Latest updates on the QCD phase diagram from lattice

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Outline

1. The QCD phase diagram: Latest results
2. Critical-end point search from lattice

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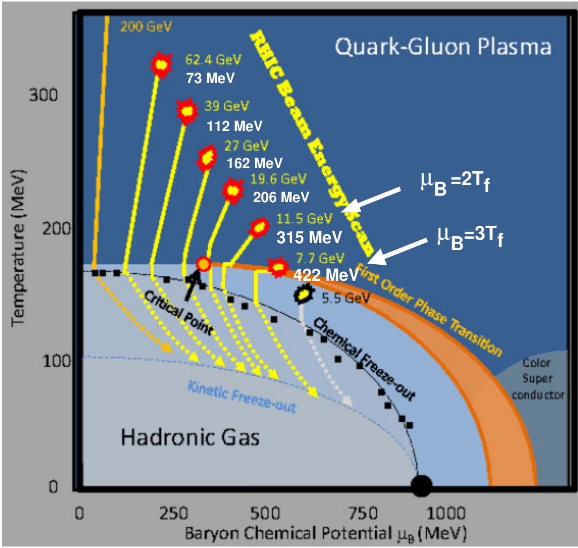
Necessary inputs from Lattice QCD



[http://www.bnl.gov/rhic/news]

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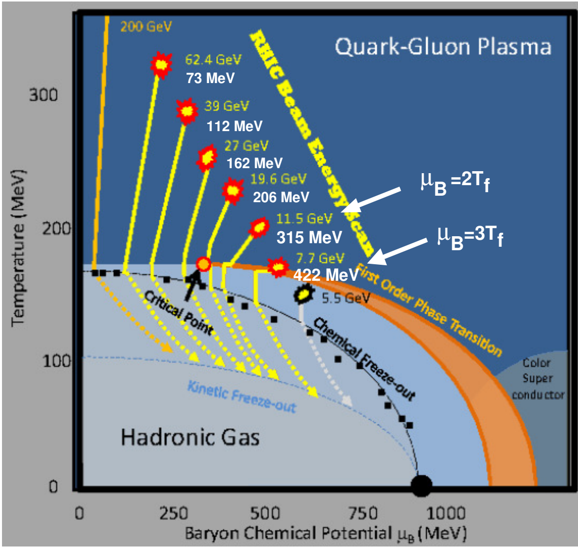

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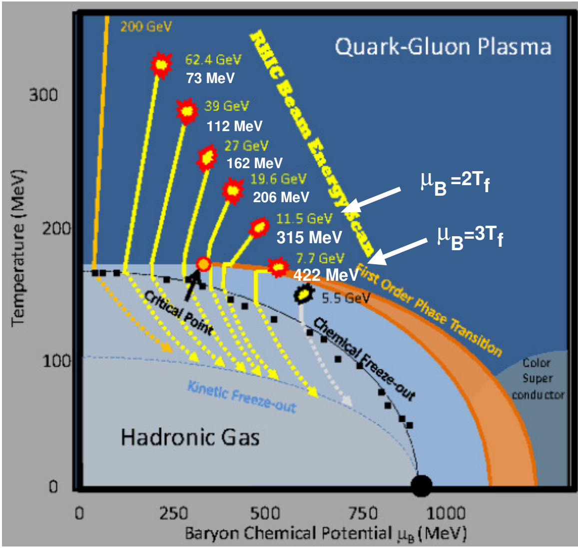

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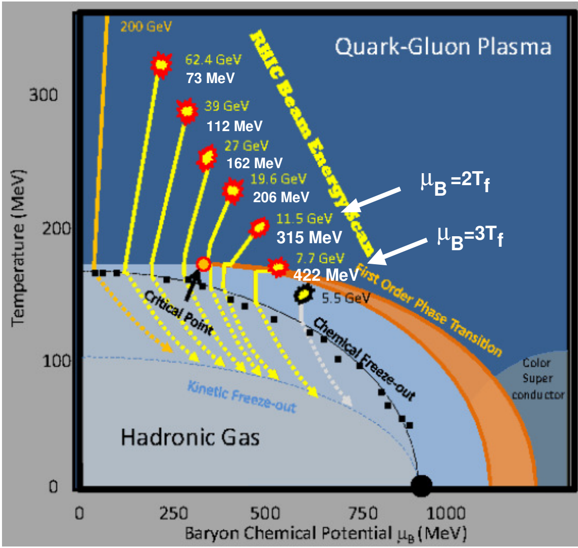

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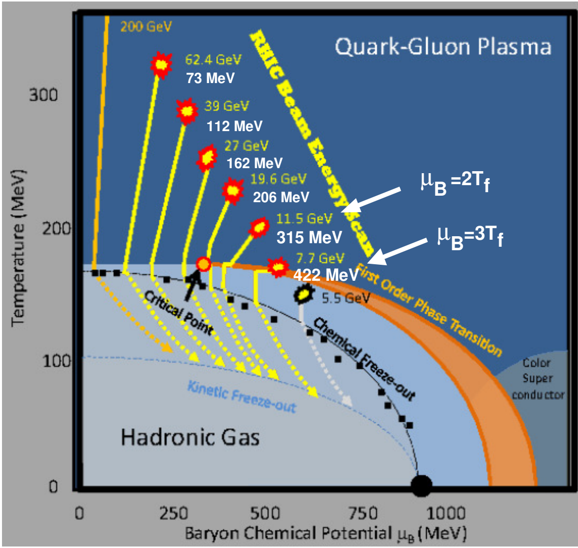
- For RHIC Beam Energy Scan-II: Equation of State for $\mu_B/T \leq 3$.
- Measure the curvature of chiral crossover line.
- Look for possible existence of critical end-point in the phase diagram.
- Relevant degrees of freedom of the phases?



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Lattice techniques at finite μ_B-I

- Conventional Monte-Carlo methods suffer from sign problem at finite μ_q.
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- Fitting it to a polynomial in μ_q analytically continue in the real-μ plane.
- Limited due to discontinuities at Roberge-Weiss end-points!.

Lattice techniques at finite μ_B-II

- Taylor expansion of physical observables around $\mu = 0$ in powers of μ/T [Bi-Swansea collaboration, 02]

$$\frac{P(\mu_B, T)}{T^4} = \frac{P(0, T)}{T^4} + \left(\frac{\mu_B}{T}\right)^2 \underbrace{\frac{\chi_2^B(0, T)}{2T^2}}_{\mathbf{P_2}} + \left(\frac{\mu_B}{T}\right)^4 \underbrace{\frac{\chi_4^B(0)}{4!}}_{\mathbf{P_4}} + \ldots$$

- The series for $\chi_2^B(\mu_B)$ should diverge at the critical point. On finite lattice χ_2^B peaks, ratios of Taylor coefficients equal, indep. of volume [Gavai& Gupta, 03]

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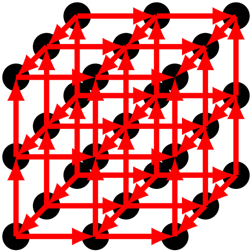
- Why extending to higher orders so difficult?
  - Matrix inversions increasing with the order
  - Delicate cancellation between a large number of terms for higher order QNS.
- A new method of introducing μ developed makes it easier to access higher order QNS. [Gavai & Sharma, 15, De Forcrand & Jaeger, 18]

Our Set-up

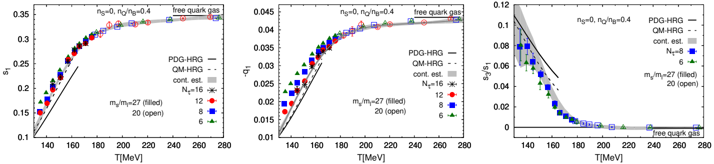


- $V = N^3 a^3$, Box size: $m_\pi \; V^{1/3} > 4$.
- $T = \frac{1}{N_\tau a}$
  We use $N_\tau = 6, 8, 12, 16$ lattices for $\chi_{2,4}$ and $N_\tau = 6, 8$ for higher order fluctuations.
- Input m_s physical and $m_\pi^G = 160$ MeV for $T > 175$ MeV and $m_\pi^G = 140$ MeV for $T <= 175$ MeV.

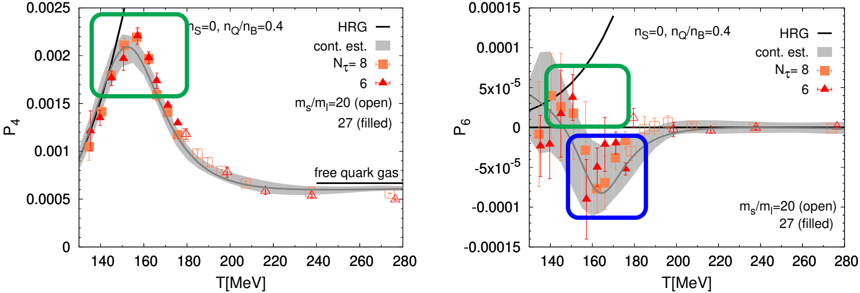
EoS in the constrained case

- In most central heavy-ion experiments typically:
  $n_S = 0$, Strangeness neutrality,
  $\frac{n_Q}{n_B} = \frac{n_P}{n_P + n_N} = 0.4$.
  [Bi-BNL collaboration, 1208.1220]
- For lower $\sqrt{s}$ collisions: Need to understand baryon stopping!
- Imposes non-trivial constraints on the variation of μ_S and μ_Q.
- Possible to vary them by only varying μ_B through

$$\mu_S = s_1 \mu_B + s_3 \mu_B^3 + s_5 \mu_B^5 + \ldots.$$
$$\mu_Q = q_1 \mu_B + q_3 \mu_B^3 + q_5 \mu_B^5 + \ldots.$$

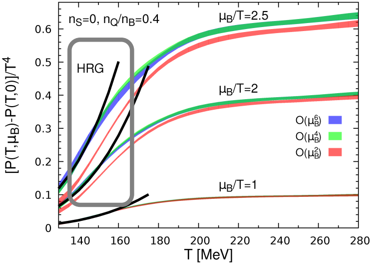


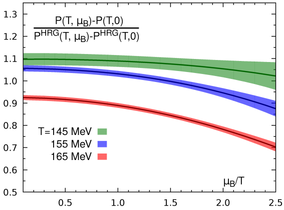
- Central values of P_4, P_6 already deviate from Hadron Resonance gas model at $T > 145$ MeV $\rightarrow$ need to analyze the errors on P_6 better.
- P_6 has characteristic structure at $T > T_c$ $\rightarrow$ remnant of the chiral symmetry due to the light quarks. Effects of $U_A(1)$ anomaly?
- Essentially non-perturbative $\rightarrow$ cannot be predicted within Hard Thermal Loop perturbation theory.



EoS in the constrained case

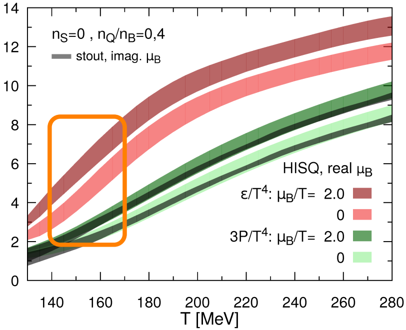
- The EoS is well under control for $\mu_B/T \sim 2.5$ with χ_6.
- Full parametric dependence for N_B on T available in arxiv: 1701.04325.
- Expanding to $\mu_B/T = 3$, need to calculate χ_8!

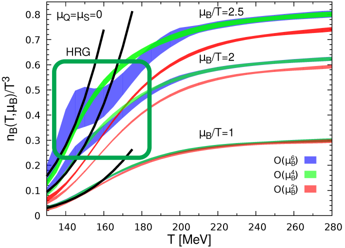




Summary for the EoS

- Continuum estimates from two different fermion discretization agree for $\mu_B/T \leq 2$.
  [Bielefeld-BNL-CCNU collaboration, 1701.04325, Borsanyi et. al, 1606.07494].
- Steeper EoS for RHIC energies compared to LHC energy.

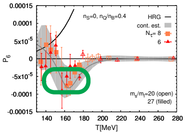


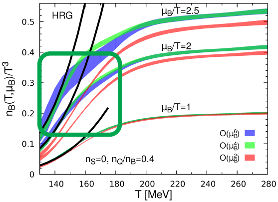


- χ_6 contribution is 30-times larger than in pressure.

$$\frac{N(\mu_B)}{T^3} = \frac{\mu_B}{T}\chi_2^B(0) + \frac{1}{2}\left(\frac{\mu_B}{T}\right)^4 \chi_4^B(0) + \frac{1}{4!}\left(\frac{\mu_B}{T}\right)^6 \chi_6^B(0) + \ldots$$

- Strongly sensitive to the singular part of χ_6^B.





- For strangeness neutral system, effect is milder.

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- Initially believed that thermodynamic properties of QCD can be very well described by a non-interacting gas of hadrons+resonances.

  [Dashen, Ma and Bernstein, 69,71]

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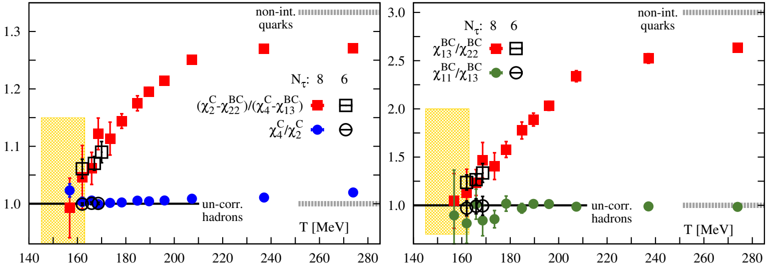
  [Dashen, Ma and Bernstein, 69,71]

- With very precise lattice data we now know HRG description breaks down much below T_c!. [A. Bazavov et. al. HotQCD coll. 1404.6511]

Why should naive HRG description break down?

- There may be many more baryon states and resonances than currently measured especially in the strangeness and charm sectors.

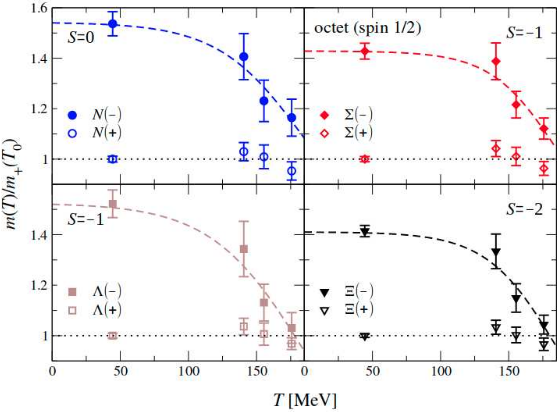
[A. Bazavov et. al. HotQCD coll. 1404.6511].



Why should naive HRG description break down?

- All baryon channels do not have resonant interactions.
- In-medium modification of baryon masses ?

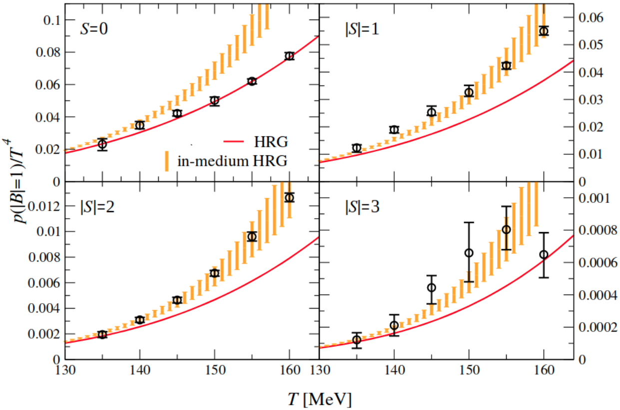
  [G. Aarts et. al., 1812.07393].



Why should naive HRG description break down?

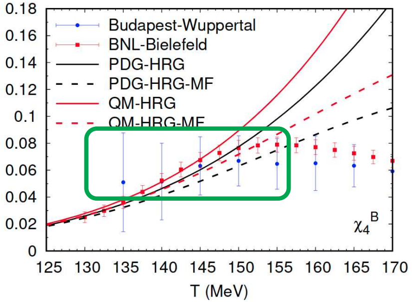
- Thermal width of the resonances?

  [G. Aarts et. al., 1812.07393]



Can $T < T_c$ be described by Hadron Resonance Gas?

- Higher order fluctuations of conserved charges are more sensitive to the departure from HRG.
- Repulsive baryon interactions?
- Lattice data for higher order baryon no. fluc. are precise enough to distinguish between diff. scenarios → support additional resonances from quark-models+interactions



[P. Huovinen, P. Petreczky, 1811.09330]

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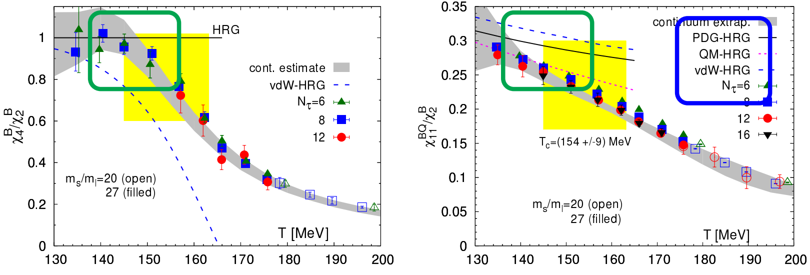


[F. Karsch, QM17 proceedings, 1706.01620]

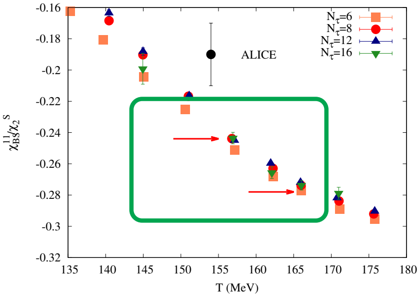
- Including Van der Waal's interaction for baryons+non-interacting mesons+resonances, new versions of HRG has been studied $\rightarrow$ significant deviation from non-interacting HRG.

  [V. Vovchenko, M. I. Gorenstein and H. Stoecker 1609.03975]

- Lattice data can constrain such models strongly!
  Currently none of these models are perfect to describe QCD at freezeout.

Cross-correlations

- Off-diagonal fluctuations are more sensitive to deviation from HRG and baryon interactions.



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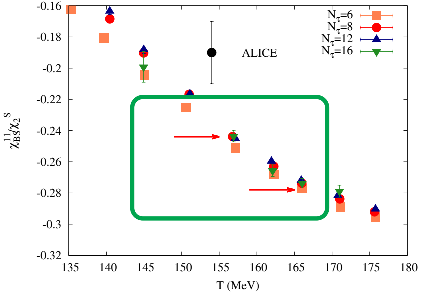

- χ^{BS}_{11}/χ^S_2 shows $\sim 15\%$ deviation between 155 and 165 MeV. Analysis with ALICE [A. Andronic et. al., 16] consistent with lattice at $T_c \sim 155$ MeV. Including $\Sigma^* \to N\bar{K}$ will make the ratio lower!

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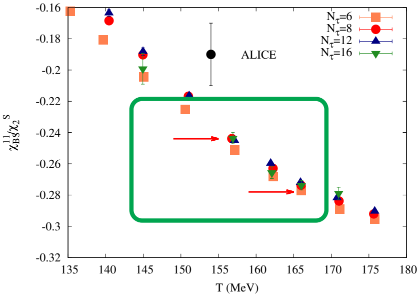

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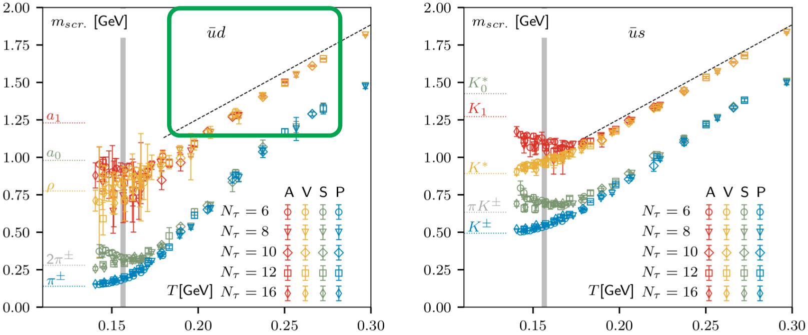
- Similar results at higher μ_B would be interesting! [A. Chatterjee et. al.,STAR collaboration, 2019]

How perturbative is the QGP medium at high T?

- Screening masses show how perturbative is the medium $\rightarrow$ less IR sensitive, more perturbative than gluonic observables.

$$C(z) = \int_0^{1/T} d\tau dxdy \langle \mathcal{O}^\dagger(x,y,z,\tau)\mathcal{O}(0,0,0,0)\rangle \sim \mathrm{e}^{-\mathrm{m}_{\mathcal{O}}\mathrm{z}}, \mathrm{z} \rightarrow \infty,$$



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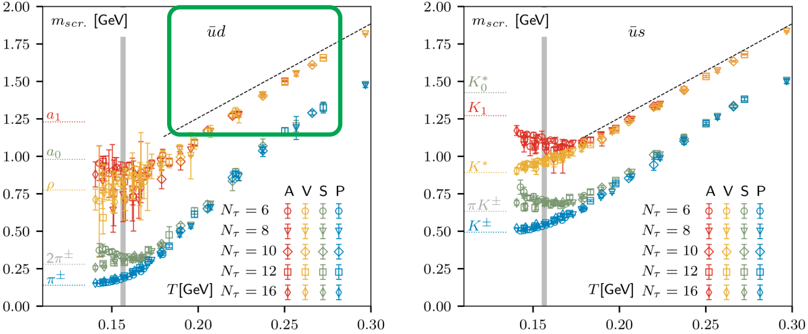
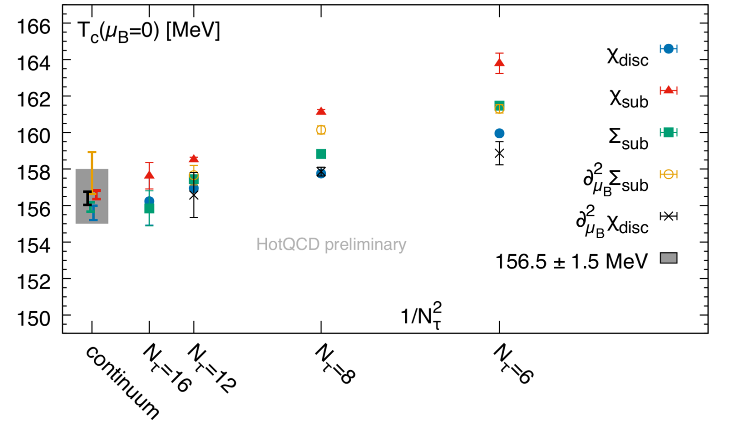
- Vector like excitations $\mathcal{O} = \bar{\psi}\gamma_\mu\psi$ reach the perturbative estimate quickly than pseudo-scalar excitations $M/T = 2\pi + \frac{g^2 C_F}{2\pi}(E_0 + 1/2)$ [HotQCD coll., 2019].



Outline

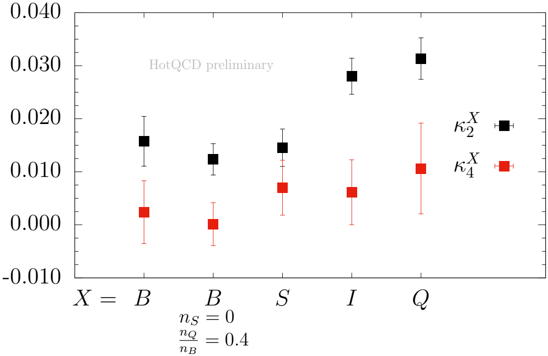
Curvature of the chiral crossover line

- Since $m_{u,d} << \Lambda_{QCD}$ the $SU_L(2) \times SU_R(2)$ is a near exact symmetry of $2+1$ flavor QCD.
- Though not strictly a phase transition, however all chiral observables show observable changes at a certain temperature. It thus makes sense to talk about a precise $T_c^{pc} = 156.5 \pm 1.5$. [HotQCD collaboration, 1812.08235]



Curvature of the chiral crossover line

- $\frac{T_c(\mu_X)}{T_c(0)} = 1 - \kappa_2^X \frac{\mu_X^2}{T_c(0)^2} - \kappa_4^X \frac{\mu_X^4}{T_c(0)^4}$
- For strangess neutral system, continuum results available!
  $\kappa_2^B = 0.012(4)$, $\kappa_4^B \sim 0$ with Taylor expansions and HISQ fermions.
  [HotQCD collaboration, 1812.08235]



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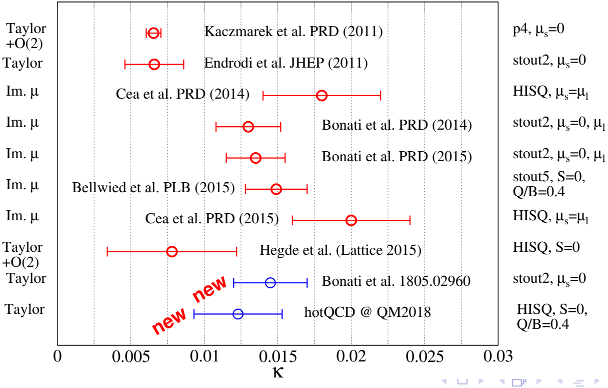
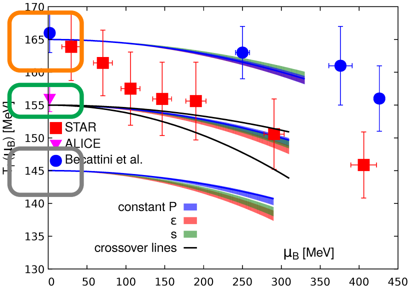
- Consistent with imaginary chemical potential method and stout fermions $\kappa_2^B = 0.0135(20)$ [C. Bonati et. al., 1805.02960]
- removes earlier tension between two methods! [courtesy M. D'Elia Quark Matter 18]



Curvature of freeze-out line vs chiral crossover line

- Different LCP's agree within 2 MeV for $\mu_B/T \leq 2$ for 3 initial choices of T_0.
- For lines $P = \text{const}$, the entropy density changes by 15% $\rightarrow$ better description of LCP for viscous medium formed in heavy-ion collisions.

  [HotQCD collaboration, 1701.04325].



- STAR results give a steeper curvature. [arXiv:1412.0499].
- Agreement with the recent ALICE results. [arXiv:1408.6403].
- Consistent with phenomenological models. [Becattini et. al., 1605.09694].

Critical-end point search from Lattice

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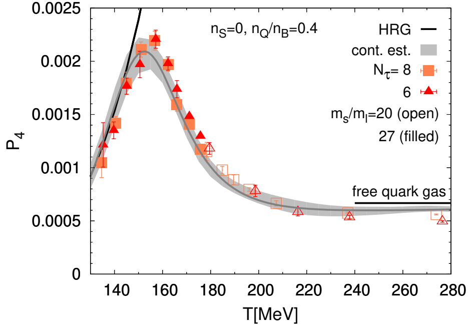
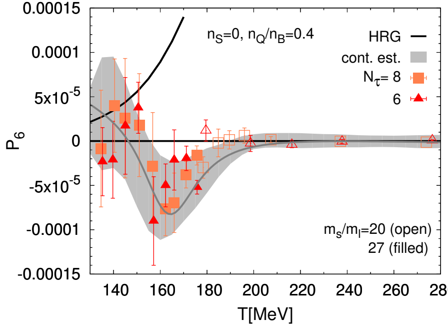
- Signal to noise ratio deteriorates for higher order χ_n^B.

Critical-end point search from Lattice

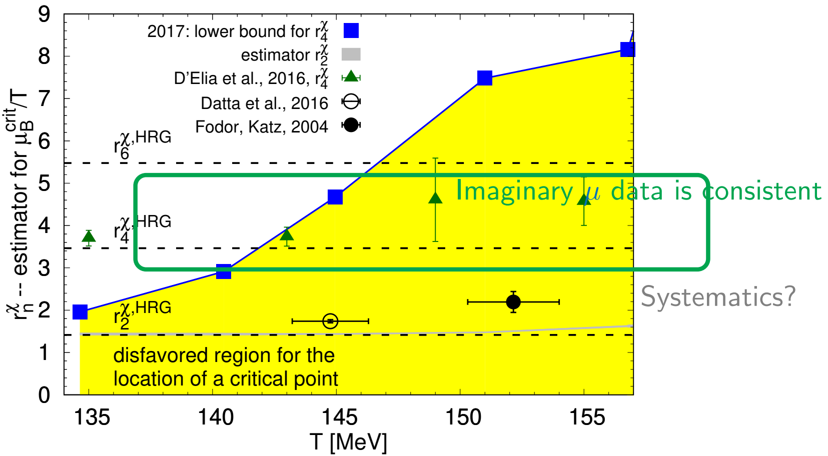
All higher order fluctuations are positive for T $\sim$ 140 MeV.



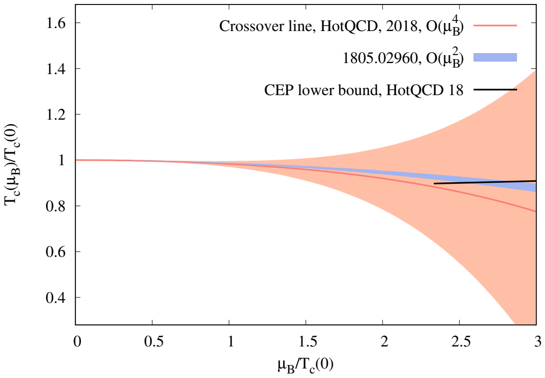


Critical-end point search from Lattice

- Current bound for CEP: $\mu_B/T > 3$ for $142 \leq T \leq 150$ MeV [HotQCD coll., 1701.04325, update 2018].
- Ultimately all estimates will agree in the continuum limit!

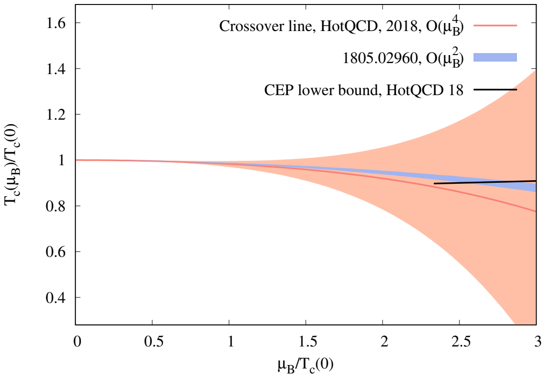


Systematics?



Steeper curvature would imply slow convergence of r_n with order n

Critical end-point and Chiral Crossover line: current status



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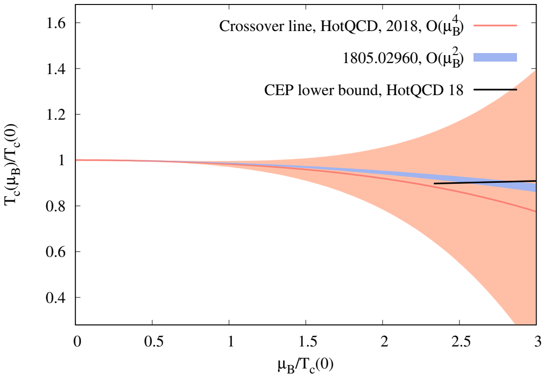

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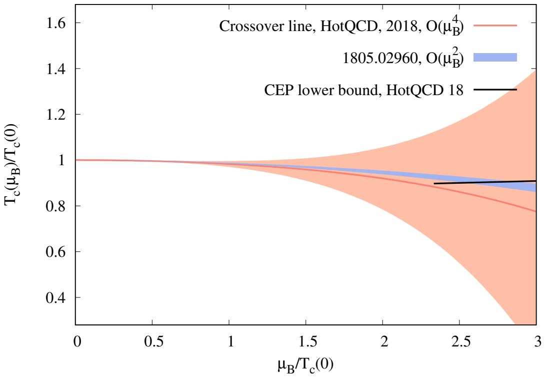

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- radius of curvature estimates tell us $T_{CEP} \sim 0.92\, T_c(0)$ and $\mu_B / T_{CEP} > 3$.
- If $\kappa_4 \sim 0.1\kappa_2$, only significantly contributes when $\mu_B / T_{CEP} > 3$ so its precise determination is imp.

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Outlook

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- Lines of constant ϵ, p consistent with LQCD estimates of curvature of chiral crossover line.
- Higher order cumulants of baryon no. will also help in bracketing the possible CEP. Recent LQCD calculations suggest $\mu_B(\text{CEP})/T \geq 3\ ,\ \ T \sim 0.9 T_c$.