to 0$, we can change the summation to an integration continuum limit

$$\sum_{\tau_f}{}' \to \frac{1}{2e_f B} \int_{2e_f B}^{\infty} d\tau_f .$$

back ups

This leads to

$$M \quad = \quad m + \frac{MGN_c}{\pi^2} \sum_f \int_0^\infty d\tau_f \Theta(\Lambda^2 - \tau_f) \tanh^{-1} \sqrt{\frac{\Lambda^2 - \tau_f}{\Lambda^2 + M^2}}$$

Note that, the presence of the step function will restrict the upper limit of the τ_f integration. Performing the remaining $d\tau_f$ integral, we are left with

$$M = m + \frac{GMN_f N_c}{\pi^2} \left[\Lambda \sqrt{\Lambda^2 + M^2} - M^2 \sinh^{-1} \left(\frac{\Lambda}{M} \right) \right]$$

which is same as the vacuum term.

back ups

NJL Lagrangian considering AMM of quarks in presence of uniform background magnetic field

$$\mathscr{L} = \bar{\psi}(x)\left(i\not{D} - m + \frac{1}{2}\hat{a}\sigma^{\mu\nu}F_{\mu\nu}\right)\psi(x) + G\left\{\left(\bar{\psi}(x)\psi(x)\right)^2 + \left(\bar{\psi}(x)i\gamma_5\tau\psi(x)\right)^2\right\}$$

$$D_\mu = \partial_\mu + iQA_\mu; \qquad \hat{Q} = \mathrm{diag}(2e/3, -e/3); \qquad \hat{a} = \hat{Q}\hat{\kappa}; \qquad \hat{\kappa} = \mathrm{diag}(\kappa_u, \kappa_d);$$
$$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]; \qquad g^{\mu\nu} = \mathrm{diag}\,(1, -1, -1, -1);$$

In MFA the Lagrangian becomes

$$\mathscr{L} = \bar{\psi}(x)\left(i\not{D} - M + \frac{1}{2}\hat{a}\sigma^{\mu\nu}F_{\mu\nu}\right)\psi(x) - \frac{(M-m)^2}{4G}$$

Regularization at finite eB

- Note that, the medium independent integral is still ultraviolet divergent

$$I_{\mathrm{div}} = \int_{-\infty}^{\infty} \frac{dp_z}{4\pi^2} \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right)$$

- First, we note that the integrands are even functions of p_z; introducing the field dependent cut-off parameter Λ_z we get,

$$I_{\mathrm{reg}} = 2\int_{0}^{\Lambda_z} \frac{dp_z}{4\pi^2} \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right)$$

where,

$$\Lambda_z = \sqrt{\Lambda^2 - \vec{p}_\perp^2}$$

while Λ being the usual three-momentum cut-off. The quantity $\vec{p}_\perp^2$ inside the square root can be identified from expression for energy

$$\begin{aligned} \vec{p}_\perp^2 &= \left(\sqrt{|q_f B|\,(2n+1-s) + M^2} - s\kappa_f q_f B\right)^2 - M^2 \\ &= |q_f B|\,(2n+1-s) + (\kappa_f q_f B)^2 - 2sM_{nfs}\kappa_f q_f B. \end{aligned}$$

$q_f B \to 0$ limit

$$M = m + 4GN_c \sum_f |q_f B| \sum_{n=0}^{\infty} \sum_{\{s\}} \int_0^{\Lambda_z} \frac{dp_z}{4\pi^2} \Theta(\Lambda^2 - \vec{p}_\perp^{\,2})\, \Theta(\vec{p}_\perp^{\,2}) \frac{M}{\omega_{nfs}} \left(1 - \frac{s\kappa_f q_f B}{M_{nfs}}\right)$$

First put all the terms containing the $\kappa_f e_f B$ equals to zero.

$$\begin{aligned} M &= m + 4GN_c \lim_{B\to 0} \sum_f |q_f B| \sum_{n=0}^{\infty} (2 - \delta_{n0}) \\ &\quad \int_0^{\sqrt{\Lambda^2 - 2n|q_f B|}} \frac{dp_z}{4\pi^2} \Theta(\Lambda^2 - 2n\,|q_f B|) \frac{M}{\sqrt{p_z^2 + 2n\,|q_f B| + M^2}} \\ &= m + \frac{MGN_c}{\pi^2} \lim_{B\to 0} \sum_f |q_f B| \sum_{n=0}^{n_{\max}} (2 - \delta_{n0}) \tanh^{-1} \sqrt{\frac{\Lambda^2 - 2n\,|q_f B|}{\Lambda^2 + M^2}} \end{aligned}$$

with $n_{\max} = \left[\Lambda^2/2\,|q_f B|\right]$. Separating out the contribution of the LLL from the above equation($\tau_f = 2n\,|q_f B|$)

$$M = m + \frac{MGN_c}{\pi^2} \lim_{B\to 0} \sum_f |q_f B| \left[\tanh^{-1} \sqrt{\frac{\Lambda^2}{\Lambda^2 + M^2}} + 2 \sum_{\tau_f = 2|q_f B|}^{\Lambda^2}{}' \tanh^{-1} \sqrt{\frac{\Lambda^2 - \tau_f}{\Lambda^2 + M^2}}\right]$$

As $q_f B \to 0$, we can change the summation to an integration continuum limit

$$\sum_{\tau_f}{}' \to \frac{1}{2q_f B} \int_{2q_f B}^{\Lambda^2} d\tau_f .$$

$q_f B \to 0$ limit

This leads to

$$M \quad = \quad m + \frac{MGN_c}{\pi^2} \sum_f \int_0^{\Lambda^2} d\tau_f \tanh^{-1} \sqrt{\frac{\Lambda^2 - \tau_f}{\Lambda^2 + M^2}}$$

Note that, the presence of the step function will restrict the upper limit of the τ_f integration. Performing the remaining $d\tau_f$ integral, we are left with

$$M = m + \frac{GMN_f N_c}{\pi^2} \left[\Lambda \sqrt{\Lambda^2 + M^2} - M^2 \sinh^{-1} \left(\frac{\Lambda}{M} \right) \right]$$

which is same as the vacuum term.

Constituent quark mass vs μ_q

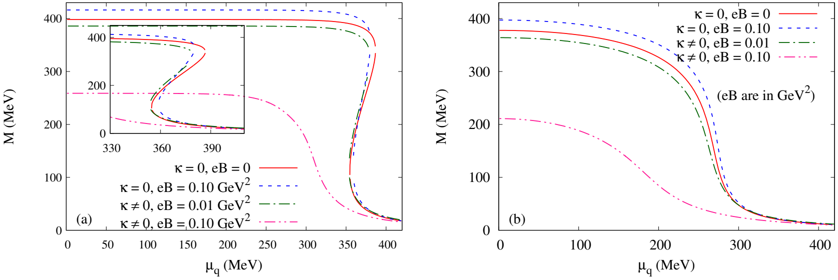


Figure: μ_q dependence of Constituent quark mass (M) at (a) $T = 30$ MeV and (b) at $T = 120$ MeV for different values of eB and κ.