Mesons in a Hot and Magnetized Medium

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Outline

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2. The NJL Model
3. The Constituent Quark Mass & The Dressed Quark Propagator
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5. Results

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  - Superconductivity of the Vacuum.
- In this work, we aim to study the properties of hot and magnetized mesons (π, σ, ρ and a_1) using the 2-flavour NJL model.

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$$\mathscr{L}_{\rm NJL} = \overline{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi + g_s\left\{(\overline{\Psi}\Psi)(\overline{\Psi}\Psi) - (\overline{\Psi}\gamma^5\vec{\tau}\Psi)\cdot(\overline{\Psi}\gamma^5\vec{\tau}\Psi)\right\}$$
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- The constituent quark mass is dynamically generated in the NJL model as a consequence of the spontaneous breaking of chiral symmetry.

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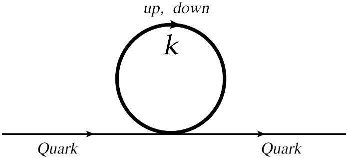

Figure: Feynman digram for one-loop quark self energy. The bold line corresponds to *'complete/dressed'* quark propagator obtained from the Dyson-Schwinger sum.

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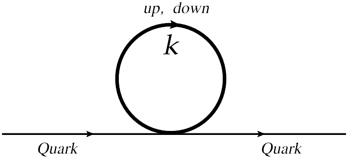

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- Mean Field Approximation (MFA): $\Sigma = \Sigma_{\text{MFA}} \mathbb{1}_{\text{Dirac}} \otimes \mathbb{1}_{\text{Flavour}} \otimes \mathbb{1}_{\text{Colour}}$.

The Gap Equation

- Solving the Dyson-Schwinger Equation:

$$S'(q,m) = S(q,M) = \frac{-(\not{q} + M)}{q^2 - M^2 + i\epsilon} \otimes \mathbb{1}_{\text{Flavour}} \otimes \mathbb{1}_{\text{Colour}}$$

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- The loop particle in the self energy graph is the complete one.
- The quark self energy is a function of M itself (since the loop particle is dressed).
- The Gap equation has to be solved self-consistently to calculate M.

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- The mostly used regulator is the momentum cutoff which breaks the Lorentz invariance and usually every symmetry of the theory.
- The momentum cutoff regulator (or any other regulator which breaks Lorentz invariance) is not useful to study the vector meson ρ or axial vector meson a_1 in the NJL model.

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- $\omega_k^{lf} = \sqrt{k_z^2 + M^2 + 2l|e_fB|}$: Landau quantized energy.
- **NO APPROXIMATION** on the strength of the magnetic field.

Outline

Polarization Functions of the mesons

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- Their propagation can be studied from the scattering of quarks in different channels using the Bethe-Salpeter approach.
- In order to calculate the meson propagators, we need to calculate the following one-loop polarization functions.
- At $T = B = 0$, they are:

$$\begin{aligned}
\Pi_\pi(q) &= -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}_{\mathrm{d,f,c}}\left[\gamma^5\tau_3 S'(q+k,m)\gamma^5\tau_3 S'(k,m)\right] \\
\Pi_\sigma(q) &= i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}_{\mathrm{d,f,c}}\left[S'(q+k,m)S'(k,m)\right] \\
\Pi_\rho^{\mu\nu}(q) &= -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}_{\mathrm{d,f,c}}\left[\gamma^\mu\tau_3 S'(q+k,m)\gamma^\nu\tau_3 S'(k,m)\right] \\
\Pi_{a_1}^{\mu\nu}(q) &= -i\int \frac{d^4k}{(2\pi)^4}\mathrm{Tr}_{\mathrm{d,f,c}}\left[\gamma^\mu\gamma^5\tau_3 S'(q+k,m)\gamma^\nu\gamma^5\tau_3 S'(k,m)\right]
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\Pi_\pi(q) &= \frac{N_c N_f}{4\pi^2}\left[\frac{1}{2}q^2\Gamma\left(0, \frac{M^2}{\Lambda^2}\right) + M^2\Gamma\left(-1, \frac{M^2}{\Lambda^2}\right)\right] \\
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\Pi_{a_1}^{\mu\nu}(q) &= -\frac{N_c N_f}{12\pi^2}\Gamma\left(0, \frac{M^2}{\Lambda^2}\right)\left[\left(q^2 - 6M^2\right)\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) - 6M^2\frac{q^\mu q^\nu}{q^2}\right] .
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- For a_1, the non-transverse piece is proportional to the constituent quark mass M.
- *Reason*: The non-conservation of the axial-vector current $J^{5\mu}=\overline{\Psi}(x)\gamma^\mu\gamma^5\Psi$ whose four-divergence is $\partial_\mu J^{5\mu}\propto M$.

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$$\mathrm{Re}\overline{\overline{\Pi}}_h(q_\parallel) = \mathrm{Re}\Pi_h(q_\parallel) + \mathrm{Re}\Pi_{h\mathrm{B}}(q_\parallel, B) - \sum_{l=0}^{\infty} \sum_{n=(l-1)}^{(l+1)} \sum_{f\in\{\mathrm{u,d}\}} \int_{-\infty}^{\infty} \frac{dk_z}{2\pi} \mathcal{P} \left[\frac{N_h^{lnf}(k^0 = -\omega_k^{lf}) f(\omega}{2\omega_k^{lf} \left\{(q^0 - \omega_k^{lf})^2 - (\omega\right.} \right.$$

$$+ \frac{N_h^{lnf}(k^0 = \omega_k^{lf}) f(\omega_k^{lf})}{2\omega_k^{lf} \left\{(q^0 + \omega_k^{lf})^2 - (\omega_p^{nf})^2\right\}} + \frac{N_h^{lnf}(k^0 = -q^0 - \omega_p^{nf}) f(\omega_p^{nf})}{2\omega_p^{nf} \left\{(q^0 + \omega_p^{nf})^2 - (\omega_k^{lf})^2\right\}} + \frac{N_h^{lnf}(k^0 = -q^0 + \omega_p^{nf}}{2\omega_p^{nf} \left\{(q^0 - \omega_p^{nf})^2 - \right.}$$

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Looks Very Nasty .... DOES NOT EVEN FIT IN THE SLIDE

$$\Pi_{\pi\mathrm{B}}(q_\parallel, B) = \frac{N_c}{4\pi^2} \sum_{f\in\{\mathrm{u,d}\}} \int_0^1 dx \left[\left(2M^2 - 3\Delta + |e_f B|\right) \ln z + 2\left(\Delta - M^2\right)\psi(z) - M^2/z \right.$$

Meson Propagators

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- For the scalar (σ) and pseudo-scalar (π) channels, it is trivial to obtain:

$$D'_h(q) = \frac{-2g_s}{1 - 2g_s \Pi_h} \quad ; \quad h \in \{\pi, \sigma\}$$

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- For vector (ρ) and axial-vector (a_1) channels: Additional complications arise because of the Lorentz indices.
- *Way out*: To find Lorentz decomposition of the polarization functions of ρ and a_1 in a suitable tensor basis.
- Constraint: The polarization tensors are symmetric

$$\Pi^{\mu\nu}_{\rho,a_1} = \Pi^{\nu\mu}_{\rho,a_1}$$

Lorentz Basis at $T = 0$ and $B = 0$

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- The available quantities to construct a tensor basis are the momentum of the meson q^μ and the metric tensor $g^{\mu\nu}$.
- Only two basis tensors can be constructed which are the following:

$$
\begin{aligned}
P_1^{\mu\nu} &= \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \\
P_2^{\mu\nu} &= \frac{q^\mu q^\nu}{q^2} .
\end{aligned}
$$

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- Apart from q^μ and $g^{\mu\nu}$, in this case we have an additional four-vector u^μ.
- Thus we can choose the following four tensors as the basis:

$$\begin{aligned}
\overline{P}_1^{\mu\nu} &= \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2} \right) \\
\overline{P}_2^{\mu\nu} &= \frac{q^\mu q^\nu}{q^2} \\
\overline{P}_3^{\mu\nu} &= \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2} \\
\overline{P}_4^{\mu\nu} &= \frac{1}{\sqrt{q^2 \tilde{u}^2}} \left(q^\mu \tilde{u}^\nu + q^\nu \tilde{u}^\mu \right)
\end{aligned}$$

where

$$\tilde{u}^\mu = u^\mu - \frac{(q \cdot u)}{q^2} q^\mu$$

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- Another four vector b^μ appears which specify the direction of the external magnetic field in the LRF. In the LRF, we have $b^\mu_{\text{LRF}} \equiv (0, 0, 0, 1)$.

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- Another four vector b^μ appears which specify the direction of the external magnetic field in the LRF. In the LRF, we have $b^\mu_{\mathrm{LRF}} \equiv (0, 0, 0, 1)$.
- Using q^μ, u^μ, b^μ and $g^{\mu\nu}$, we can construct the following seven orthogonal tensors:

$$
\begin{aligned}
\overline{\overline{P}}^{\mu\nu}_1 &= \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2} - \frac{\tilde{b}^\mu \tilde{b}^\nu}{\tilde{b}^2} \right) \\
\overline{\overline{P}}^{\mu\nu}_2 &= \frac{q^\mu q^\nu}{q^2} \\
\overline{\overline{P}}^{\mu\nu}_3 &= \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2} \\
\overline{\overline{P}}^{\mu\nu}_4 &= \frac{\tilde{b}^\mu \tilde{b}^\nu}{\tilde{u}^2} \\
\overline{\overline{P}}^{\mu\nu}_5 &= \frac{1}{\sqrt{q^2 \tilde{u}^2}} \left(q^\mu \tilde{u}^\nu + q^\nu \tilde{u}^\mu \right) \\
\overline{\overline{P}}^{\mu\nu}_6 &= \frac{1}{\sqrt{q^2 \tilde{b}^2}} \left(q^\mu \tilde{b}^\nu + q^\nu \tilde{b}^\mu \right) \\
\overline{\overline{P}}^{\mu\nu}_7 &= \frac{1}{\sqrt{\tilde{u}^2 \tilde{b}^2}} \left(\tilde{u}^\mu \tilde{b}^\nu + \tilde{u}^\nu \tilde{b}^\mu \right)
\end{aligned}
$$

where, $\tilde{b}^\mu = b^\mu - \frac{(q \cdot b)}{q^2} q^\mu - \frac{(\tilde{u} \cdot b)}{\tilde{u}^2} \tilde{u}^\mu$.

ρ and a_1 Propagators

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- The ρ and a_1 polarization functions and propagators can be expanded in the constructed basis as:

$$\overline{\overline{\Pi}}_H^{\mu\nu} = \sum_{i=1}^{7} \overline{\overline{\Pi}}_{H\,i} \overline{\overline{P}}_i^{\mu\nu} \quad ; \quad \overline{\overline{D}}_H'^{\mu\nu} = \sum_{i=1}^{7} \overline{\overline{D}}_{H\,i} \overline{\overline{P}}_i^{\mu\nu}$$

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- where,

$$\begin{aligned}
\overline{\overline{D}}_{H1} &= \left(\frac{2g_v}{1 + 2g_v \overline{\overline{\Pi}}_{H1}}\right) \\
\overline{\overline{D}}_{H2} &= \frac{1}{\mathcal{A}_{TB}} 2g_v \left[\left(1 + 2g_v \overline{\Pi}_{H3}\right)\left(1 + 2g_v \overline{\Pi}_{H4}\right) - \left(2g_v \overline{\Pi}_{H7}\right)^2\right] \\
\overline{\overline{D}}_{H3} &= \frac{1}{\mathcal{A}_{TB}} 2g_v \left[\left(1 + 2g_v \overline{\Pi}_{H2}\right)\left(1 + 2g_v \overline{\Pi}_{H4}\right) - \left(2g_v \overline{\Pi}_{H6}\right)^2\right] \\
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\overline{\overline{D}}_{H5} &= \frac{1}{\mathcal{A}_{TB}} 2g_v \left[\left(2g_v \overline{\Pi}_{H6}\right)\left(2g_v \overline{\Pi}_{H7}\right) - \left(1 + 2g_v \overline{\Pi}_{H4}\right)\left(2g_v \overline{\Pi}_{H5}\right)\right] \\
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\overline{\overline{D}}_{H7} &= \frac{1}{\mathcal{A}_{TB}} 2g_v \left[\left(2g_v \overline{\Pi}_{H5}\right)\left(2g_v \overline{\Pi}_{H6}\right) - \left(1 + 2g_v \overline{\Pi}_{H2}\right)\left(2g_v \overline{\Pi}_{H7}\right)\right]
\end{aligned}$$

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\mathcal{A}_{TB} = & \left(1 + 2g_v \overline{\overline{\Pi}}_{H2}\right)\left(1 + 2g_v \overline{\overline{\Pi}}_{H3}\right)\left(1 + 2g_v \overline{\overline{\Pi}}_{H4}\right) - \left(1 + 2g_v \overline{\overline{\Pi}}_{H2}\right)\left(2g_v \overline{\overline{\Pi}}_{H7}\right)^2 - \left(1 + 2g_v \overline{\overline{\Pi}}_{H3}\right)\left(2g \right. \\
& - \left(1 + 2g_v \overline{\overline{\Pi}}_{H4}\right)\left(2g_v \overline{\overline{\Pi}}_{H5}\right)^2 + \left(2g_v \overline{\overline{\Pi}}_{H5}\right)\left(2g_v \overline{\overline{\Pi}}_{H6}\right)\left(2g_v \overline{\overline{\Pi}}_{H7}\right).
\end{aligned}$$

Outline

Constituent Quark Mass

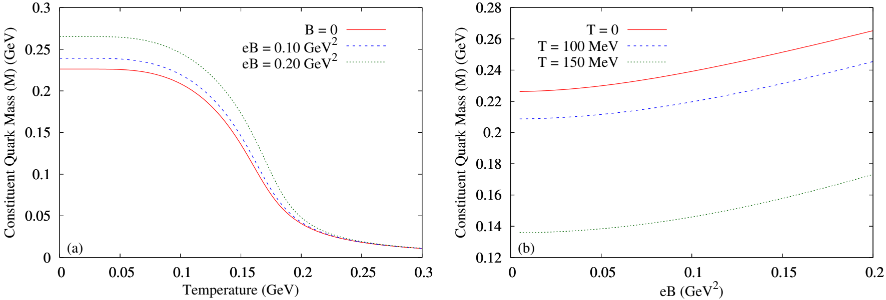

Figure: Variation of the constituent quark mass (M) as a function of (a) temperature for different values of external magnetic field and (b) external magnetic field for different values of temperature.

Susceptibilities

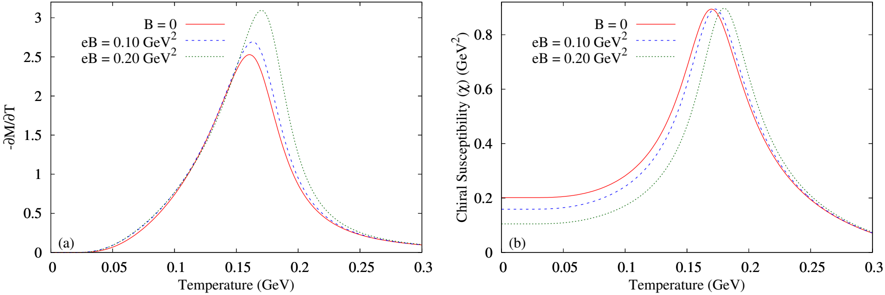

Figure: (a) Variation of the $-\partial M/\partial T$ and (b) the chiral susceptibility $(\chi = \frac{1}{2g_s}\left(\frac{\partial M}{\partial m} - 1\right))$ as a function of temperature for different values of external magnetic field.

Spectral Functions of π and σ

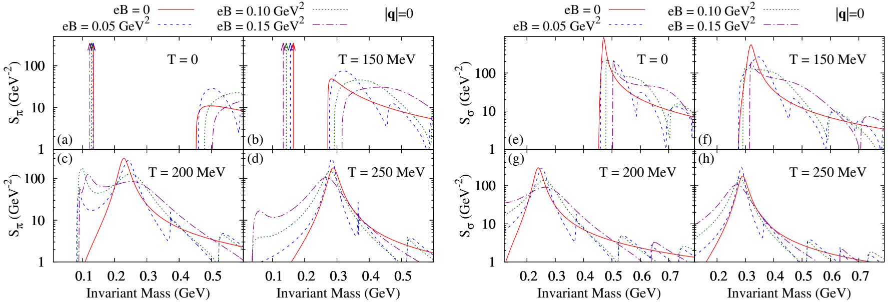

Figure: Spectral function of π^0 and σ mesons as a function of invariant mass for $\vec{q} = \vec{0}$ at different values of temperature and external magnetic field. The arrows represent Dirac delta functions.

Spectral Functions of ρ and a_1

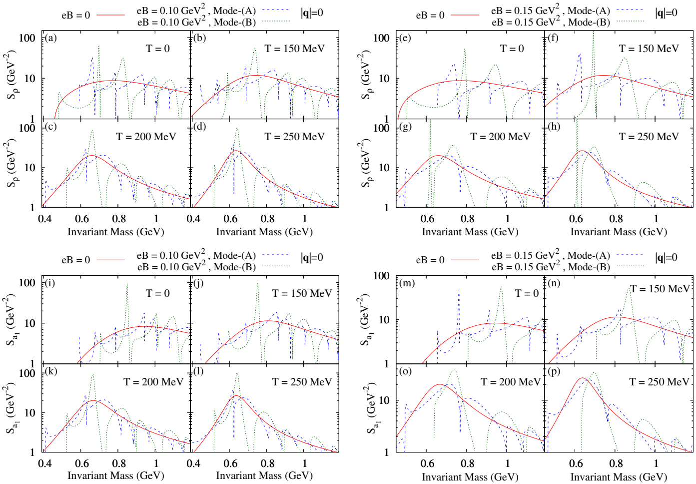

Figure: Spectral function of ρ^0 and a_1^0 mesons as a function of invariant mass for $\vec{q} = \vec{0}$ at different values of temperature and external magnetic field.

Comparison of Spectral Functions: Chiral Symmetry Restoration

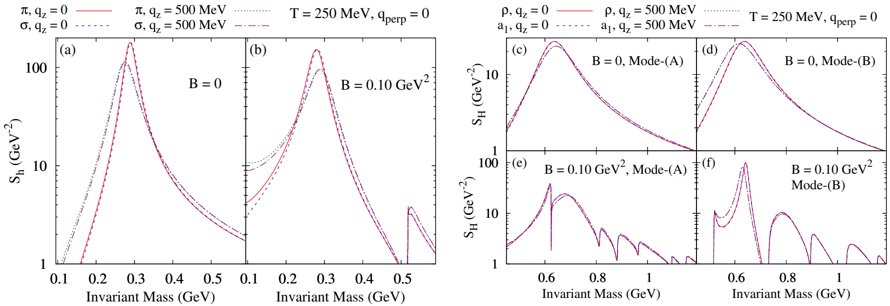

Figure: Comparison of the spectral functions of π^0 with σ and ρ^0 with a_1^0 at $T = 250$ MeV, $q_\perp = 0$ for different values of their longitudinal momentum ($q_z = 0$ and 500 MeV).

Meson Masses

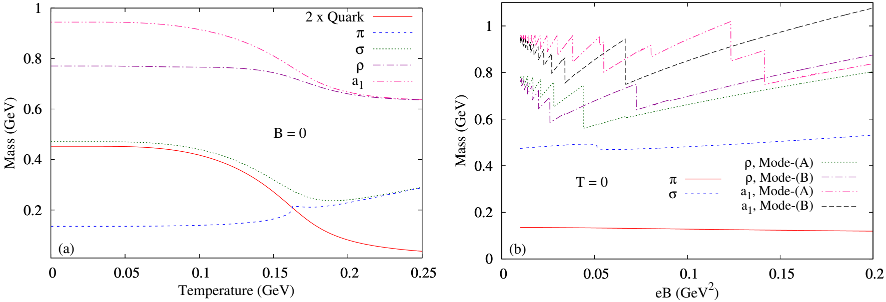

Figure: Variation of masses of π^0, σ, ρ^0 and a_1^0 as a function of (a) temperature at $B = 0$ and (b) external magnetic field at $T = 0$. The variation of twice of the constituent quark mass is also shown in (a).

Meson Masses

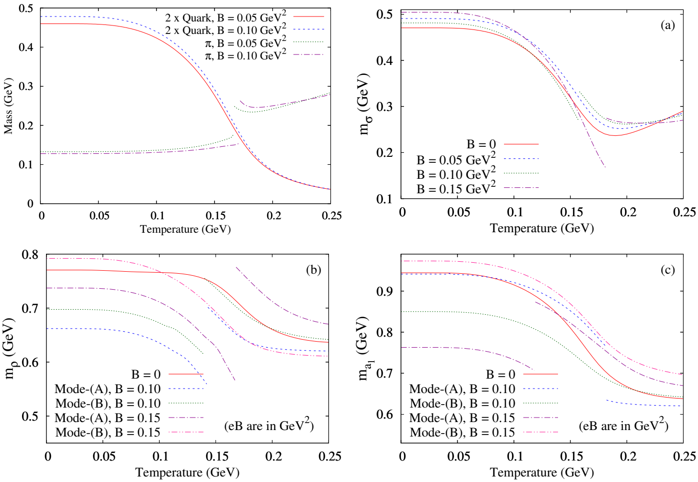

Figure: Variation of masses of (a) π^0, (b) σ, (c) ρ^0 and (d) a_1^0 as a function of temperature for different values of external magnetic field.

Dispersion Curves of π and σ

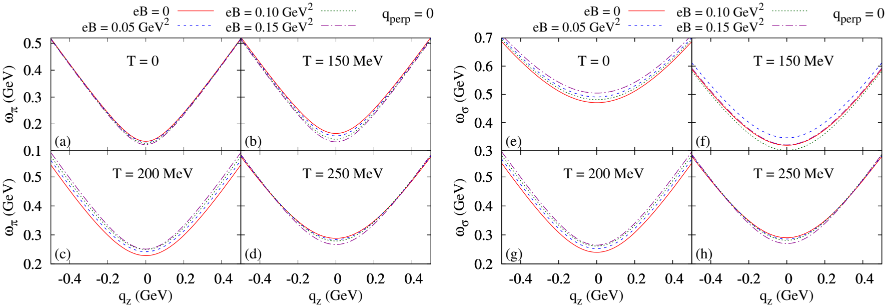

Figure: The dispersion curves of π^0 and σ meson with vanishing transverse momentum ($q_\perp = 0$) for different values of temperature and external magnetic field.

Dispersion Curves of ρ and a_1

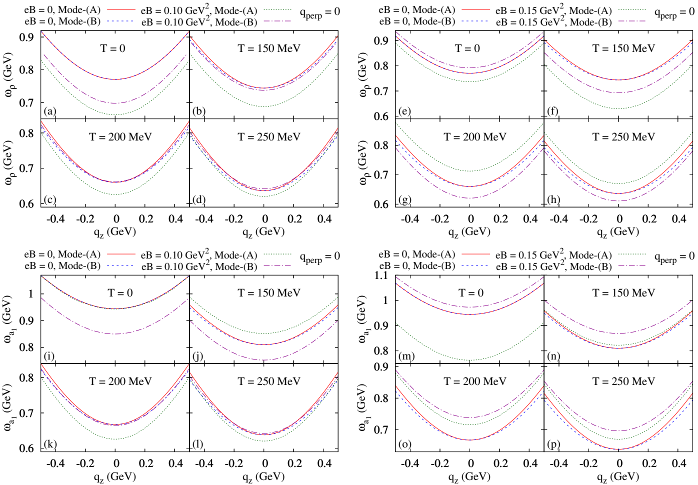

Figure: The dispersion curves of ρ^0 and a_1^0 meson with vanishing transverse momentum ($q_\perp = 0$) for different values of temperature and external magnetic field.

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Thank You