

van der Waals hadron resonance gas and QCD phase diagram

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Based on : N. Sarkar and P. Ghosh Phys. Rev. C 98, 014907 (2018)

Outline

Motivation

Ideal Hadron Resonance Gas Model

Extension of the ideal HRG Model: hadrons interaction

Result

Summary

Motivation

One of the main aim of heavy ion experiments is to study the QCD phase boundary and to search for the possible QCD critical point.

According to the present understanding of the QCD phase diagram in (T, μ_B) -plane, at vanishing μ_B and high temperature, the changes between partonic and hadronic phases occur through a crossover.

First order phase transition takes place at high μ_B and low temperature.

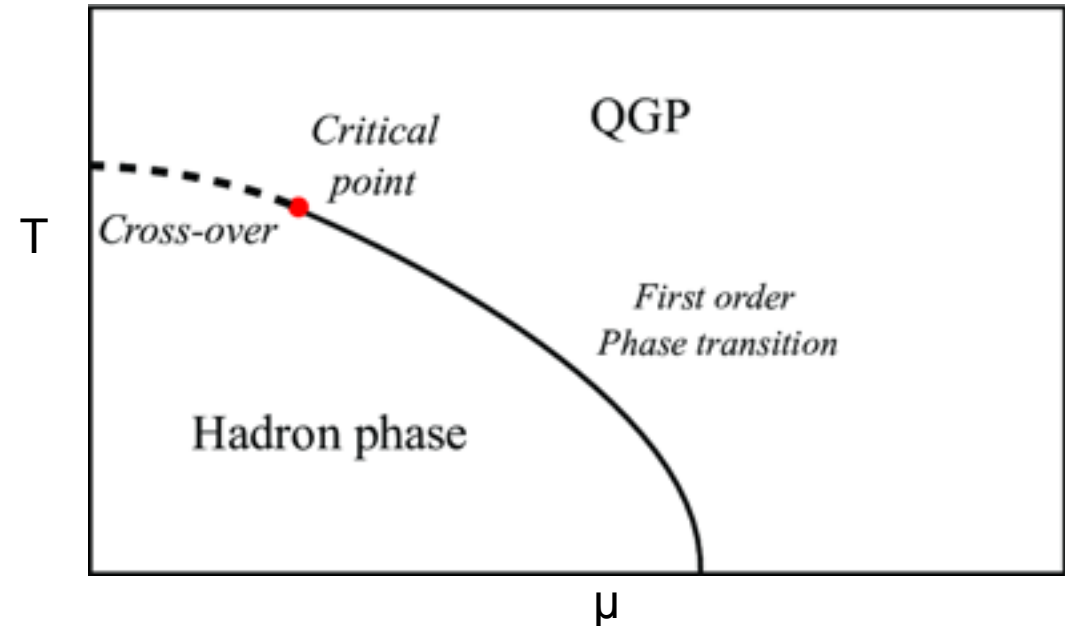
There is a critical point some where in the QCD phase diagram, where the First order phase transition end.

In the absence of ab initio calculations for the baryon-rich QCD matter, one can use a phenomenological EoS for the hadronic phase of QCD matter to study the QCD phase diagram.

A study of hadronic phase of strongly interacting matter with Hadron Resonance Gas Model

● LQCD simulations at finite temperature reveal existence of a deconfined partonic phase at high temperature and a confined hadronic phase at low temperature.

● The hadronic phase of the QCD medium is successfully addressed also by Hadron Resonance Gas (HRG) model.



We consider, hadron resonance gas (HRG) model with van der Waals type of interaction to study the thermodynamic of hadronic phase of the QCD medium and find the probable location of the critical point in (T, μ_B) -plane

The Hadron Resonance Gas Model

The HRG model calculations reproduces the LQCD results on thermodynamic properties of hadronic phase of strongly interacting matter, produced in ultra-relativistic heavy-ion collisions.

The partition function for a grand canonical ensemble, in the simplest form (for non-interacting multicomponent system), in the hadron gas model, is given by:

$$\ln Z^{\text{id}} = \sum_{i=1} \ln Z_i^{\text{id}}$$

“id” stands for ideal, “ i ” indicates the ith Hadron

$$\ln Z_i^{\text{id}} = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 dp \ln \{1 \pm \exp[-(E_i - \mu_i)/T]\}$$

$$E_i = \sqrt{p^2 + m_i^2} \quad \text{and} \quad \mu_i = B_i \mu_B + S_i \mu_s + Q_i \mu_Q.$$

Thermodynamic variables in HRG model

A. Andronic, P. B. Munzinger, J. Stachel, and M. Winn, Phys. Lett. B 718, 80 (2012).

$$P^{id}(T, \mu) = \pm \sum_i \frac{g_i T}{2\pi^2} \int_0^\infty p^2 dp \ln \{1 \pm \exp[-(E_i - \mu_i)/T]\}$$

$$n^{id}(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1}$$

$$\epsilon^{id}(T, \mu) = \sum_i \frac{g_i}{2\pi^2} \int_0^\infty \frac{p^2 dp}{\exp[(E_i - \mu_i)/T] \pm 1} E_i$$

$$s^{id}(T, \mu) = \pm \sum_i \frac{g_i}{2\pi^2} \int_0^\infty p^2 dp [\ln \{1 \pm \exp[-(E_i - \mu_i)/T]\} \pm \frac{(E_i - \mu_i)}{T(\exp[(E_i - \mu_i)/T] \pm 1)}]$$

$$I^{id}(T) = \frac{(\epsilon^{id} - 3P^{id})}{T^4}$$

“T” is the temperature and “μ” is the chemical potential of the system.

“+” and “-” corresponds to fermions and bosons respectively.

Extension of the ideal HRG Model: hadrons interaction

The effect of repulsive interactions at the short distances, are incorporated by Van der Waals Excluded Volume (EV) method.

D. H. Rischke, M. I. Gorenstein, H. Stocker, and W. Greiner Z.Phys.C51,485(1991)

$$P^{EV}(T, \mu_1, \mu_2, \dots) = \sum_i P_i^{id}(T, \bar{\mu}_1, \bar{\mu}_2, \dots)$$

$$\bar{\mu}_i = \mu_i - V_{ev,i} P^{EV}(T, \mu_1, \mu_2, \dots)$$

Where, $V_{ev,i} = \frac{16\pi}{3} r^3$ **r is the hadron radius**

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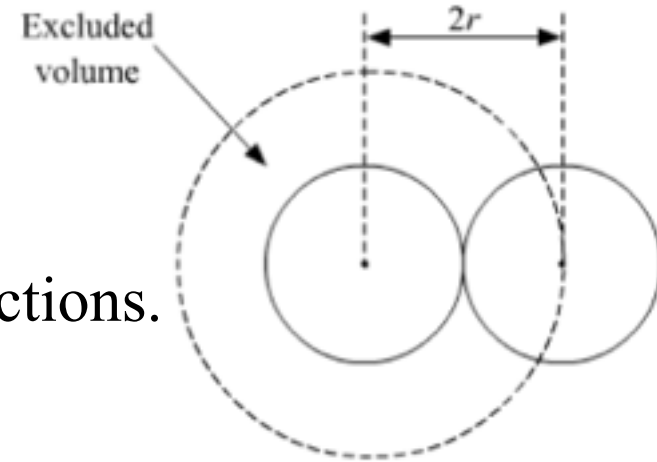
$$n^{EV}(T, \mu_1, \mu_2, \dots) = \frac{\sum_i n^{id}(T, \bar{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \bar{\mu}_k)}$$

$$s^{EV}(T, \mu_1, \mu_2, \dots) = \frac{\sum_i s^{id}(T, \bar{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \bar{\mu}_k)}$$

$$\epsilon^{EV}(T, \mu_1, \mu_2, \dots) = \frac{\sum_i \epsilon^{id}(T, \bar{\mu}_i)}{1 + \sum_k V_{ev,k} n_k^{id}(T, \bar{\mu}_k)}$$

van der Waals hadron resonance gas and QCD phase diagram

$$P(T, V, N) = \frac{NT}{V - bN} - a \frac{N^2}{V^2}$$



- VDW contains **attractive** and **repulsive** interactions.
- Contain **1st order phase transition**.
- Contains **critical point**.

Two ingredients:

1) Short-range repulsion: excluded volume (EV) procedure,

$$V \rightarrow V - bN, b = \frac{16\pi r^3}{3}$$

2) Intermediate range attraction:

$$P \rightarrow P - an^2$$

Hadron resonance gas with **VDW** type of interaction

Grand Canonical formalism of VDW Gas for baryons

Pressure:
$$p(T, \mu) = p^{id}(T, \bar{\mu}) - an^2(T, \mu)$$

Number density:
$$n(T, \mu) = \frac{n^{id}(T, \bar{\mu})}{1 + bn^{id}(T, \bar{\mu})}$$

Energy density:
$$\epsilon(T, \mu) = \frac{\epsilon^{id}(T, \bar{\mu})}{1 + bn^{id}(T, \bar{\mu})} - an^2(T, \mu)$$

Modified chemical potential:

$$\bar{\mu} = \mu - bp(T, \mu) - abn^2(T, \mu) + 2an(T, \mu)$$

Model properties:•

Reduces to ideal HRG-EV for $a=0$.

Reduces to ideal HRG(No interaction) for $a=0$ and $b=0$.

Good agreement with LQCD results on thermodynamic property at

(V.V., M. Gorenstein, H. Stoecker, *Phys. Rev. Lett.* 118, 182301 (2017))

$$\mu_B = 0$$

Transport coefficient: Shear viscosity

Shear viscosity using kinetic theory

$$\eta_{VDW/EV}(T, \mu) = \frac{5}{64\sqrt{8}} \sum_{i \in (M, B, \bar{B})} \frac{\langle |P_i| \rangle n_{VDW/EV,i}(T, \mu_i)}{n_{VDW/EV}(T, \mu) r_i^2}$$

M. Gorenstein, M. Hauer, O. Moroz, Phys. Rev.C77,6024911 (2008)

Average momentum of the i th hadron

$$\langle |P_i| \rangle = \frac{\int_0^\infty \frac{p^3 dp}{\exp[(\sqrt{p^2 + m_i^2} - \bar{\mu}_i)/T] \pm 1}}{\int_0^\infty \frac{p^2 dp}{\exp[(\sqrt{p^2 + m_i^2} - \bar{\mu}_i)/T] \pm 1}}$$

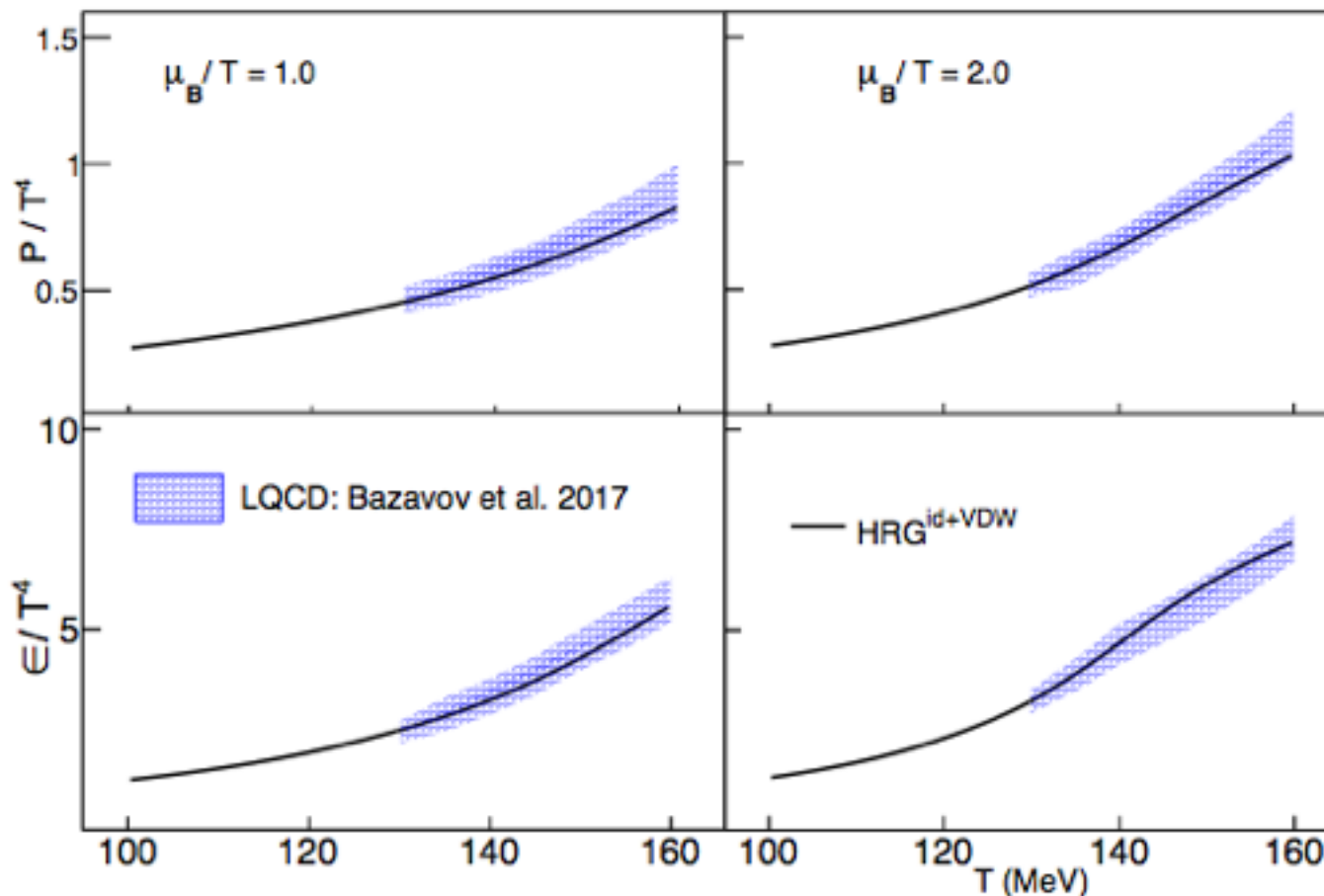
$$\eta(T, \mu) = \eta_M(T, \mu) + \eta_B(T, \mu) + \eta_{\bar{B}}(T, \mu)$$

Hadron resonance gas with **VDW** type of interaction

$$p(T, \mu) = p_M(T, \mu) + p_B(T, \mu) + p_{\bar{B}}(T, \mu)$$

EV Type Interaction

VDW Type Interaction



$$a = 926 \text{ MeV fm}^3$$

$$b = 4.08 \text{ fm}$$

Fit Value of a and b

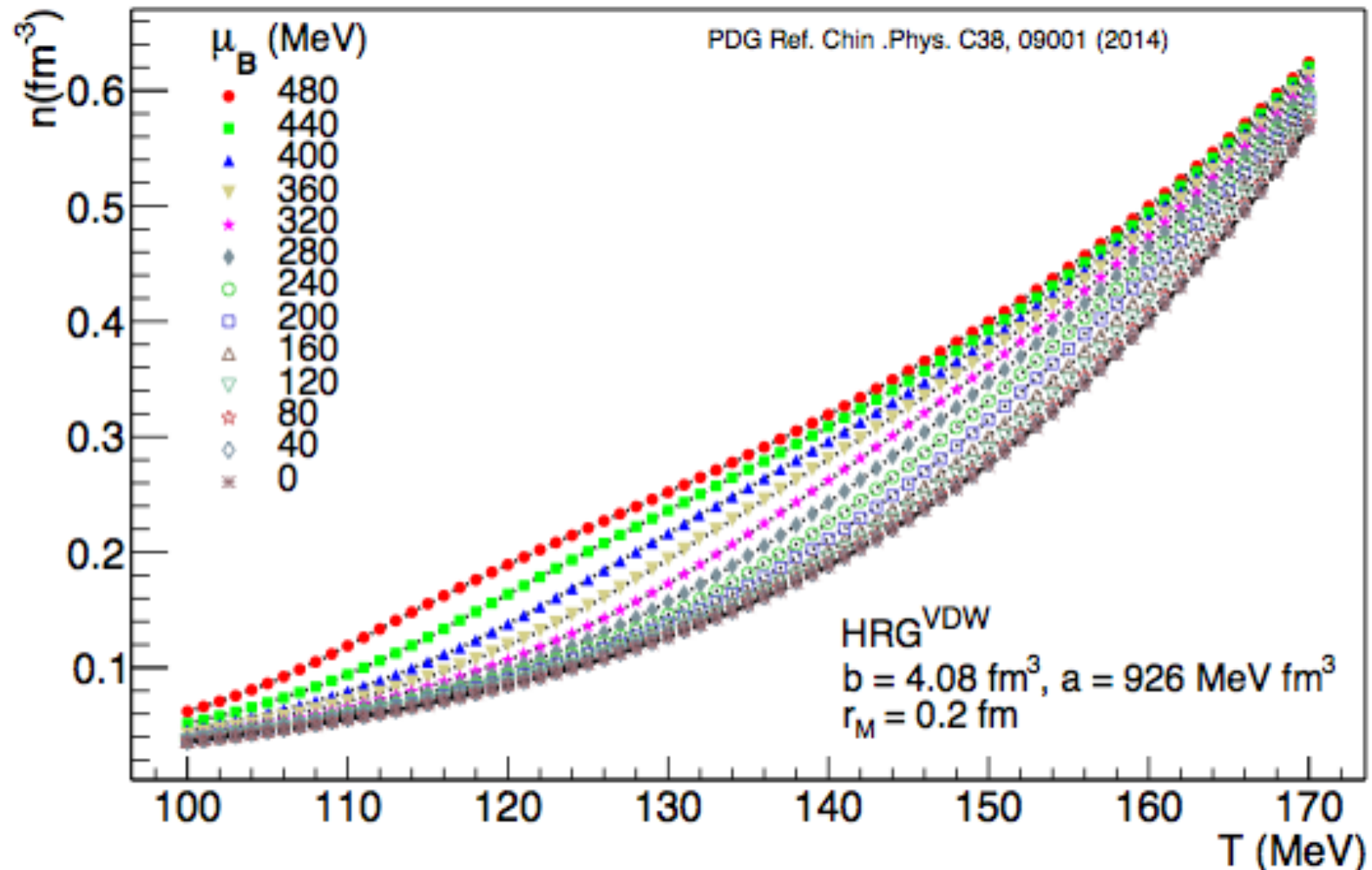
$$r_m = 0.2 \text{ fm}$$

VDW constants (a ,b) were obtained by simultaneous fitting

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Low μ_B and high T region

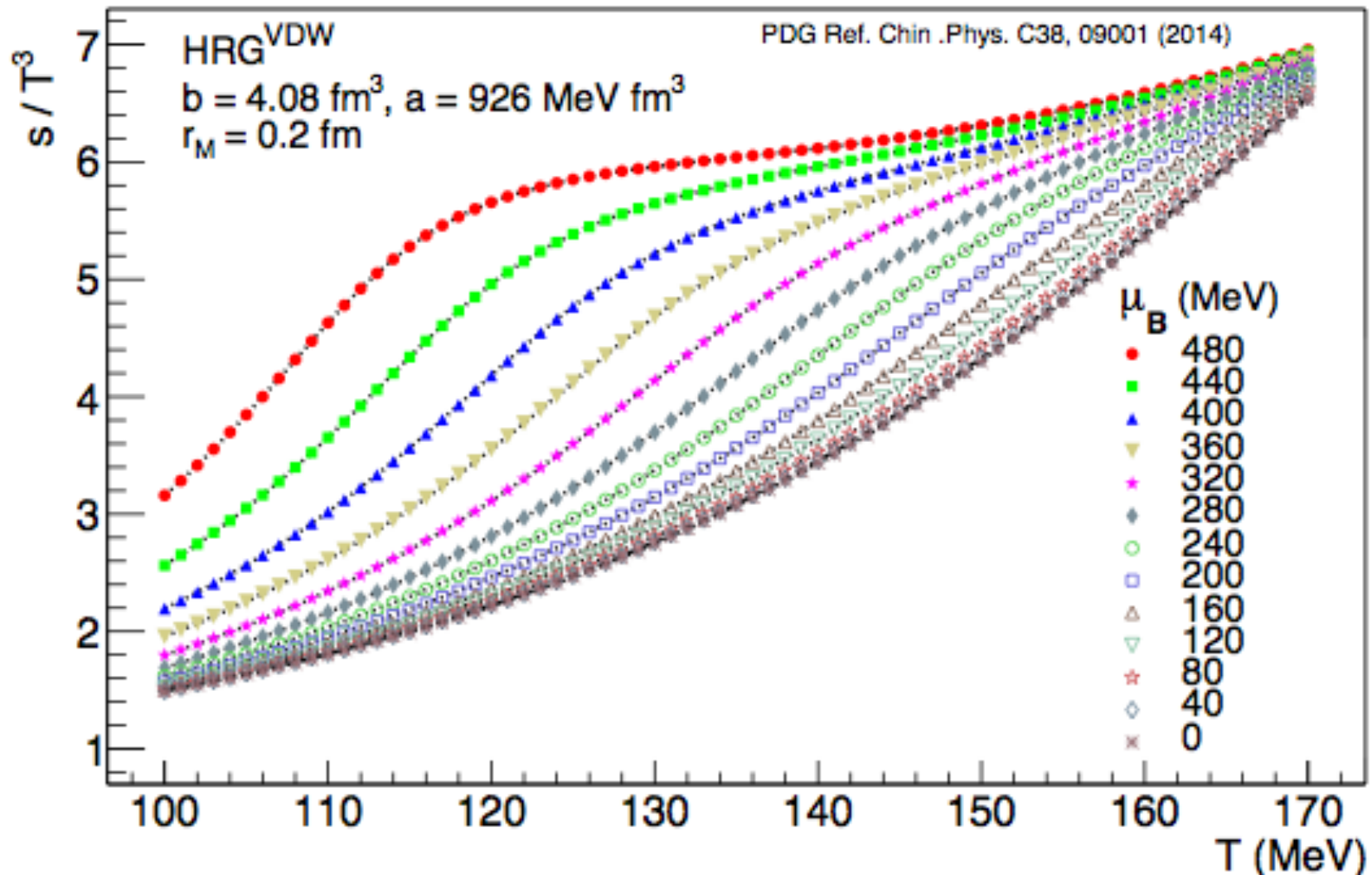
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The temperature dependent number density for hadron resonance gas with VDW form of interactions between (anti)baryons at different μ_B in the range, $\mu_B \sim 0$ to 480 MeV, covering the RHIC-BES program.

Low μ_B and high T region

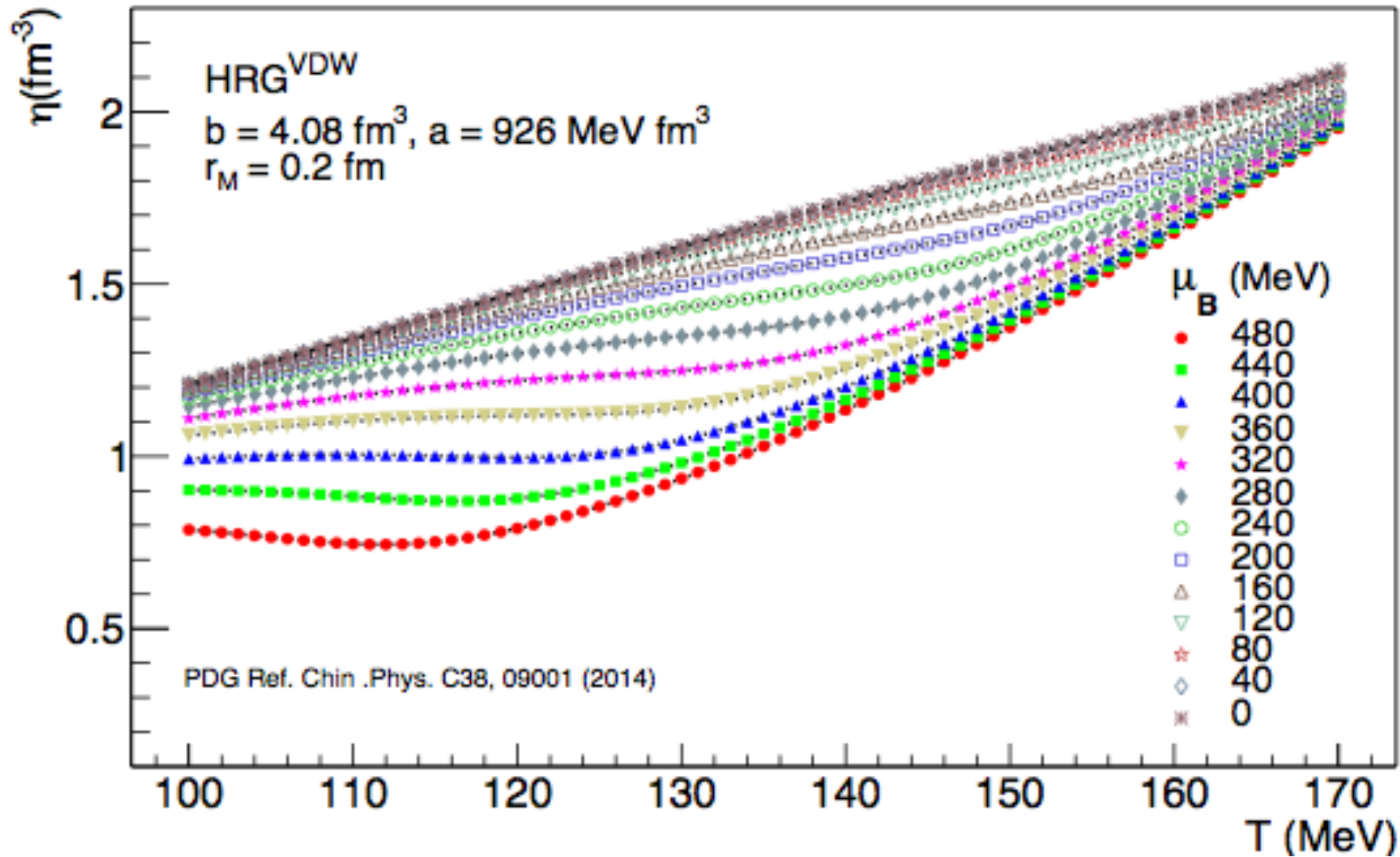
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The temperature dependent s/T^3 density for hadron resonance gas with VDW form of interactions between (anti)baryons at different μ_B in the range, $\mu_B \sim 0$ to 480 MeV, covering the RHIC-BES program.

Low μ_B and high T region

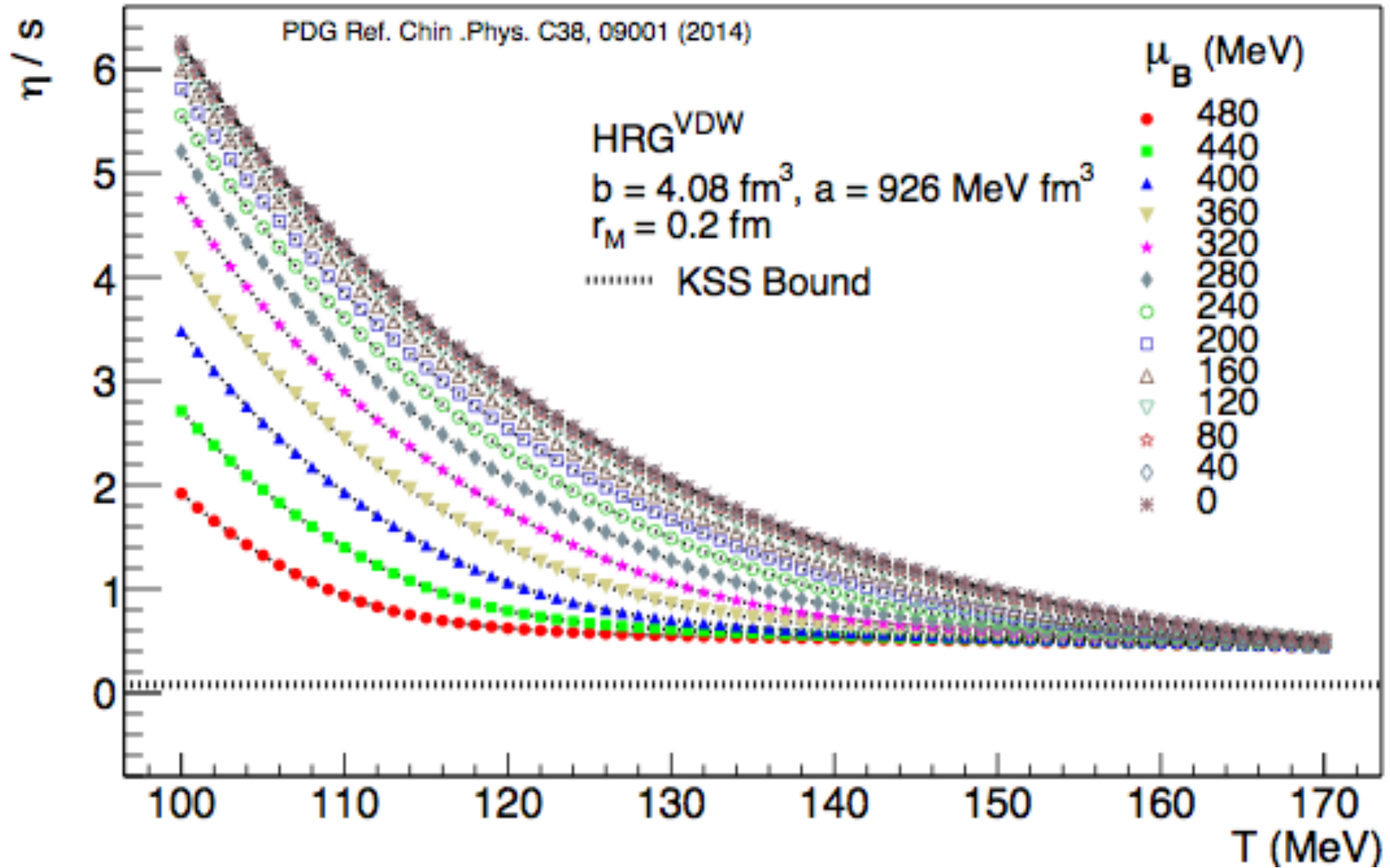
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The temperature dependent transport coefficient μ_B in the range, $\mu_B \sim 0$ to 480 MeV,

Low μ_B and high T region

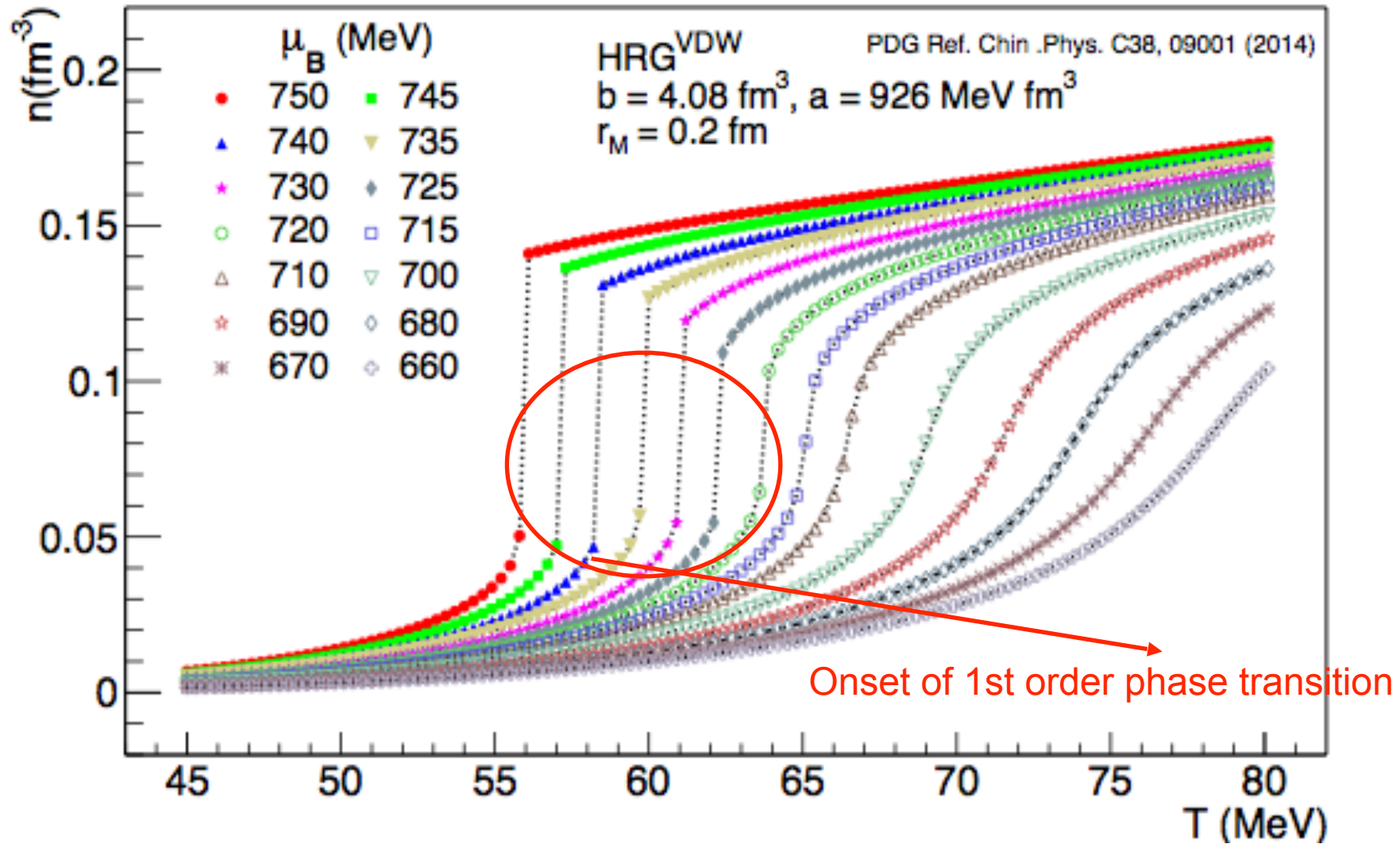
N. Sarkar and P. Ghosh Phys. Rev. C 98, 014907 (2018)



The temperature dependent η/s μ_B in the range, $\mu_B \sim 0$ to 480 MeV,

High μ_B and Low T region

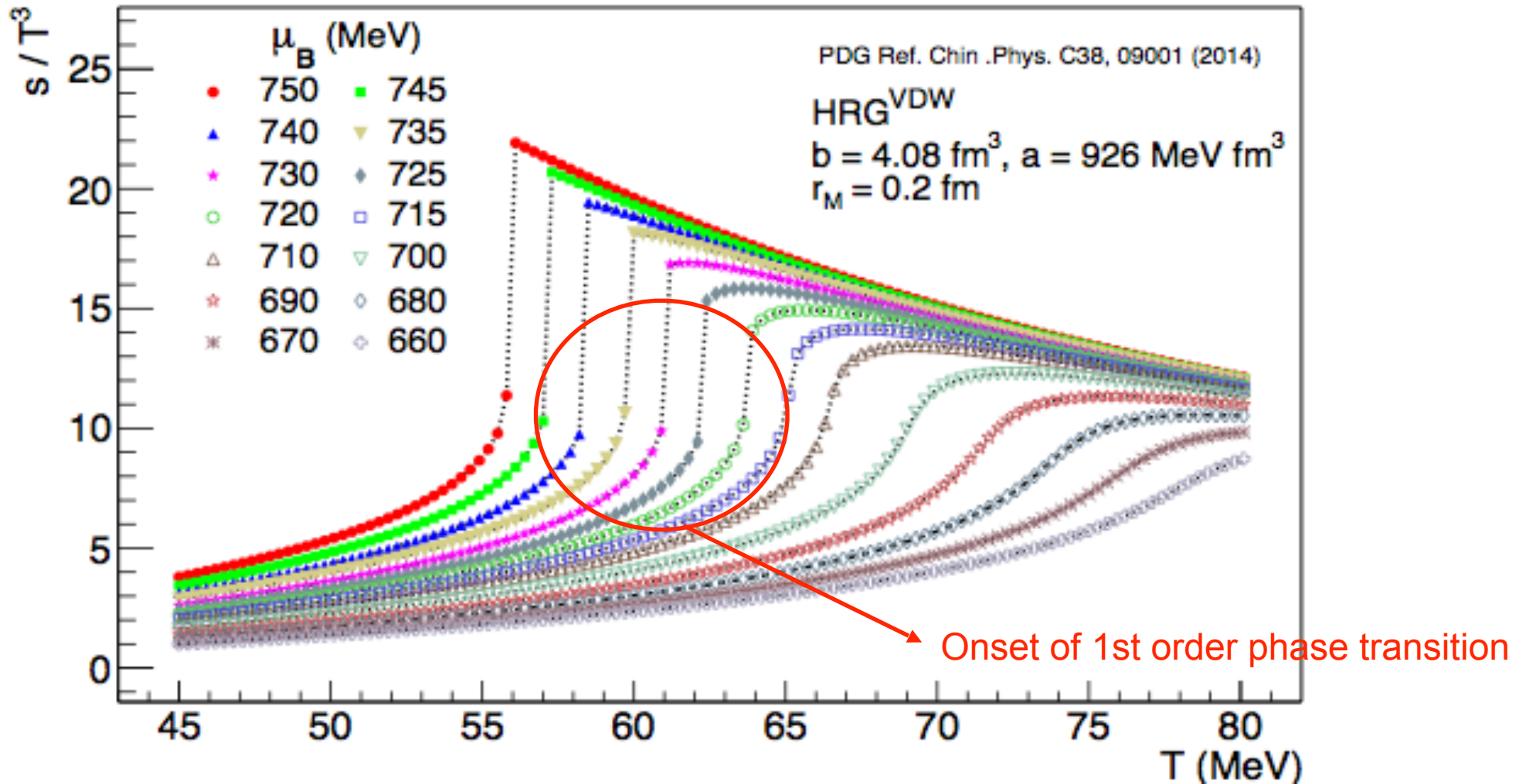
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The temperature dependent number density for hadron resonance gas with VDW form of interactions between (anti)baryons at different μ_B in the range, $\mu_B \sim 660$ to 750 MeV,

High μ_B and Low T region

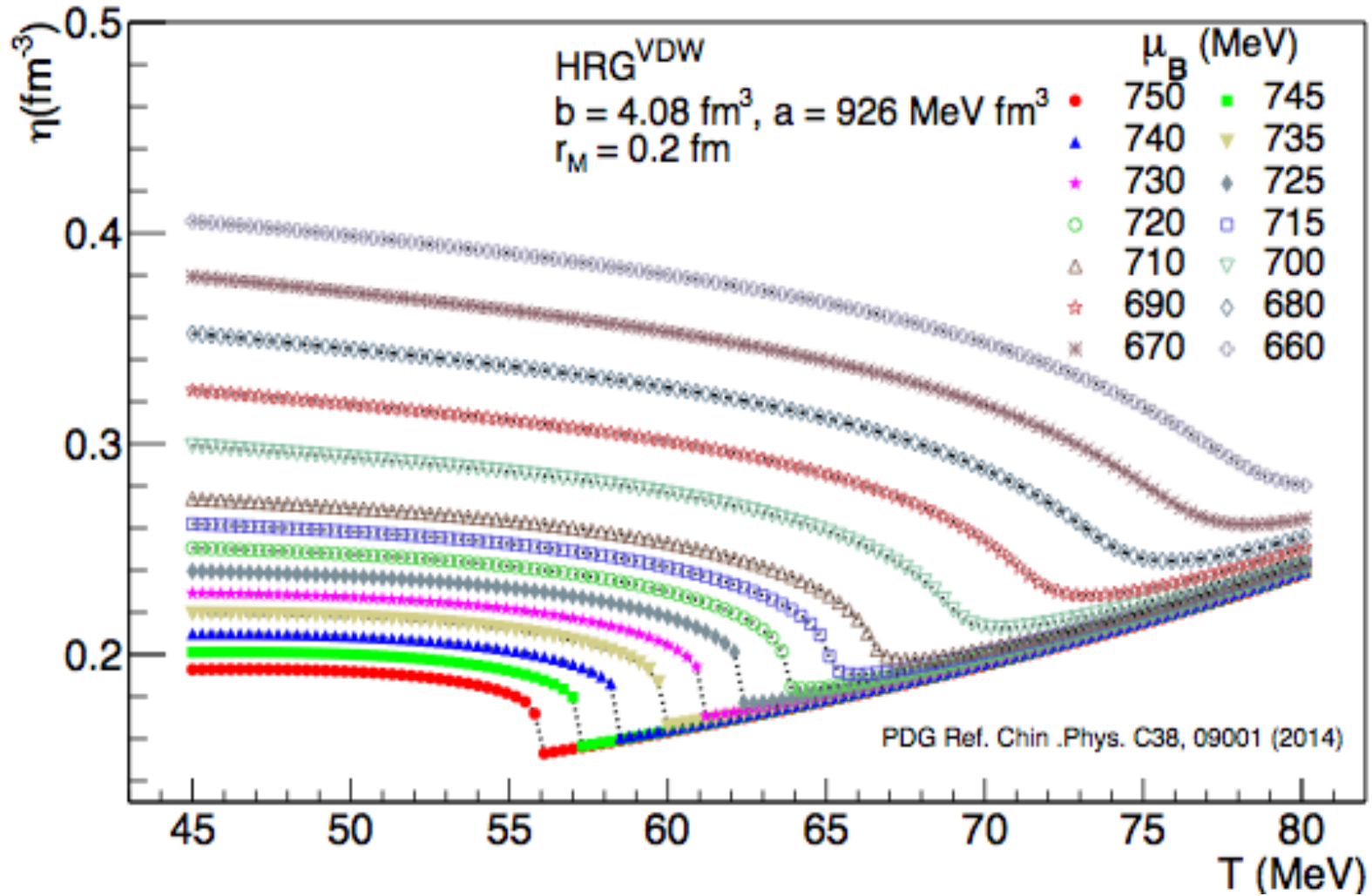
N. Sarkar and P. Ghosh Phys. Rev. C 98, 014907 (2018)



The temperature dependent S/T^3 for hadron resonance gas with VDW form of interactions between (anti)baryons at different μ_B in the range, $\mu_B \sim 660$ to 750 MeV,

High μ_B and Low T region

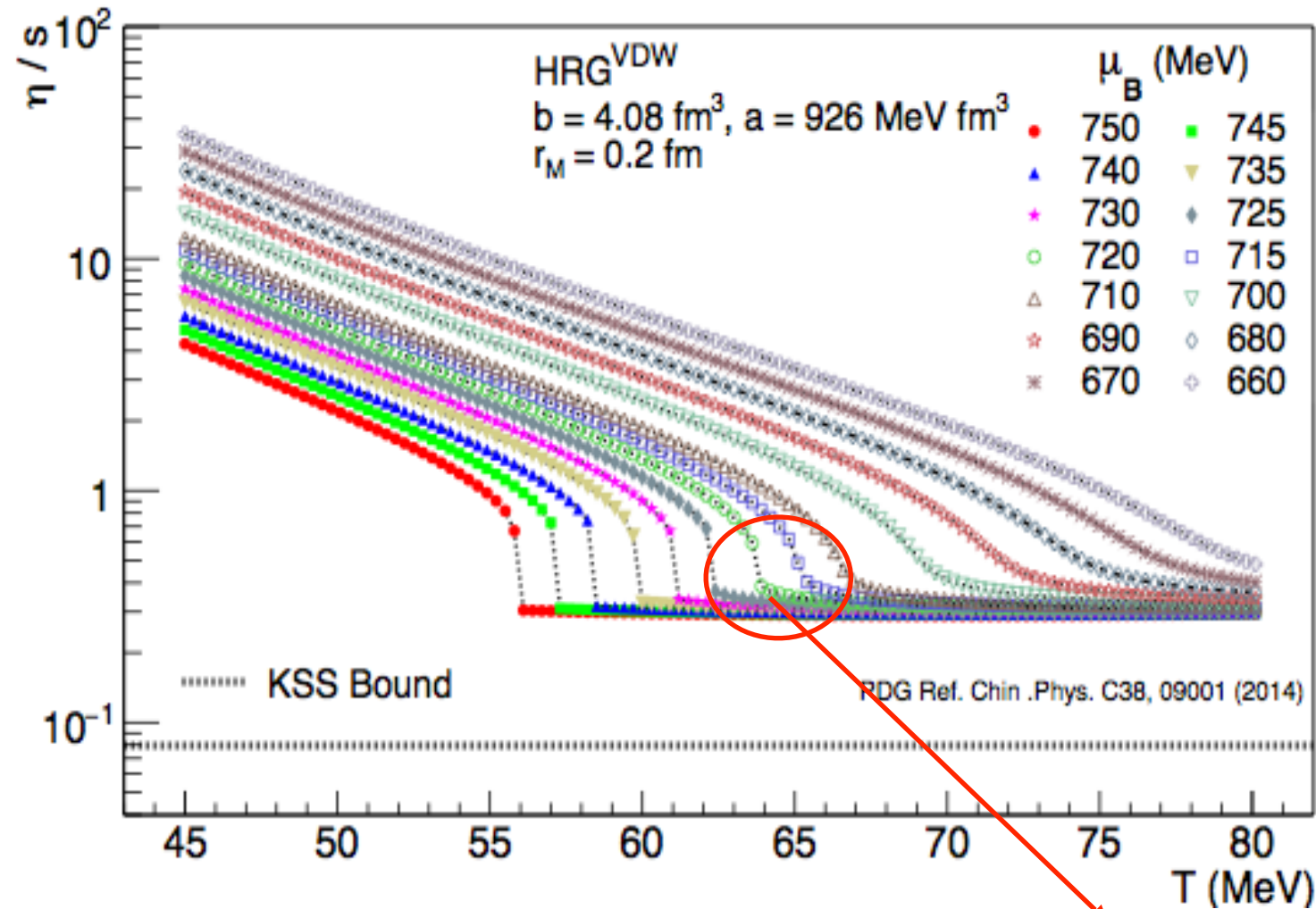
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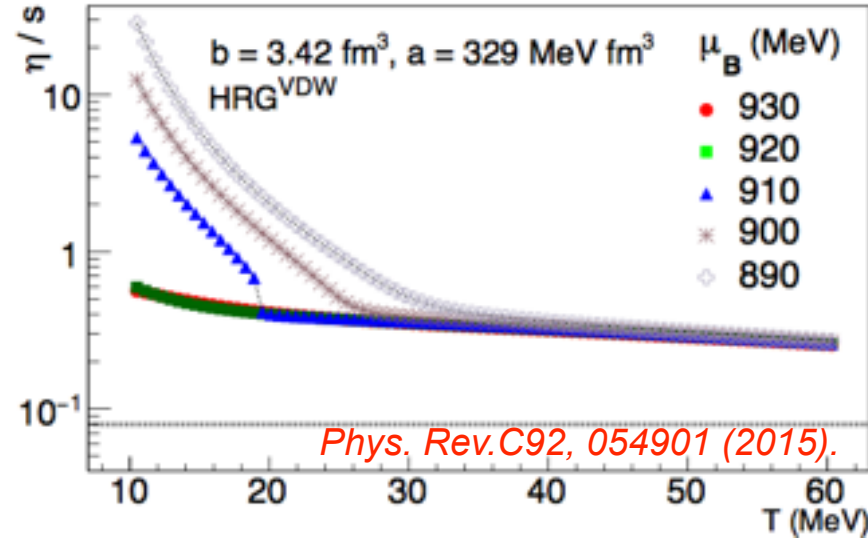
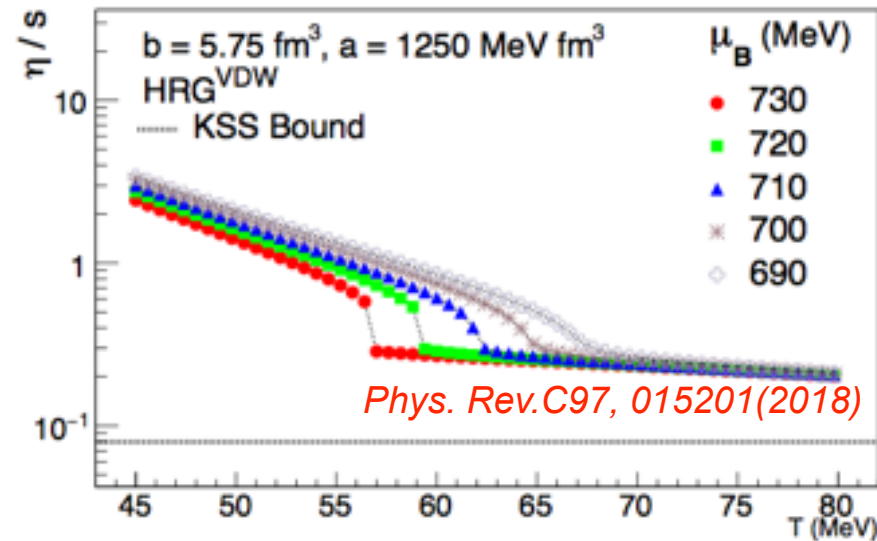


Possible location of the QCD critical point
($T \sim 65 \text{ MeV}$, $\mu_B \sim 715 \text{ MeV}$)

The temperature dependent η/s μ_B in the range, $\mu_B \sim 660$ to 750 MeV ,

High μ_B and Low T region

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The striking similarities of the possible location of critical point in terms of discontinuity in η/s with the previous studies

Summary

We have studied the thermodynamical property and transport coefficients of the hadronic phase of QCD matter using hadron resonance as model

We have Implemented van der Waals (VDW) like the equation of state (EoS) for the grand canonical ensemble of fermions.

The van der Waals coefficients a and b are obtained from simultaneous fitting of temperature-dependent energy and pressure with lattice data.

We find the possible location of the QCD critical point lies around $T \sim 65$ MeV and $\mu_B \sim 715$ MeV.

We found the striking similarities of the possible location of critical point in terms of discontinuity in η/s with the previous studies

Implementing system size with Multiple Reflection Expansion (MRE)

MRE formalism modifies the density of states for finite spherical droplet

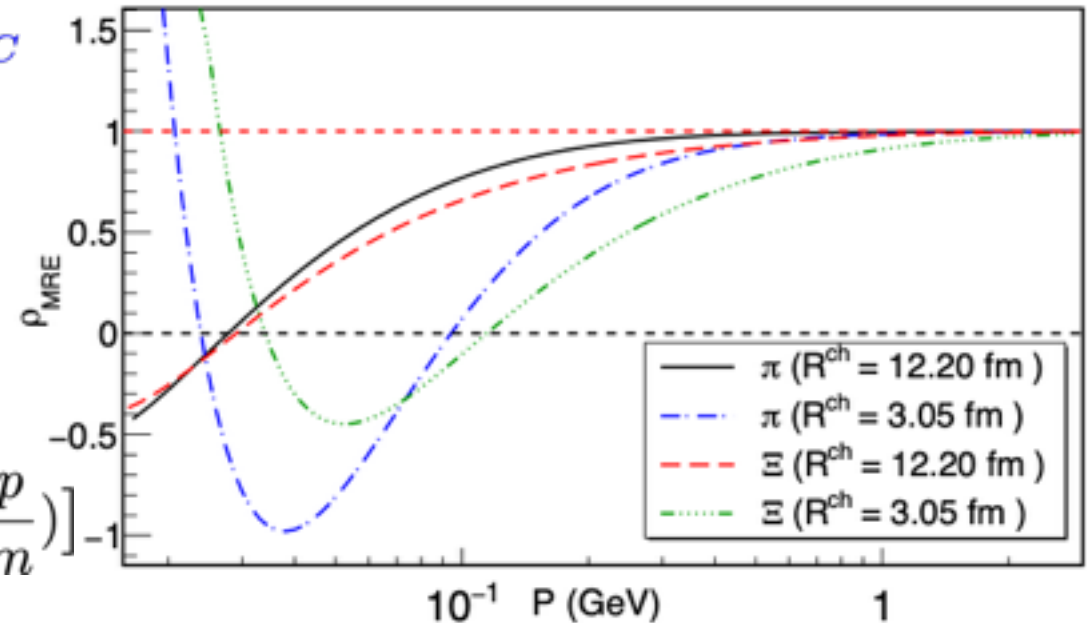
$$\rho_{MRE}(p, m, R) = 1 + \frac{6\pi^2}{pR} f_S + \frac{12\pi^2}{(pR)^2} f_C$$

Surface Term

$$f_S(m, p) = -\frac{1}{8\pi} \left(1 - \frac{2}{\pi} \arctan \frac{p}{m} \right)$$

Curvature Term

$$f_C(m, p) = \frac{1}{12\pi^2} \left[1 - \frac{3p}{2m} \left(\frac{2}{\pi} - \arctan \frac{p}{m} \right) \right]$$

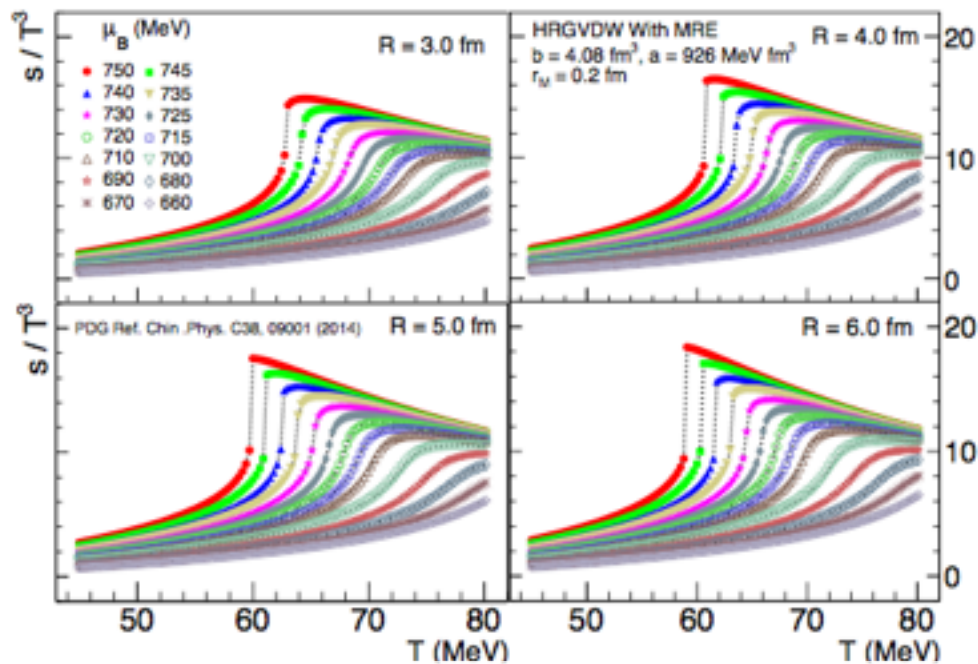


Modification of momentum integral with MRE

$\Lambda_{MRE}(m, R)$ is largest solution for momentum of $\rho_{MRE}(p, m, R) = 0$

$$\int_0^\infty \cdots p^2 dp \rightarrow \int_{\Lambda_{MRE}(m, R)}^\infty \cdots \rho_{MRE} p^2 dp$$

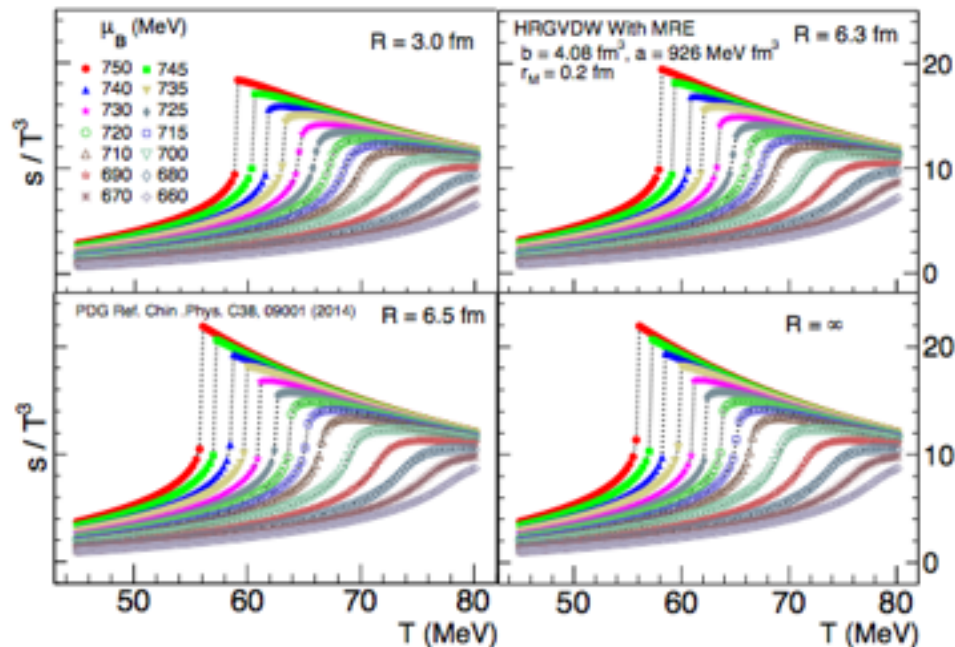
Effect of system size on probable location of critical point



With decreasing system size critical point shifted to the higher μ_B and low temperature region.

Above 6.5 fm, there is no effect of system size on the location of critical point.

System of radius less than 6.5 f(approximately) is not in the thermodynamic limit.



To search for the reliable location of critical point the size of a thermodynamic system at chemical freeze-out should be more than $\sim 6.5 \text{ fm}$ (approximately).