

Fluctuations and correlations of conserved charges in hadron resonance gas model

Subhasis Samanta

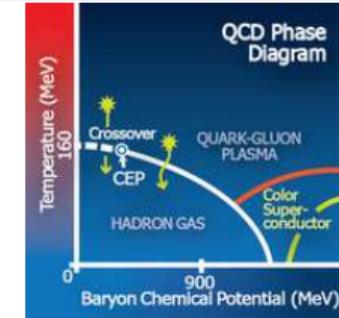
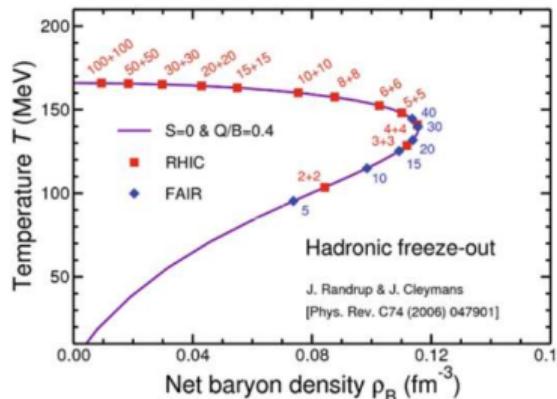
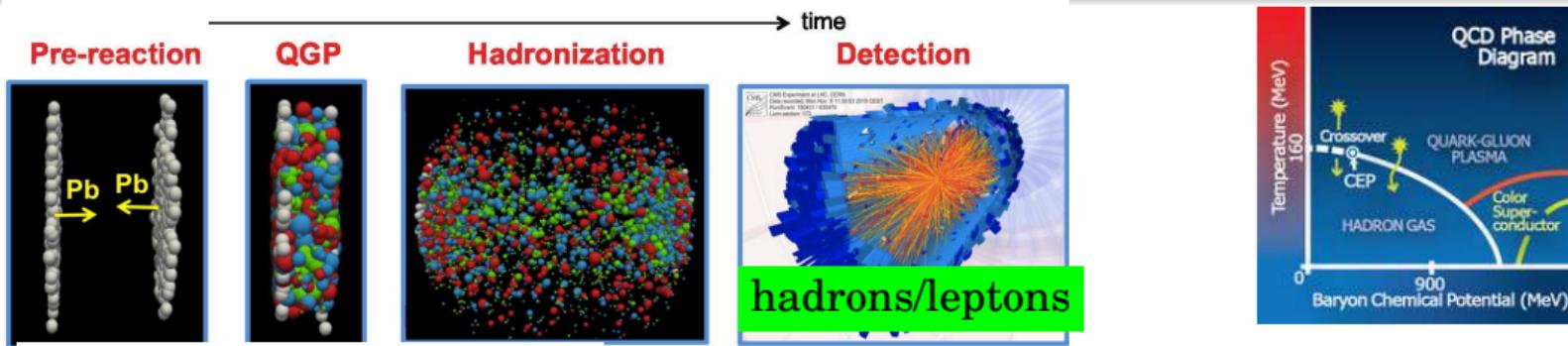
National Institute of Science Education and Research, HBNI, Jatni, India

Outline

- ★ Introduction
- ★ HRG models
 - Ideal
 - S-matrix formalism
 - VDWHRG
- ★ Summary



Introduction



The major goals

- ★ The mapping of QCD phase diagram
- ★ Locating the QCD critical point

| Facility | $\sqrt{s_{NN}}$ (GeV) | μ_B (MeV) | Status |
|------------|-----------------------|---------------|---------|
| LHC | 2760 | 0 | Running |
| RHIC | 7.7 - 200 | 420-20 | Running |
| NA61/SHINE | 8 | 400 | Running |
| FAIR | 2.7-4.9 | 800-500 | Future |
| NICA | 4-11 | 600-300 | Future |

HRG models have been used to study hadronic phase

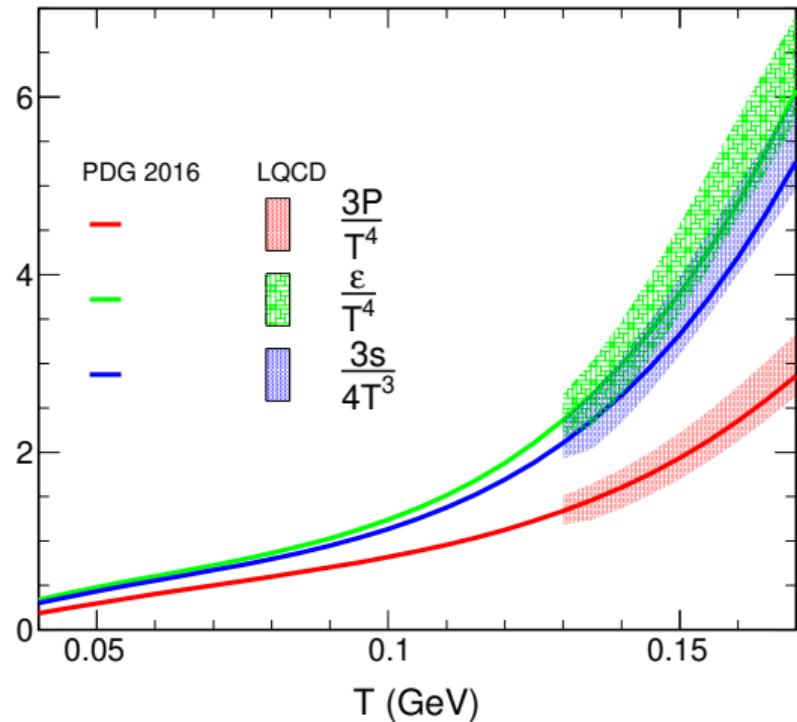
Ideal Hadron Resonance Gas model

- ★ System consists of all the hadrons including resonances (non-interacting point particles)
- ★ Hadrons are in thermal and chemical equilibrium
- ★ The grand canonical partition function of a hadron resonance gas: $\ln Z = \sum_i \ln Z_i$
- ★ For i th hadron/resonance,

$$\ln Z_i^{id} = \frac{Vg_i}{2\pi^2} m_i^2 T \sum_{j=1}^{\infty} (\pm 1)^{j-1} (z^j / j^2) K_2(jm_i/T), \quad z = \exp(\mu/T),$$
$$\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$$

- The + (-) sign refers to bosons (fermions)
- The first term ($j = 1$) corresponds to the classical ideal gas
- Width of the resonances are ignored

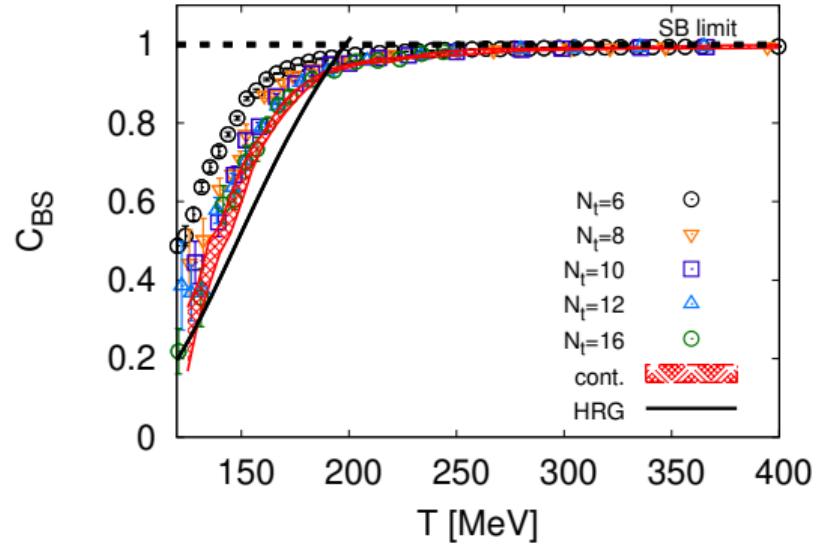
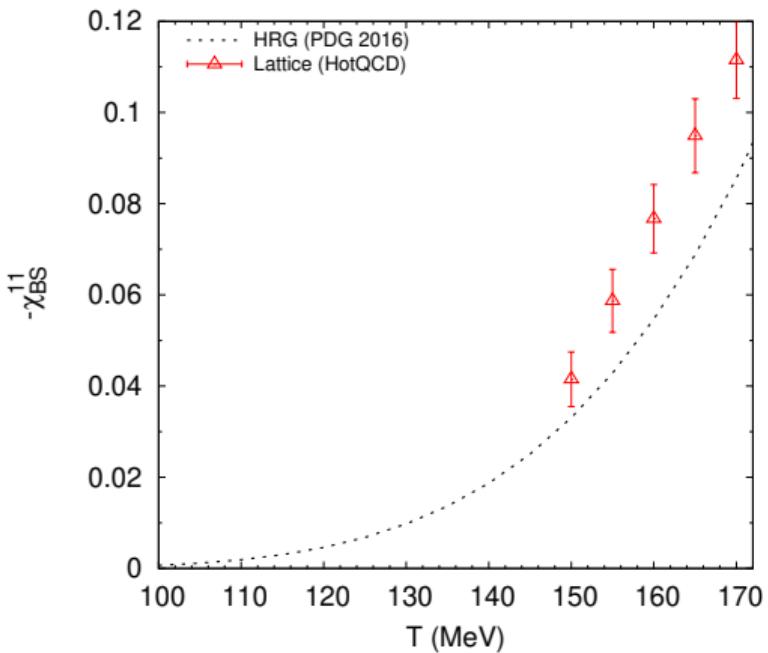
EOS of IDHRG at $\mu = 0$



- * IDHRG provides a satisfactory description in the hadronic phase of continuum LQCD data

S. Samanta et al. JPG 46, 065106 (2019); LQCD data: A. Bazavov et al. (HotQCD), PRD 90, 094503 (2014)

Problem to quantify χ_{BS} , C_{BS}



Ref: A. Borsanyi et al., JHEP01, 138 (2012)

- * IDHRG fails to describe χ_{BS} , $C_{BS} = -3\chi_{BS}^{11}/\chi_S^2$
- ⇒ Interaction is needed

Classical Virial Expansion (Non-relativistic)

$$P = \frac{NT}{V} \left(1 + \left(\frac{N}{V} \right) B(T) + \left(\frac{N}{V} \right)^2 C(T) + \dots \right)$$

- * The first term in the expansion corresponds to an ideal gas
- * The second term is obtained by taking into account the interaction between pairs of particles and subsequent terms involve the interaction between groups of three, four, etc. particles
- * B, C, \dots are called second, third, etc., virial coefficients

Second virial coefficient

$$B(T) = \frac{1}{2} \int (1 - e^{-U_{12}/T}) dV$$

U_{12} is the two body interaction energy

Relativistic Virial Expansion

$$\ln Z = \ln Z_0 + \sum_{i_1, i_2} z_1^{i_1} z_2^{i_2} b(i_1, i_2)$$

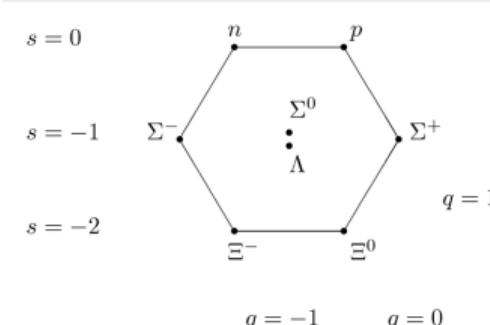
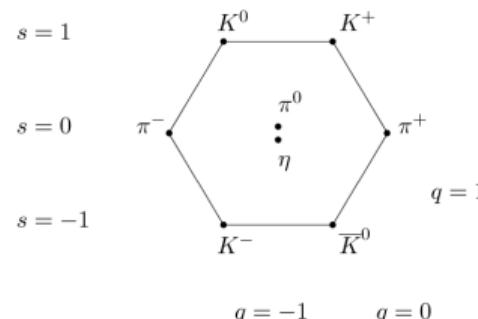
$$b(i_1, i_2) = \frac{V}{4\pi i} \int \frac{d^3 p}{(2\pi)^3} \int d\varepsilon \exp \left(-\beta(p^2 + \varepsilon^2)^{1/2} \right) \left[\left\{ S^{-1} \frac{\partial S}{\partial \varepsilon} - \frac{\partial S^{-1}}{\partial \varepsilon} S \right\} \right]$$

$aa \rightarrow R \rightarrow aa, ab \rightarrow R \rightarrow ab, aab \rightarrow R \rightarrow aab$ etc.

- * z_1 and z_2 are fugacities of two species ($z = e^{\beta\mu}$)
- * The labels i_1 and i_2 refer to a channel of the S-matrix which has an initial state containing $i_1 + i_2$ particles

Second virial coefficient

$$b_2 = b(i_1, i_2)/V \text{ where } i_1 = i_2 = 1$$



$\pi\pi \rightarrow R \rightarrow \pi\pi$
 $\pi K \rightarrow R \rightarrow \pi K$
 $KK \rightarrow R \rightarrow KK$
 $\pi N \rightarrow R \rightarrow \pi N$
etc.

Interacting part of pressure

b_2 in terms of phase shift

$$b_2 = \frac{1}{2\pi^3 \beta} \int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{I,l} {}' g_{I,l} \frac{\partial \delta_l^I(\varepsilon)}{\partial \varepsilon}$$

$$\begin{aligned} P_{\text{int}} &= \frac{1}{\beta} \frac{\partial \ln Z_{\text{int}}}{\partial V} = \frac{1}{\beta} z_1 z_2 b_2 \\ &= \frac{z_1 z_2}{2\pi^3 \beta^2} \int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{I,l} {}' g_{I,l} \frac{\partial \delta_l^I(\varepsilon)}{\partial \varepsilon} \end{aligned}$$

- * Interaction is attractive (repulsive) if derivative of the phase shift is positive (negative)

K-matrix formalism (Attractive part of the interaction)

Scattering amplitude: $S_{ab \rightarrow cd} = \langle cd | S | ab \rangle$

Scattering operator (matrix)

$$S = I + 2iT$$

S is unitary

$$SS^\dagger = S^\dagger S = I$$

$$(T^{-1} + iI)^\dagger = T^{-1} + iI$$

$K^{-1} = T^{-1} + iI$, $K = K^\dagger$ (i.e., K matrix is real and symmetric)

Phase shift in K-matrix formalism

$$\operatorname{Re} T = K(I + K^2)^{-1}, \quad \operatorname{Im} T = K^2(I + K^2)^{-1} \Rightarrow \operatorname{Im} T / \operatorname{Re} T = K$$

$$K_{ab \rightarrow R \rightarrow ab} = \sum_R \frac{m_R \Gamma_{R \rightarrow ab}(\sqrt{s})}{m_R^2 - s}$$

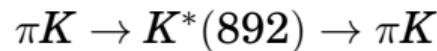
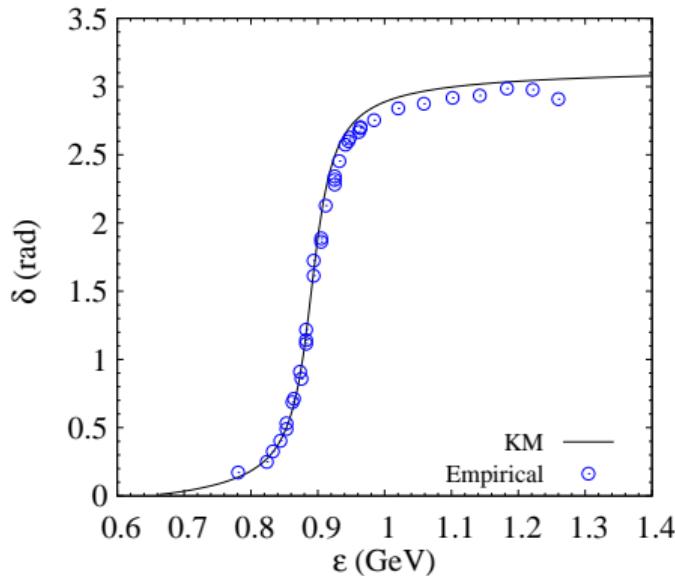
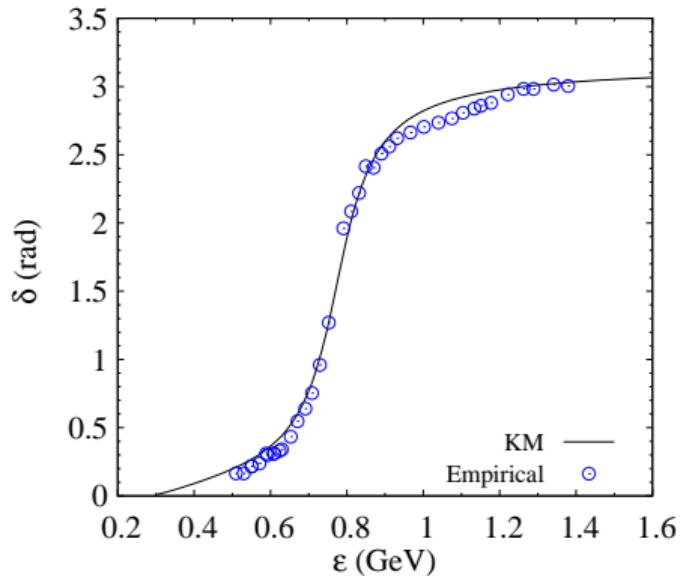
Resonances appear as sum of poles in the K matrix

Partial wave decomposition

$$\begin{aligned} S_l &= \exp(2i\delta_l) = 1 + 2iT_l \\ \Rightarrow T_l &= \exp(i\delta_l) \sin(\delta_l) \\ \operatorname{Re} T_l &= \sin(\delta_l) \cos(\delta_l), \quad \operatorname{Im} T_l = \sin^2(\delta_l) \end{aligned}$$

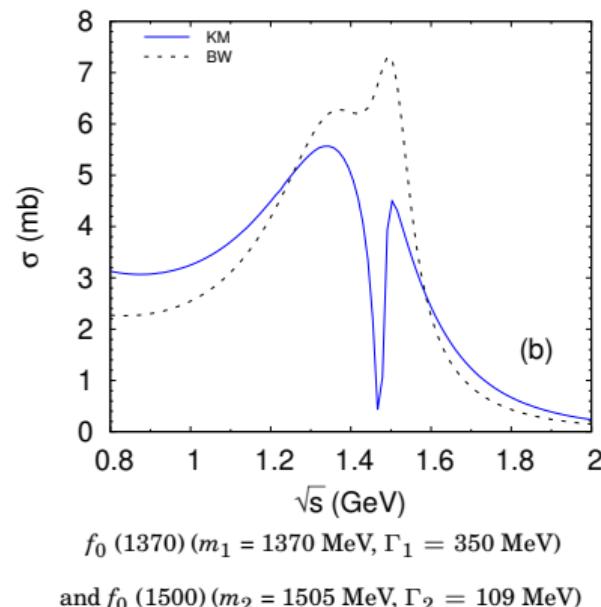
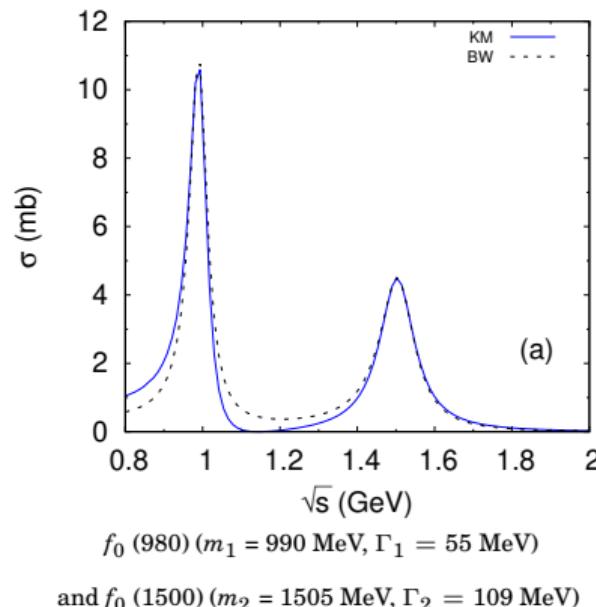
$$K = \tan(\delta_l), \quad \delta_l = \tan^{-1}(K)$$

Phase shift: Empirical vs KM



- * Good agreement between the empirical phase shifts of resonances and the K-matrix approach

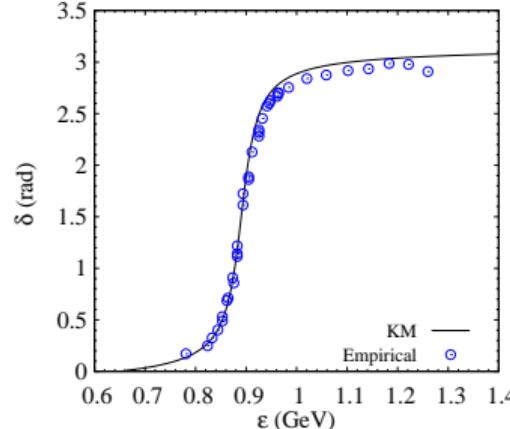
Comparison between K-matrix and Breit-Wigner approach



- * KM formalism preserves the unitarity of the S matrix and neatly handles overlapping resonances

Ideal gas limit

- For a narrow resonance, δ_l^I changes rapidly through π radian around $\varepsilon = m_R$
- δ_l^I can be approximated by a step function: $\delta_l^I \sim \Theta(\varepsilon - m_R)$
- $\partial\delta_l^I/\partial\varepsilon \approx \pi\delta(\varepsilon - m_R)$

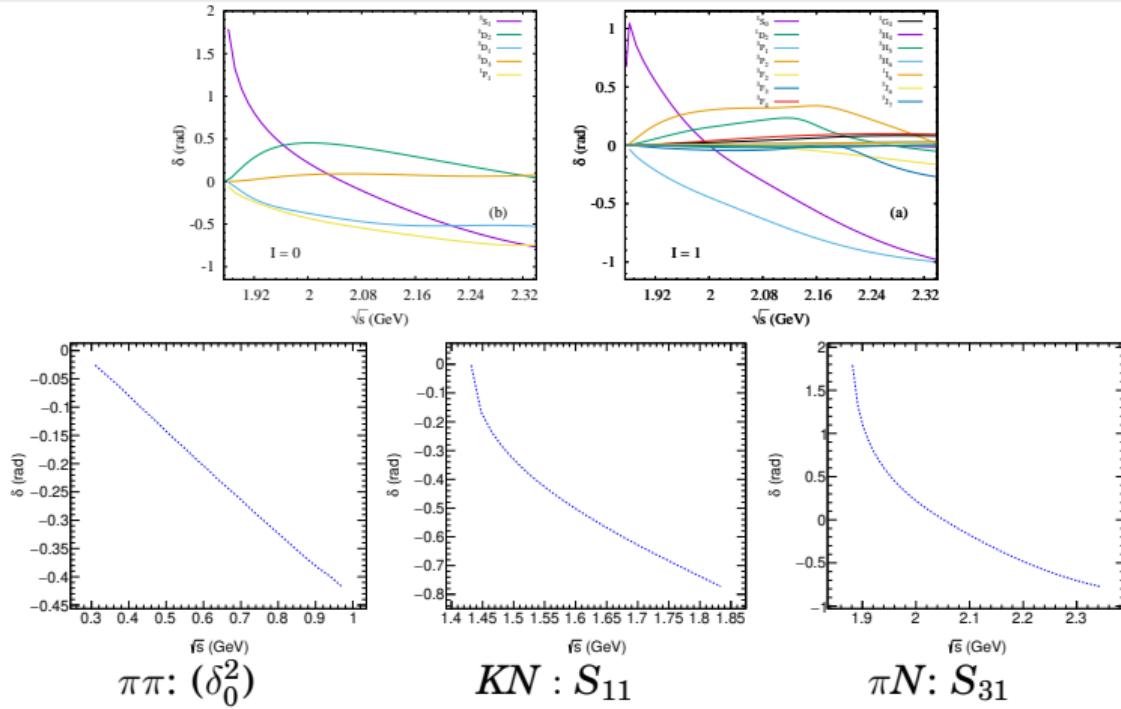


$$\begin{aligned} b_2 &= \frac{1}{2\pi^3\beta} \int_M^\infty d\varepsilon \varepsilon^2 K_2(\beta\varepsilon) \sum_{l,I} g_{I,l} \frac{\partial\delta_l^I(\varepsilon)}{\partial\varepsilon} \\ &= \frac{g_{I,l}}{2\pi^2} m_R^2 T K_2(\beta m_R) \end{aligned}$$

$$P_{\text{int}} = T z_1 z_2 b_2 = P_{\text{id}}^R$$

- Pressure exerted by an ideal (MB) gas of particles of mass m_R
- This establishes the fundamental premise of the IDHRG model

Repulsive interaction from experimental data of phase shift

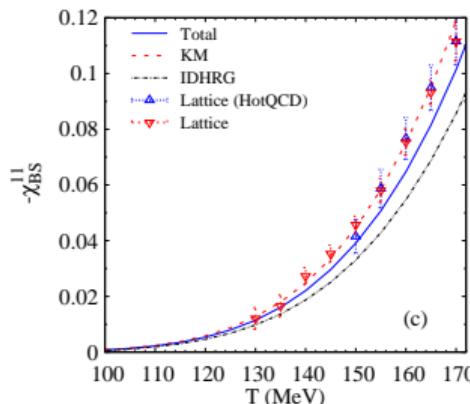
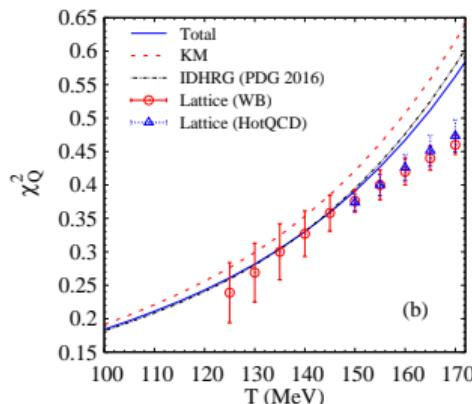
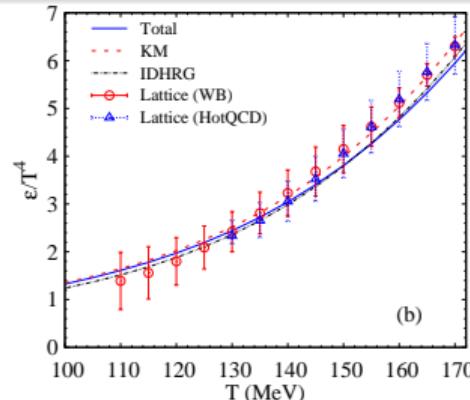
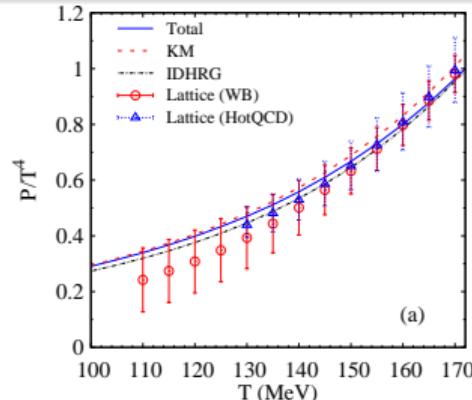


- ★ NN interaction:
All available data
- ★ $\pi\pi$ repulsive
interaction: δ_0^2
- ★ KN repulsive
interaction:
 $S_{11}(l_{I,2J})$ ($\Sigma(1660)$)
- ★ πN repulsive
interaction: S_{31}
($l_{2I,2J}$) ($\Delta(1620)$),
 $\Delta(1910)$, $N(1720)$
etc.

- ★ $\Sigma(1660)$, $\Sigma(1750)$, $\Sigma(1915)$, $\Delta(1620)$), $\Delta(1910)$, $\Delta(1930)$, $N(1720)$ etc. are included in the repulsive part

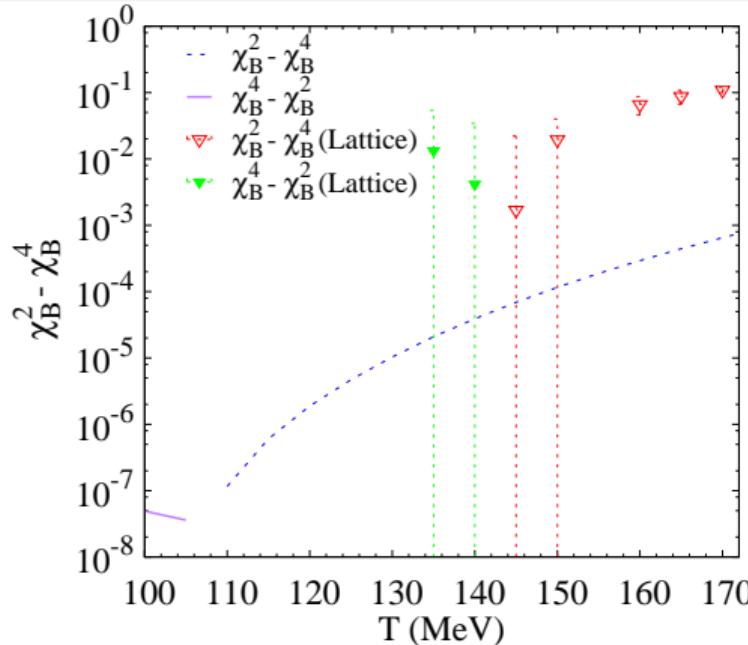
Ref: SAID [<http://gwdac.phys.gwu.edu>]

Results

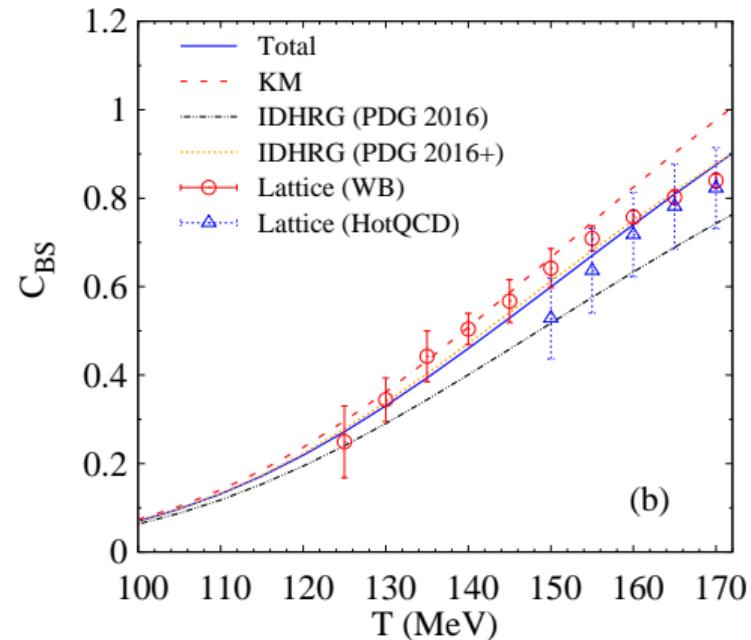


- ★ KM: Attractive interaction
- ★ Total: Attractive + repulsive
- ★ Both KM and Total contain non-interacting part as well
- ★ Repulsive interactions suppress the bulk variables

$\chi_B^2 - \chi_B^4$ and C_{BS}

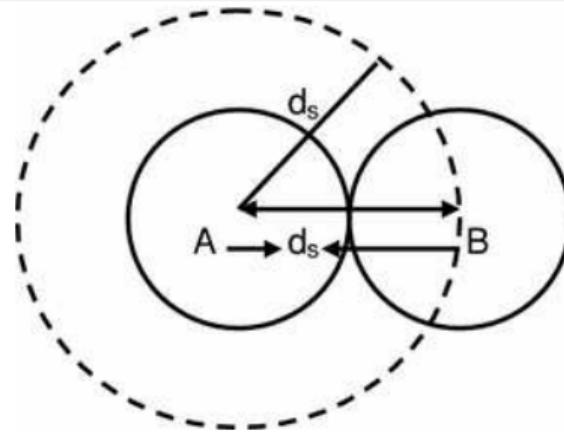


- ★ $\chi_B^2 - \chi_B^4$ is non-zero
- ★ For C_{BS} : Improvement compared to IDHRG



(b)

Excluded volume hadron resonance gas model



- Hadrons have finite hard-core radii. ($P(V - Nb) = NT$)
- $b = V_{ex} = \frac{16}{3}\pi R^3$ is the volume excluded for the hadron.
- Pressure and chemical potential in EVHRG model:

$$P(T, \mu_1, \mu_2, \dots) = \sum_i P_i^{id}(T, \hat{\mu}_1, \hat{\mu}_2, \dots),$$

$$\hat{\mu}_i = \mu_i - V_{ev,i} P(T, \mu_1, \mu_2, \dots)$$

van der Waals interaction in HRG model (VDWHRG model)

$$\left(P + \left(\frac{N}{V} \right)^2 a \right) (V - Nb) = NT,$$

$$P(T, n) = \frac{NT}{V - bN} - a \left(\frac{N}{V} \right)^2 \equiv \frac{nT}{1 - bn} - an^2$$

where $n \equiv N/V$ is the number density of particles.

$$P(T, \mu) = P_{id}(T, \mu^*) - an^2, \quad \mu^* = \mu - bP(T, \mu) - abn^2 + 2an$$

$$n = \frac{n_{id}(T, \mu^*)}{1 + bn_{id}(T, \mu^*)}$$

- * $a = 0 \Rightarrow$ EVHRG
- * $a = b = 0 \Rightarrow$ IDHRG

Extraction of parameters a and b

$$a = 1250 \pm 150 \text{ MeV fm}^3, r = 0.7 \pm 0.05 \text{ fm}$$

$$\chi^2 = \sum_{i,j} \frac{(R_{i,j}^{LQCD}(T_j) - R_{i,j}^{model}(T_j))^2}{(\Delta_{i,j}^{LQCD}(T_j))^2},$$

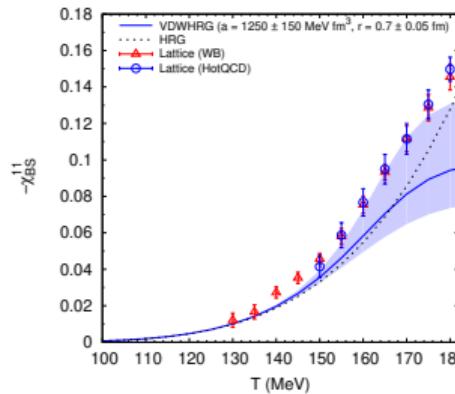
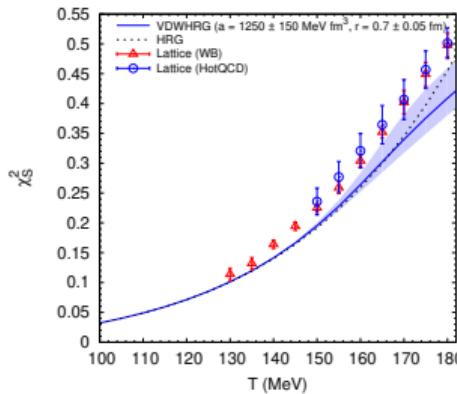
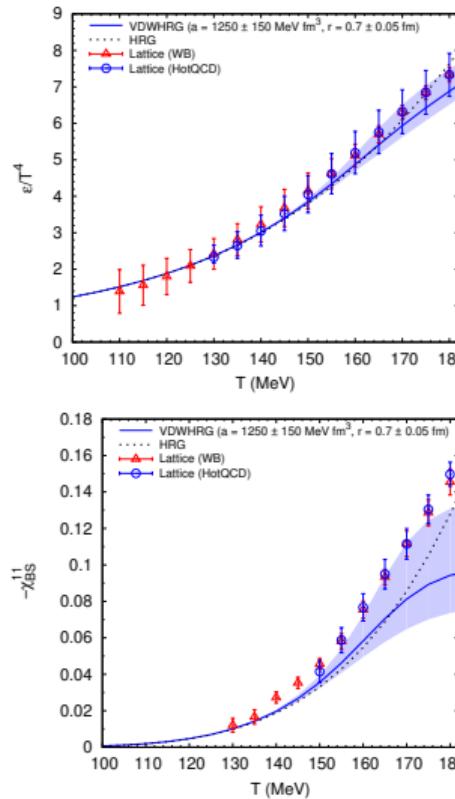
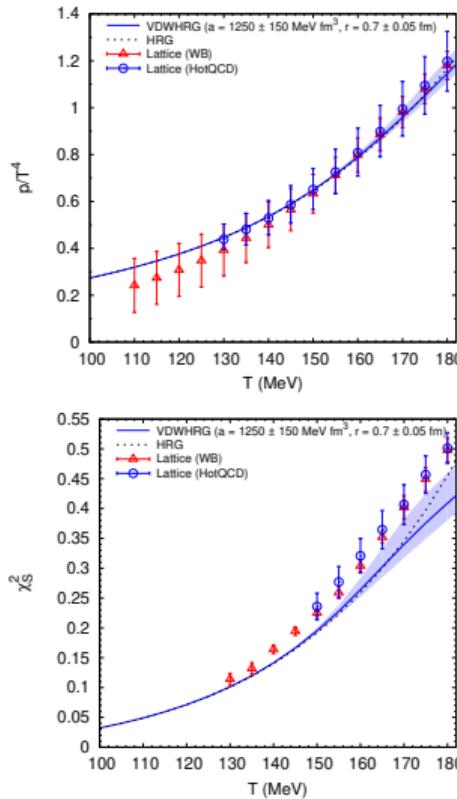
LQCD data of P/T^4 , ε/T^4 , s/T^3 , C_V/T^3 and χ_B^2 at $\mu = 0$ have been used to calculate χ^2

$$a = 329 \text{ MeV fm}^3, r = 0.59 \text{ fm}$$

By reproducing the properties of the nuclear matter ($n_0 = 0.16 \text{ fm}^{-3}$, $E/N = -16 \text{ MeV}$) at zero temperature

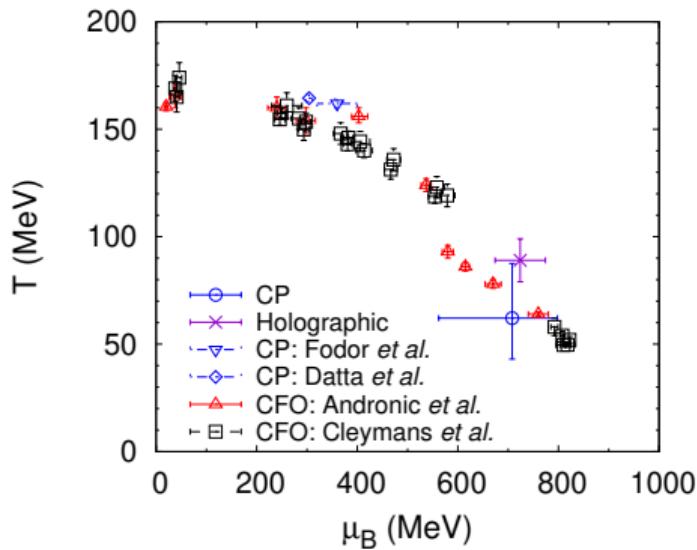
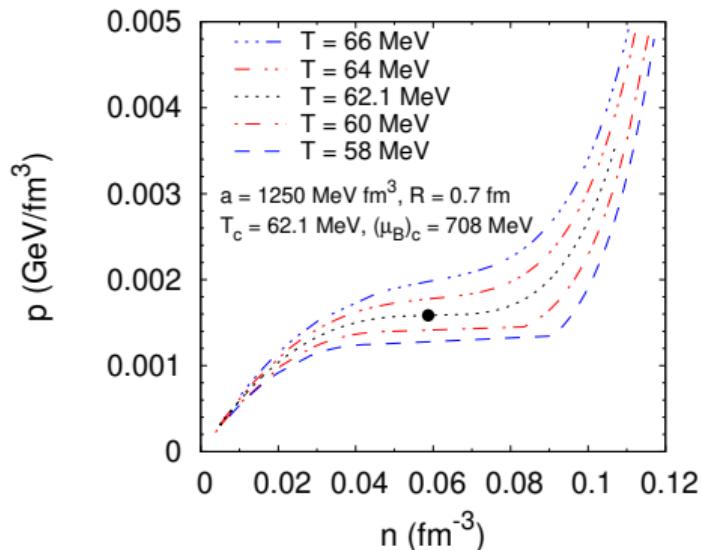
Ref: V. Vovchenko et al., PRC **91**, 064314 (2015)

Results- VDWHRG model



- ★ Agreement between LQCD and VDWHRD

Phase transition in VDWHRG model



- ★ Observed first order phase transition
- ★ Critical point at $T = 62.1$ MeV, $\mu_B = 708$ MeV
- ★ Comparable the CP obtained by using the holographic gauge/gravity correspondence

Observables for CP search

Cumulants

$$C_1 = \langle N_q \rangle, \quad C_2 = \langle (\delta N_q)^2 \rangle, \quad C_3 = \langle (\delta N_q)^3 \rangle,$$
$$C_4 = \langle (\delta N_q)^4 \rangle - 3 \langle (\delta N_q)^2 \rangle^2$$

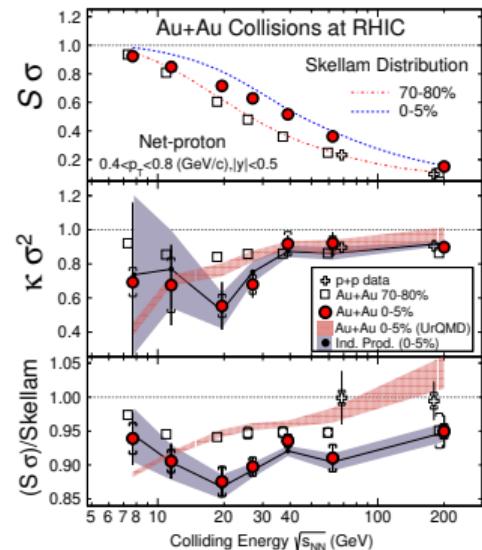
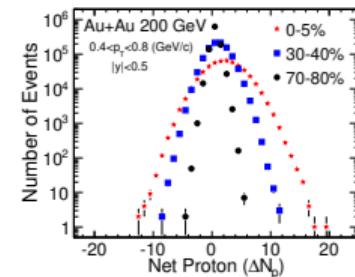
$N_q = N_{q+} - N_{q-}$ and $\delta N_q = N_q - \langle N_q \rangle$ q can be any conserved quantum number

Mean, variance, skewness, kurtosis

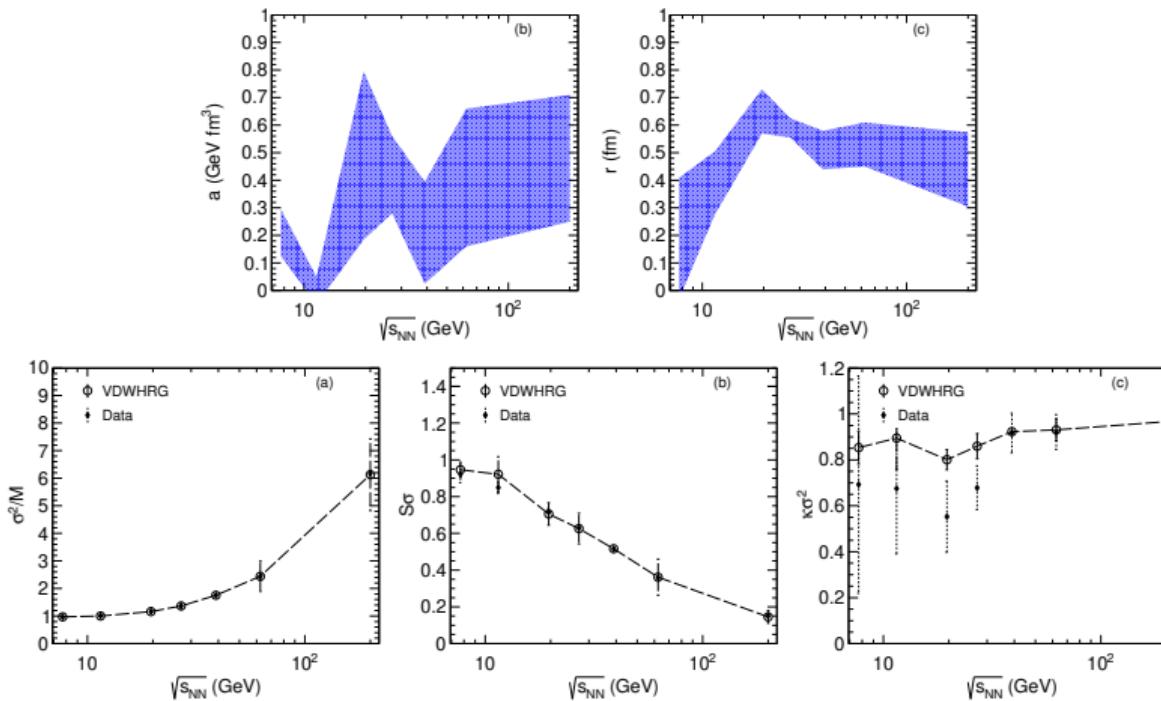
$$M = C_1, \quad \sigma^2 = C_2, \quad S = \frac{C_3}{\sigma^3}, \quad \kappa = \frac{C_4}{\sigma^4}$$

- Higher moments are sensitive to correlation length
 $\langle (\delta N_q)^2 \rangle \sim \zeta^2, \quad \langle (\delta N_q)^3 \rangle \sim \zeta^{4.5}, \quad \langle (\delta N_q)^4 \rangle \sim \zeta^7$
- $\sigma^2/M = C_2/C_1 = \chi^2/\chi^1, S\sigma = C_3/C_2 = \chi^3/\chi^2,$
 $\kappa\sigma^2 = C_4/C_2 = \chi^4/\chi^2$
- Non-monotonic variations of $S\sigma, \kappa\sigma^2$ with beam energy are believed to be good signatures of CP

Ref: STAR: PRL 112, 032302 (2014); PRL 102, 032301 (2009)

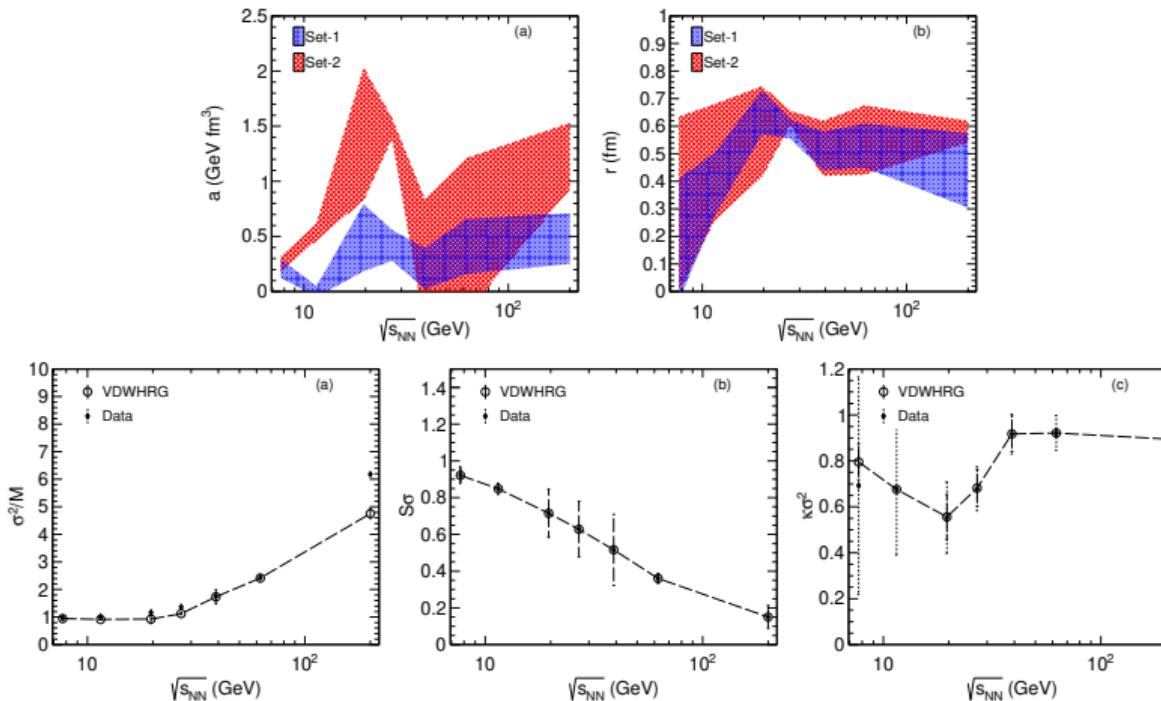


VDW parameters from data of σ^2/M , $S\sigma$ and $\kappa\sigma^2$ of np (Set-1)



- ★ Unable to describe $\kappa\sigma^2$

VDW parameters extracted from data of $S\sigma$ and $\kappa\sigma^2$ of np (Set-2)



- ★ Large a 's are needed to describe non-monotonic behaviors of $S\sigma$ and $\kappa\sigma^2$

Summary

- ★ An extension of HRG model is constructed to include interactions using relativistic virial expansion of partition function (S-matrix formalism)
- ★ Interacting part of the partition function depends on the derivative of the phase shift
- ★ The attractive part of the interaction is calculated by parameterizing the two body phase shifts using K-matrix formalism
- ★ The repulsive part is included by fitting to experimental phase shifts
- ★ Effect of interaction is more visible in $\chi_Q^2, \chi_B^2 - \chi_B^4, C_{BS}$
- ★ We find a good agreement for the C_{BS} (without adding extra resonances) and lattice QCD simulations
- ★ Critical point is observed in VDWHRG model
- ★ Large a 's are needed to describe non-monotonic behaviors of higher order moments of net-proton

Prof. Bedangadas Mohanty
Mr. Ashutosh Dash

Thank you