Systematic uncertainties of chemical freeze-out

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Evolution of the fireball

Experimental data accumulated from the relativistic heavy-ion collisions indicate formation of thermally equilibrated quark gluon matter.

Expansion and cooling

The system would most likely make a transition from the partonic phase to the hadronic phase in thermal and chemical equilibrium.

Inter-particle distance increase

The chemical composition of the hadrons would freeze out.

Phenomenological framework

The underlying consideration behind several Statistical models are :

- Hadronic interactions cease at some point of evolution of the fireball,
- The hadrons populate the phase space according to statistical distribution,

Hadron Resonance Gas (HRG) model

From the partition function, the number density n_i is obtained as,

$$\boxed{n_i = \frac{T}{V}\left(\frac{\partial \ln Z_i}{\partial \mu_i}\right)_{V,T} = \frac{g_i}{(2\pi)^3}\int \frac{d^3p}{\exp[(E_i - \mu_i)/T] \pm 1}}$$

where, $\mu_i = B_i\mu_B + S_i\mu_S + Q_i\mu_Q$, with B_i, S_i and Q_i denoting its baryon number, strangeness and electric charge.

Continued . . .

Relation between number density for the i'th detected hadron to the corresponding rapidity density is,

$$\boxed{\left.\frac{dN_i}{dy}\right|_{Det} \simeq \left.\frac{dV}{dy} n_i^{Tot}\right|_{Det}}$$

where the subscript *Det* denotes the detected hadrons. Here,

$$\begin{aligned} n_i^{Tot} &= n_i(T, \mu_B, \mu_Q, \mu_S) + \\ \sum_j & n_j(T, \mu_B, \mu_Q, \mu_S) \times Branch\ Ratio(j \to i) \end{aligned}$$

where the summation is over the unstable resonances j that decay to the i^{th} hadron.

Parametrization of chemical freeze-out hyper surface

Chemical freeze-out parameters, temperature T and the three chemical potentials μ_B, μ_Q and μ_S, are obtained from the statistical model calculations by performing a χ^2 fit with the available experimental multiplicity data.

χ^2 fitting method

$$\boxed{\chi^2 = \sum_{\alpha} \frac{(R_\alpha^{exp} - R_\alpha^{therm})^2}{(\sigma_\alpha^{exp})^2}}$$

Lets, revisit previous analyses of the chemical freeze-out parameters from the χ^2 fit of experimental multiplicity data ratios using the HRG.

Conventional χ^2 analysis

Common approach is to first fix two of the parameters, say, μ_Q and μ_S using the constraint relations,

$$\frac{\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)Q_i}{\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)B_i} = constant,$$

and

$$\sum_i n_i(T, \mu_B, \mu_S, \mu_Q)S_i = 0.$$

The problem is now reduced to obtaining the best fit value of the other two parameters by minimizing the χ^2.

This approach is successful in explaining the yield of experimentally observed hadrons produced in heavy ion collisions.

Systematics discussed in literature

Andronic et al., Nucl. Phys. A772, 167 (2006); STAR collaboration, Phys. Rev. C96, 044904 (2017)

The systematic uncertainties in the experimental data are expected to be reduced when considering the ratios of hadron yields.

Andronic et al., Nucl. Phys. A772, 167 (2006)

Whenever a given experimental yield is used several times in the ratios there may be some correlation uncertainties in particle ratios.

STAR collaboration, Phys. Rev. C96, 044904 (2017)

An estimate of such uncertainties was found to be less than 5 percent.

Systematics discussed in literature

Andronic et al., Nucl. Phys. A772, 167 (2006); F. Becattini, arXiv:0707.4154 [nucl-th]

- There are $N-1$ statistically independent ratios from the total number $N(N-1)/2$ of ratios, which can be constructed from N experimentally measured hadron yields.
- Choice of particle ratios, may introduce a bias for the extracted freeze-out parameters.

F. Becattini, arXiv:0707.4154 [nucl-th]; J.Manninen et al., Phys. Rev. C78, 054901(2008)

Choosing $(N-1)$ number of statistically independent ratios may lead to the information loss.

Quantifying systematic uncertainties due to choice of hadron ratios

For N given hadron yields there are $N(N-1)/2$ possible ratios, from which $(N-1)$ ratios may be chosen in ${}^{N(N-1)/2}C_{N-1}$ ways. One can obtain the freeze-out parameters for each of these sets of independent ratios. The variation of the parameters with the choice of these different sets would then be an useful measure of the concerned systematic uncertainty.

The number of independent sets would grow very large with the increase in number of hadron yields !!

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We have performed this study using χ^2 analysis for 9 such sets of independent yield ratios and obtain the variation of the resulting freeze-out parameters and as well as in hadron multiplicity ratios.

Systematics discussed in literature

STAR collaboration, Phys. Rev. C96, 044904 (2017)

The variations in the systematics were obtained by choosing several conditions like,

- $\mu_Q = 0$,
- μ_Q constrained by net baryon to net charge ratio,
- μ_Q constrained to that ratio along with vanishing net strangeness.

L. V. Bravina et al., Phys. Rev. C60, 024904 (1999)

The possibilities of net strangeness being non-zero in mid rapidity region.

Systematics due to relaxing the net charge constraints

The constraints should definitely be satisfied globally, there is no imperative reasons to enforce these conditions in a given rapidity bin. It seems natural to relax the constraints because for higher collisional energies, baryon stopping in the central rapidity region is negligible. The system may simply be composed of the secondary particles produced, and would have both the net charge and net baryon number to vanish

The thermodynamic parameters will be obtained from the four χ^2 minimization equations,

$$\frac{\partial \chi^2}{\partial T} = 0,$$

$$\frac{\partial \chi^2}{\partial \mu_x} = 0, \quad \text{where}, x = B, Q, S.$$

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It would be interesting to study the systematics of the net baryon to net charge ratio and the net strangeness with varying $\sqrt{s}$.

Data analysis

- We have used the mid-rapidity data of most central collision of Au-Au nuclei for $\sqrt{S}$ of AGS, SPS, RHIC, LHC,
- The yields of the hadrons used to obtain the freeze-out parameters are, $\pi^{\pm}$ (139.57 MeV), $k^{\pm}$ (493.68 MeV), p,$\bar{p}$ (938.27 MeV), Λ,$\bar{\Lambda}$ (1115.68 MeV), $\Xi^{\mp}$ (1321.71 MeV),
- LHC data set does not have the $\bar{\Lambda}$, and we assumed it to be same as that reported for Λ,
- For the AGS at $\sqrt{s} = 4.85$ GeV, Ξ data was not available,
- For the lower AGS energies we used $\pi^{\pm}$, $k^{\pm}$, p and Λ data,
- To derive uncertainty σ, we have quadratically add statistical and systematic errors of measured yields,
- Error of ratios have been calculated from error propagation method.

Results

Freeze-out Parameters

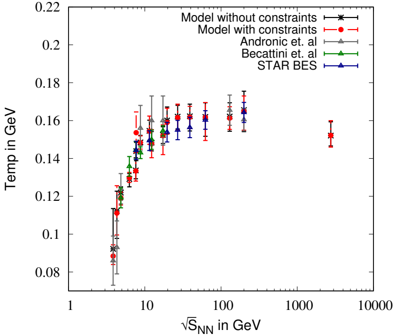

Figure: Variation of T, μ_B, μ_Q, μ_S with $\sqrt{s}$. We have compared our freeze-out parameters with Andronic et. al, Becattini et. al, STAR BES

Reduced χ^2 : the goodness of fit

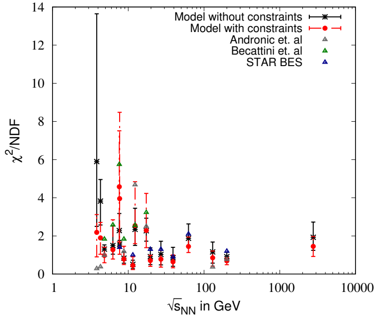

Figure: Variation of χ^2/NDF with $\sqrt{s}$. We have compared our χ^2/NDF with Andronic et. al, Becattini et. al, STAR BES

Our results are in close proximity to the results already reported in the literature.

Necessary condition

Reduced χ^2 for the variety of independent sets of hadron yield ratios should have a χ^2 distribution with the mean value close to the number of degrees of freedom.

It should not be considered alarming if the reduced χ^2 value is far from 1.

Hadron Yield Ratios

- We have obtained various hadron yield ratios with fitted freeze- out parameters and compare them with the experimental data.
- We have demonstrated the estimated systematic uncertainties in hadron yield ratios due to the choice of different hadron ratio sets.
- We have shown the effect of relaxing constraint relation on hadron yield ratios.

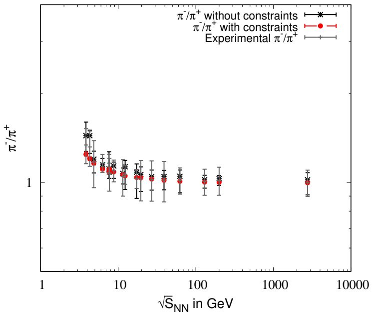

Figure: Variation of π^-/π^+ with $\sqrt{s}$

Hadron Yield Ratios

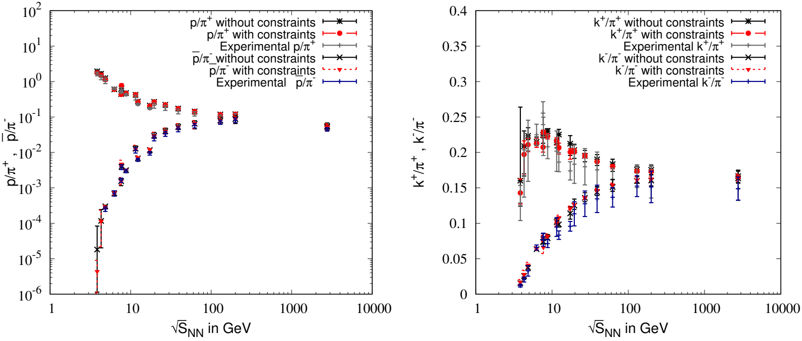

Figure: Variation of p/π^+, $\bar{p}/\pi^-$, k^+/π^+ and k^-/π^- with $\sqrt{s}$

Hadron Yield Ratios

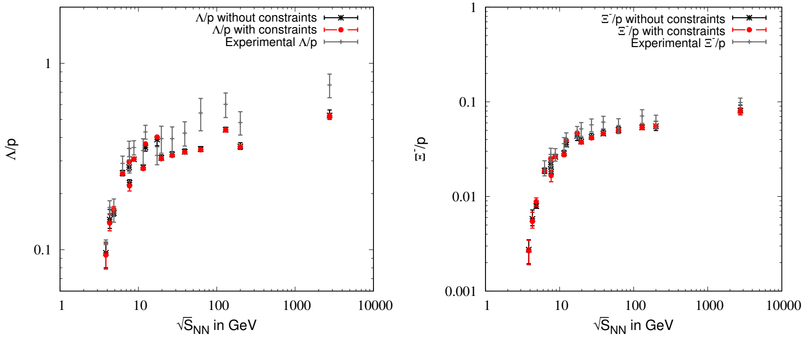

Figure: Variation of Λ/p and Ξ^-/p with $\sqrt{s}$

Prediction of multi strange hadron yield ratios

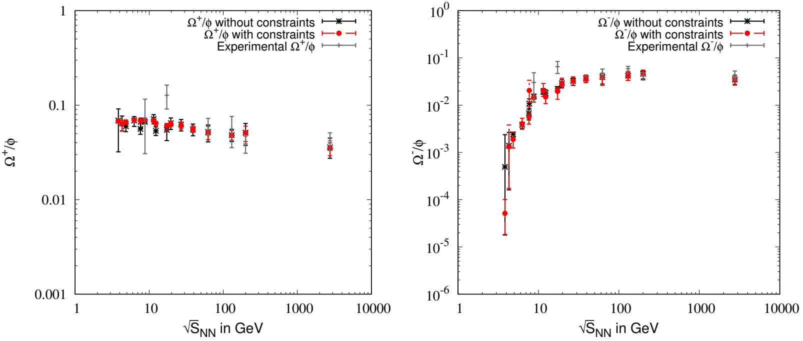

Figure: Variation of Ω^+/ϕ and Ω^-/ϕ with $\sqrt{s}$

Effects of removing the net charge constraints

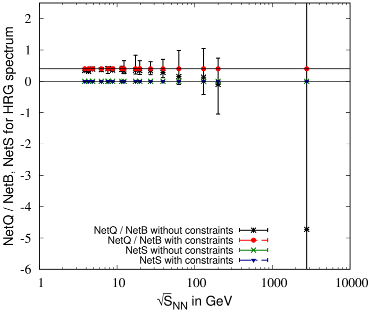

Figure: Variation of Net$Q$/Net$B$ and Net$S$ with $\sqrt{s}$

- Net$S$ remains consistent with zero for the full range of collision energy,
- The constraints Net$Q$/Net$B$ hold in the central rapidity region when baryon stopping is significant,
- beyond $\sqrt{s} \sim 40$ GeV, baryon stopping is no longer favourable,

Effects of removing the net charge constraints

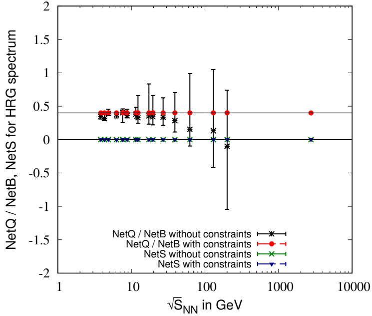

Figure: Zoomed version: Variation of Net$Q$/Net$B$ and Net$S$ with $\sqrt{s}$

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Conclusion

- We discuss systematic uncertainties in the chemical freeze-out parameters from the χ^2 analysis of hadron multiplicity ratios in the heavy-ion collision experiments.
- We quantify the sensitivity of the bias introduced due to the choice of specific hadron ratios and found the systematics are within control as they lie within the experimental uncertainties. *This indeed indicate that system is not far from equilibrium*.
- The variations obtained by removing the usual constraints on the conserved charges show similar behaviour.
- Beyond $\sim$ 40 GeV the uncertainties in the $NetB/NetQ$ ratio gradually increases, possibly *indicating the reduction in baryon stopping*.

Thank You!

Extra slides

Set 1	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^+}$,	$\frac{k^-}{\pi^+}$,	$\frac{p}{\pi^+}$,	$\frac{\bar{p}}{\pi^+}$,	$\frac{\Lambda}{\pi^+}$,	$\frac{\bar{\Lambda}}{\pi^+}$,	$\frac{\Xi^-}{\pi^+}$,	$\frac{\Xi^+}{\pi^+}$
Set 2	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{\pi^-}$,	$\frac{p}{\pi^-}$,	$\frac{\bar{p}}{\pi^-}$,	$\frac{\Lambda}{\pi^-}$,	$\frac{\bar{\Lambda}}{\pi^-}$,	$\frac{\Xi^-}{\pi^-}$,	$\frac{\Xi^+}{\pi^-}$
Set 3	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^+}$,	$\frac{\bar{p}}{k^+}$,	$\frac{\Lambda}{k^+}$,	$\frac{\bar{\Lambda}}{k^+}$,	$\frac{\Xi^-}{k^+}$,	$\frac{\Xi^+}{k^+}$
Set 4	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{k^-}$,	$\frac{\Lambda}{k^-}$,	$\frac{\bar{\Lambda}}{k^-}$,	$\frac{\Xi^-}{k^-}$,	$\frac{\Xi^+}{k^-}$
Set 5	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{p}$,	$\frac{\Lambda}{p}$,	$\frac{\bar{\Lambda}}{p}$,	$\frac{\Xi^-}{p}$,	$\frac{\Xi^+}{p}$
Set 6	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{p}$,	$\frac{\Lambda}{\bar{p}}$,	$\frac{\bar{\Lambda}}{\bar{p}}$,	$\frac{\Xi^-}{\bar{p}}$,	$\frac{\Xi^+}{\bar{p}}$
Set 7	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{p}$,	$\frac{\Lambda}{\bar{p}}$,	$\frac{\bar{\Lambda}}{\Lambda}$,	$\frac{\Xi^-}{\Lambda}$,	$\frac{\Xi^+}{\Lambda}$
Set 8	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{p}$,	$\frac{\Lambda}{\bar{p}}$,	$\frac{\bar{\Lambda}}{\Lambda}$,	$\frac{\Xi^-}{\bar{\Lambda}}$,	$\frac{\Xi^+}{\bar{\Lambda}}$
Set 9	$\frac{\pi^-}{\pi^+}$,	$\frac{k^+}{\pi^-}$,	$\frac{k^-}{k^+}$,	$\frac{p}{k^-}$,	$\frac{\bar{p}}{p}$,	$\frac{\Lambda}{\bar{p}}$,	$\frac{\bar{\Lambda}}{\Lambda}$,	$\frac{\Xi^-}{\bar{\Lambda}}$,	$\frac{\Xi^+}{\Xi^-}$

Table: Set of sets of hadron yield ratios for AGS ($\sqrt{s} = 4.85$ GeV), SPS, RHIC and LHC data.

Extra slides

Set 1	$\frac{\pi^-}{\pi^+}, \frac{k^+}{\pi^+}, \frac{k^-}{\pi^+}, \frac{p}{\pi^+}, \frac{\Lambda}{\pi^+}$
Set 2	$\frac{\pi^-}{\pi^+}, \frac{k^+}{\pi^-}, \frac{k^-}{\pi^-}, \frac{p}{\pi^-}, \frac{\Lambda}{\pi^-}$
Set 3	$\frac{k^-}{\pi^+}, \frac{k^-}{\pi^-}, \frac{k^-}{k^+}, \frac{p}{k^-}, \frac{\Lambda}{k^-}$
Set 4	$\frac{p}{\pi^+}, \frac{p}{\pi^-}, \frac{p}{k^+}, \frac{p}{k^-}, \frac{\Lambda}{p}$
Set 5	$\frac{\Lambda}{\pi^+}, \frac{\Lambda}{\pi^-}, \frac{\Lambda}{k^+}, \frac{\Lambda}{k^-}, \frac{\Lambda}{p}$

Table: Set of sets of hadron yield ratios for lower AGS energies