ELECTRICAL AND HALL CONDUCTIVITY OF QUARK GLUON PLASMA

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OUTLINE

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Motivation

- Huge magnetic field ~ m_π^2 (RHIC) and ~ 10 m_π^2 (LHC) is produced due to the initial overlap of two nuclei.
- This initial magnetic field decays rapidly in the absence of conducting medium.
- This magnetic field may survive in QGP phase due to induced currents.
- The magnetic field may be as large as maximum magnetic field produced.
- This is important to estimate the electrical and Hall conductivity of QGP in presence of magnetic field.

Boltzmann Kinetic Equation

- f(x,p,t): Single particle phase space distribution function.

- Rate of change of the distribution function df/dt = C[f]

$$\frac{\partial f}{\partial t} + (V.\nabla) f + (F.\nabla_p) f = C[f]$$

collision: $1 + 2 \rightarrow 1' + 2'$

Rate of change of the distribution function of particle type 1:

$$\frac{\partial f_1}{\partial t} + (V.\nabla) f_1 + (F.\nabla_P) f_1 = c[f] = \int d^3 p_2 d^3 p_{1'} d^3 p_{2'} \delta^4 (p_{1'} + p_{2'} - p_1 - p_2) \left| \mathrm{M}_{fi} \right|^2 (f_{1'} f_{2'} - f_1 f_2)$$

This is an integro-differential equation which is very difficult to solve.

Relaxation time approximation

- The distribution function goes to equilibrium distribution function in a relaxation time τ.

- Assumptions: i. Magnetic field is not the dominant scale.

ii. Landau quantization of energy levels has not been considered.

Collision integral =

$$C[f] = -\frac{f - f_0}{\tau} = -\delta f/\tau$$

$$f = f_0 + \delta f$$

$$\frac{\partial f}{\partial t} + \vec{V} \cdot \frac{\partial f}{\partial \vec{r}} + e\left[\vec{E} + \vec{V} \times \vec{B}\right] \cdot \frac{\partial f}{\partial \vec{p}} = -\nu(f - f_0)$$

- In the static and homogeneous case

$$-e\left[\vec{E}+\vec{V}\times\vec{B}\right]\cdot\frac{\partial f}{\partial\vec{p}}=\nu(f-f_0)$$

- Ansatz : $f(p)=f_0-\frac{1}{\nu}e\vec{E}\cdot\frac{\partial f_0(p)}{\partial\vec{p}}-\vec{a}\cdot\frac{\partial f_0(p)}{\partial\vec{p}}$
- Lets take electric and magnetic field along X and Z axis.

$$a_x=eE\cdot\frac{\omega_c^2}{\nu(\nu^2+\omega_c^2)}\qquad a_y=-eE\cdot\frac{\omega_c}{(\nu^2+\omega_c^2)}\qquad a_z=0$$

$\omega_c=eB/\varepsilon$ Cyclotron frequency

Electric current is given by: $j^i=e\int\frac{d^3p}{(2\pi)^3}V^i\delta f=\sigma_{ij}E_j=\sigma^{el}\delta_{ij}E_J+\sigma^H\varepsilon_{ij}E_j$

$$\sigma^{el}=\frac{e^2}{3T}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{\varepsilon^2}\frac{\nu}{(\nu^2+\omega_c^2)}f_0=\frac{e^2\tau}{3T}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{\varepsilon^2}\frac{1}{1+(\omega_c\tau)^2}f_0$$

$$\sigma^H=\frac{e^2}{3T}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{\varepsilon^2}\frac{\omega_c}{(\nu^2+\omega_c^2)}f_0=\frac{e^2\tau}{3T}\int\frac{d^3p}{(2\pi)^3}\frac{p^2}{\varepsilon^2}\frac{\omega_c\tau}{1+(\omega_c\tau)^2}f_0$$

Quasi Particle Model 1 for QGP

- QGP is described by an ideal gas of quasiparticles having temperature-dependent mass arising from the interactions with the surrounding quarks and gluons.
- Effective mass : $m^2 = m_0^2 + \sqrt{2} m_0 m_{th} + m_{th}^2$
- Thermal mass : $m_{th}^2(T,\mu) = g^2(T,\mu) T^2 \frac{N_c^2 - 1}{8 N_c}\left(1 + \frac{\mu^2}{\pi^2 T^2}\right)$
- Effective coupling :

$$g^2(T,\mu) = \frac{24\pi^2}{(33-2n_f)\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}}\right)}\left(1 - \frac{3(153-19n_f)}{(33-2n_f)^2}\frac{\ln\left(2\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}}\right)\right)}{\ln\left(\frac{T}{\Lambda_T}\sqrt{1+a\frac{\mu^2}{T^2}}\right)}\right)$$

Relaxation time:

$$\tau_{q\bar{q}} = \frac{1}{5.1T\alpha_s^2 \ln(\frac{1}{\alpha_s})(1+0.12(2n_f+1))}$$

$$\tau_g = \frac{1}{22.5T\alpha_s^2 \ln(\frac{1}{\alpha_s})(1+0.06n_f)}$$

Quasi Particle model 2 for QGP

- Lattice QCD EOS can be reproduced in terms of non-interacting quasiparticles having effective fugacities (z_q, z_g) which encodes all the interaction effects of the particles in the system.

- Distribution function for gluon : $$f_0^g = \frac{z_g \exp(-\beta p)}{1 - z_g \exp(-\beta p)}$$

- Distribution function for quark or antiquark :

$$f_0^{q/\bar{q}} = \frac{z_q \exp(-\beta \varepsilon(p))}{1 + z_q \exp(-\beta \varepsilon(p))}$$

- Single particle energy : $$\omega_p^g = p + T^2 \partial_T \ln(z_g)$$

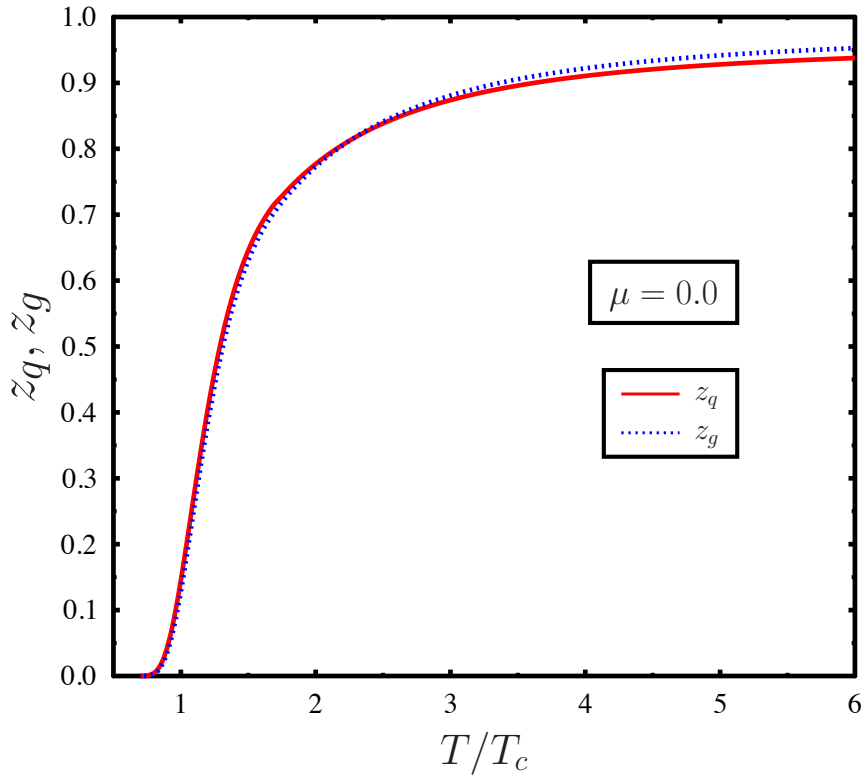
$$\omega_p^q = \sqrt{p^2 + m^2} + T^2 \partial_T \ln(z_q)$$

- Temp dependence of fugacities:

$$z_{q,g} = a_{q,g} \exp(-b_{q,g} / x^5), \text{ for } x < x_{q,g}$$

$$z_{q,g} = a'_{q,g} \exp(-b'_{q,g} / x^5), \text{ for } x > x_{q,g}, x_{q,g} \equiv T_{q,g} / T_c \approx 1.70, 1.68$$

$z_{g,q}$	$a_{g,q}$	$b_{g,q}$	$a'_{g,q}$	$b'_{g,q}$
Gluon	0.803 ± 0.009	1.837 ± 0.039	0.978 ± 0.007	0.942 ± 0.035
Quark	0.810 ± 0.010	1.721 ± 0.040	0.960 ± 0.007	0.846 ± 0.033



Relaxation time in QPM 2

$$\tau_g^{-1} = g_g \int \frac{d^3\vec{p}_g}{(2\pi)^3} f_0^g (1+f_0^g)\left(\frac{9g_{eff}^4}{16\pi\langle s\rangle_{gg}}\left[\ln\frac{\langle s\rangle_{gg}}{k^2}-1.267\right]\right)$$

$$+ g_q \int \frac{d^3\vec{p}_q}{(2\pi)^3} f_0^q (1-f_0^q)\left(\frac{g_{eff}^4}{16\pi\langle s\rangle_{gq}}\left[\ln\frac{\langle s\rangle_{gq}}{k^2}-1.287\right]\right)$$

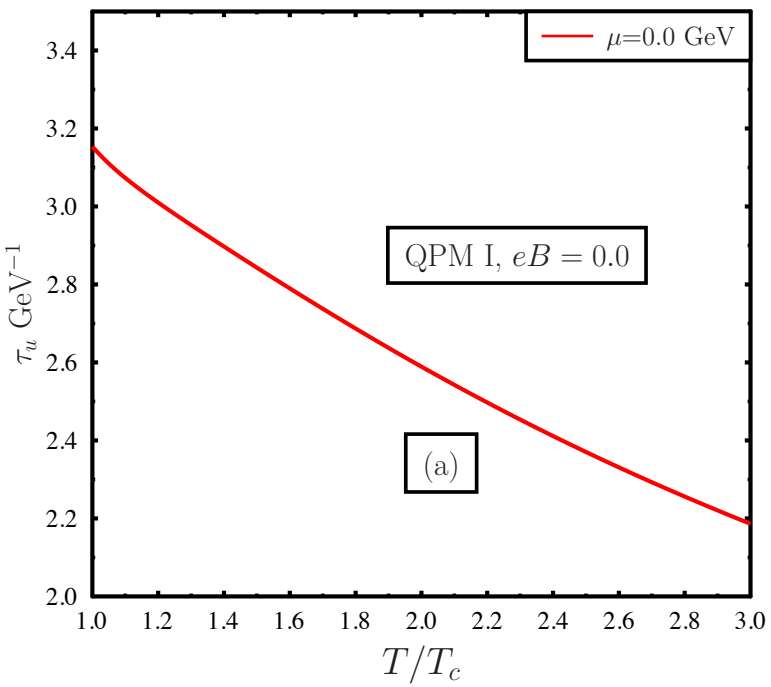
$$+ g_{\bar{q}} \int \frac{d^3\vec{p}_{\bar{q}}}{(2\pi)^3} f_0^{\bar{q}} (1-f_0^{\bar{q}})\left(\frac{9g_{eff}^4}{16\pi\langle s\rangle_{g\bar{q}}}\left[\ln\frac{\langle s\rangle_{g\bar{q}}}{k^2}-1.287\right]\right)$$

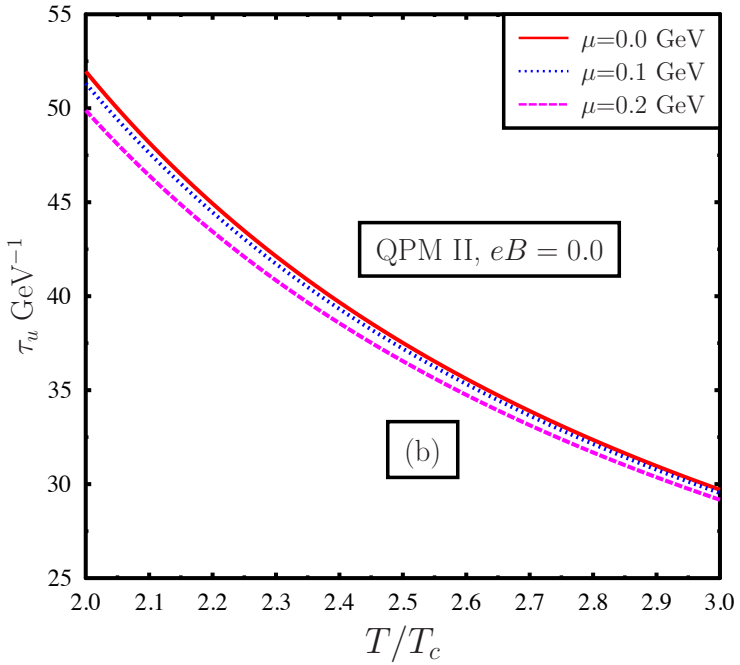
$$\tau_q^{-1} = g_g \int \frac{d^3\vec{p}_g}{(2\pi)^3} f_0^g (1+f_0^g)\left(\frac{g_{eff}^4}{16\pi\langle s\rangle_{gq}}\left[\ln\frac{\langle s\rangle_{gq}}{k^2}-1.287\right]\right)$$

$$+ g_q \int \frac{d^3\vec{p}_q}{(2\pi)^3} f_0^q (1-f_0^q)\left(\frac{g_{eff}^4}{9\pi\langle s\rangle_{qq}}\left[\ln\frac{\langle s\rangle_{qq}}{k^2}-1.417\right]\right)$$

$$+ g_{\bar{q}} \int \frac{d^3\vec{p}_{\bar{q}}}{(2\pi)^3} f_0^{\bar{q}} (1-f_0^{\bar{q}})\left(\frac{9g_{eff}^4}{9\pi\langle s\rangle_{q\bar{q}}}\left[\ln\frac{\langle s\rangle_{q\bar{q}}}{k^2}-1.417\right]\right)$$

$$\alpha_{s_{eff}}(T,\mu) = \frac{g_{eff}^2}{4\pi} = \alpha_s(T,\mu)\frac{\frac{2N_c}{\pi^2}\mathrm{PolyLog}[2,z_g] - \frac{2N_f}{\pi^2}\mathrm{PolyLog}[2,-z_q] + \tilde{\mu}^2\left(\frac{N_f}{\pi^2}\frac{z_q}{1+z_q}\right)}{\left(\frac{N_c}{3}+\frac{N_f}{6}\right)+\tilde{\mu}^2\frac{N_f}{2\pi^2}}$$

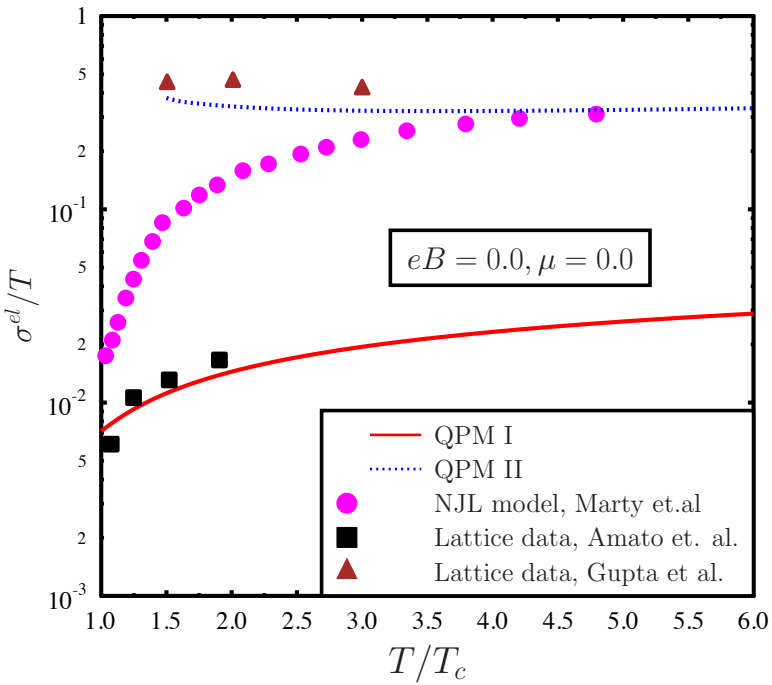




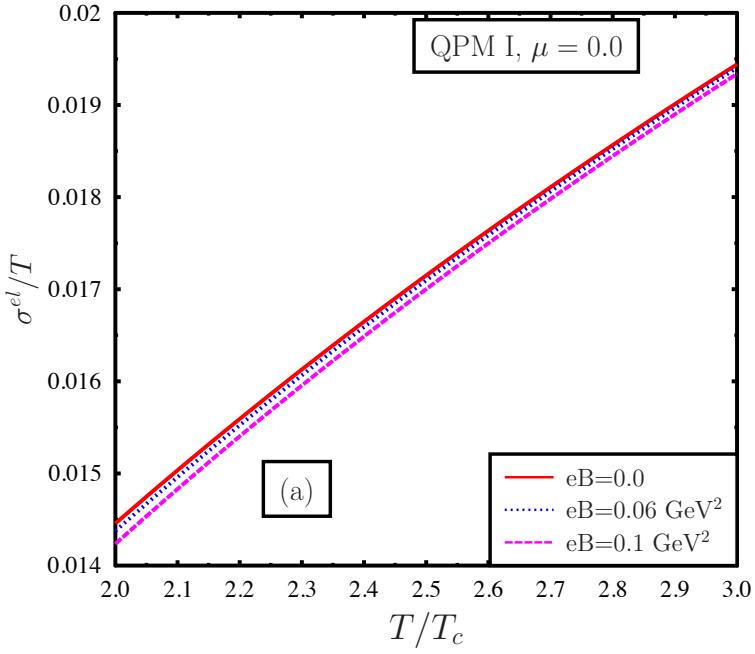
Relaxation time of QPM 2 is one order magnitude is larger than QPM 1.

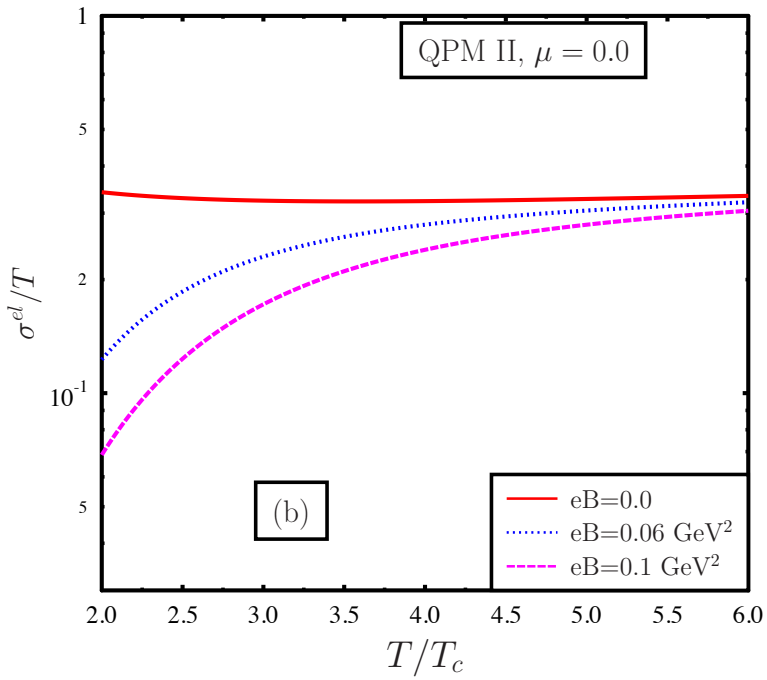
It decreases as temp increases since number density increases as temp increases

RESULTS



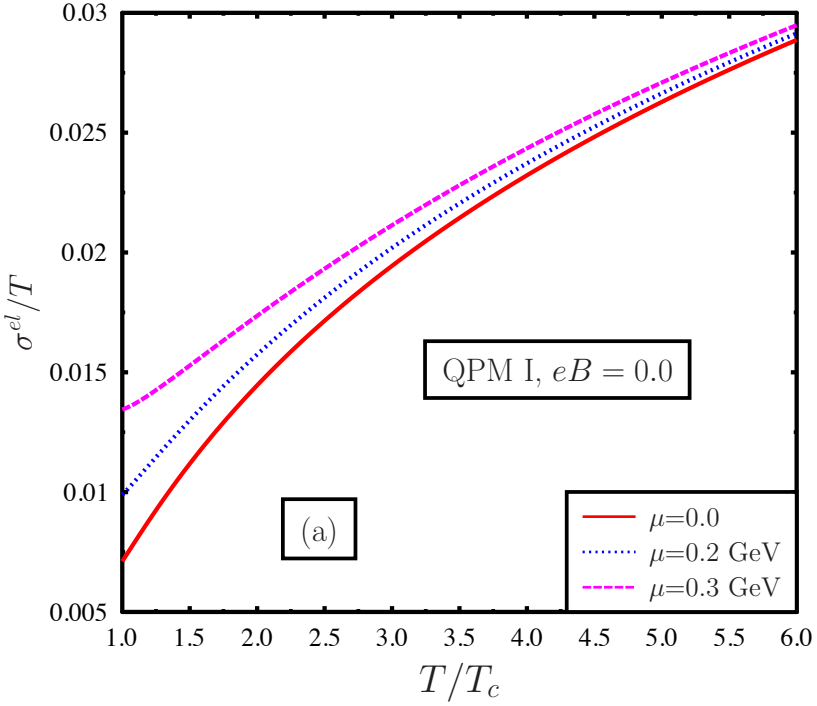
σ_{el} /T for QPM 2 is one order larger than QPM 1 since relaxation time is one order magnitude larger in QPM 2.

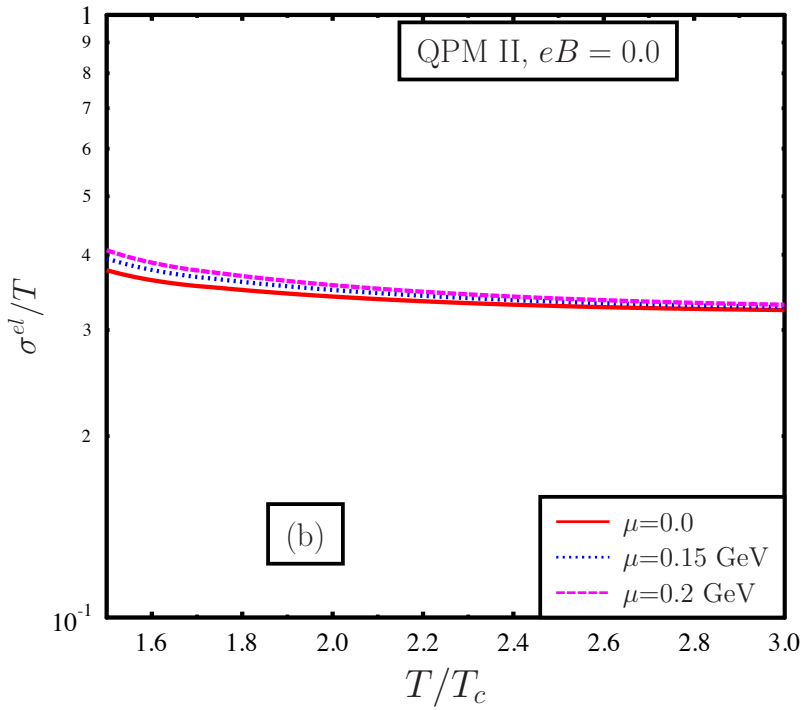




It decreases with B at a fixed temp. But the rate of decrease in QPM 2 is larger than QPM 1.

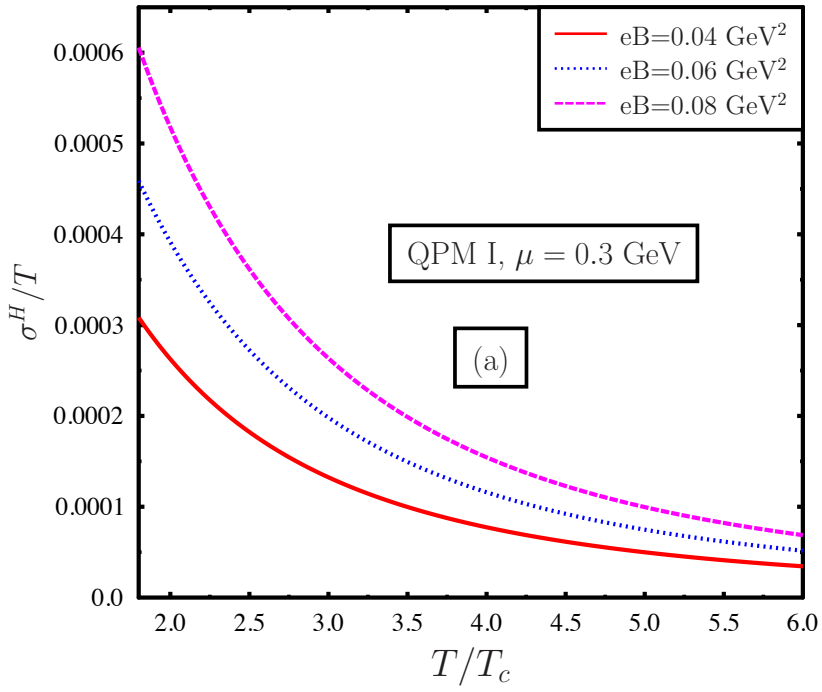
Since in QPM 1 value of the relaxation time is order of magnitude smaller than the relaxation time in QPM II, $\omega\tau$ in the denominator is larger for QPM 2.

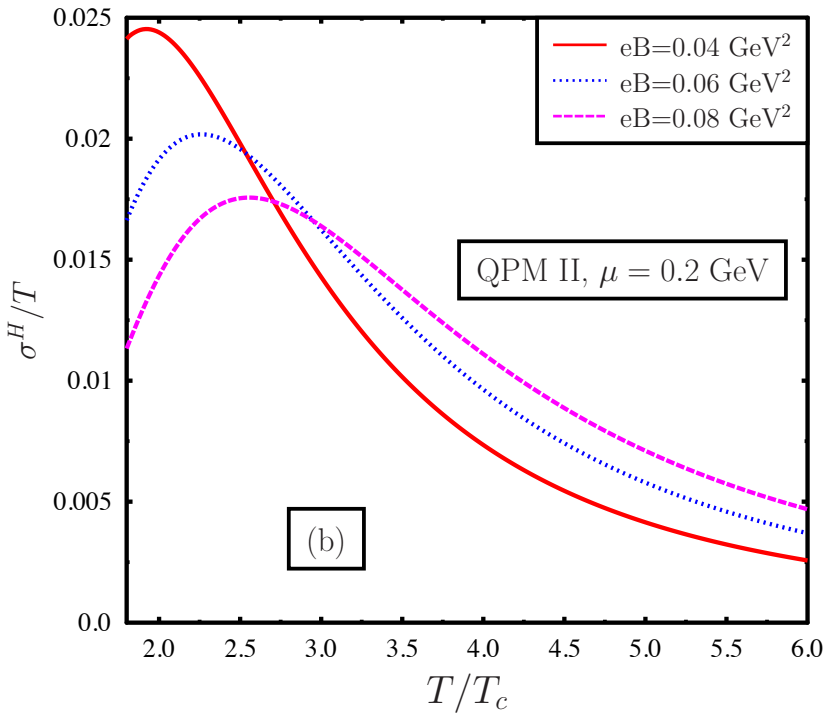




It increases with chemical potential due to the increase in distribution function.

Variation of normalized electrical conductivity with quark chemical potential is connected with the variation of relaxation time with quark chemical potential and the Boltzmann factor in the equilibrium distribution function.





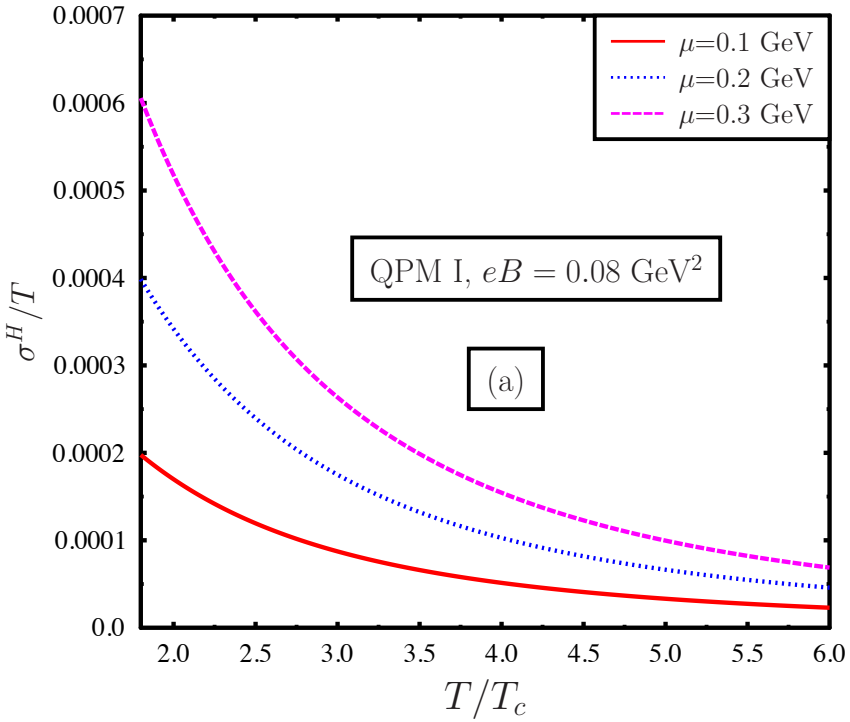
For QPM 1 Hall conductivity increases with magnetic field.

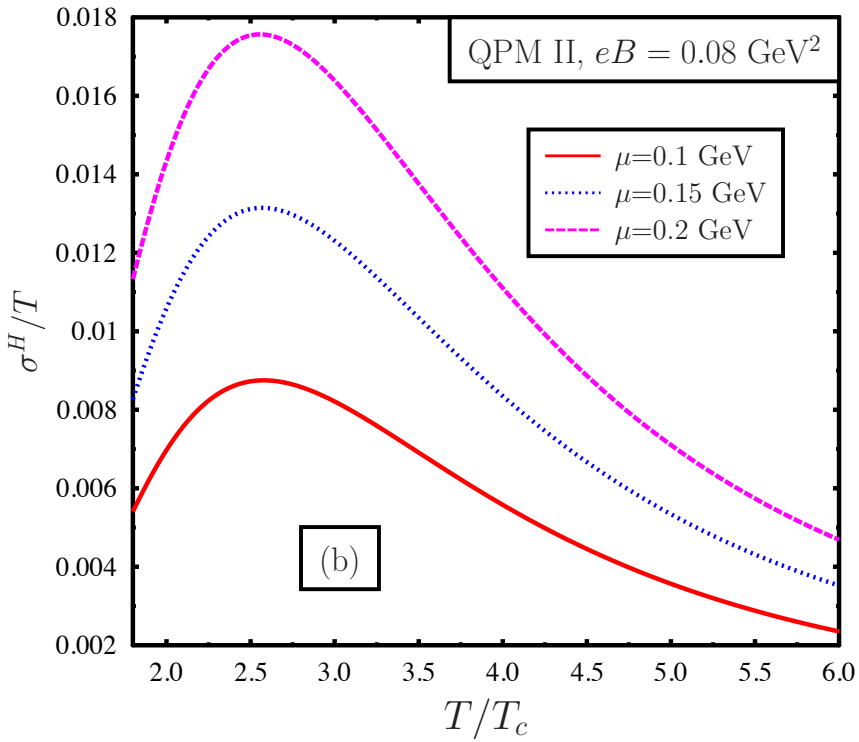
At a fixed B, it decreases with temp due to decrease in relaxation time.

However for QPM 2, it has a non monotonic behavior with temperature, where at small temperature it decreases with increase in magnetic field and at high temperature it increases with increase in magnetic field.

At relatively small temperature relaxation time is large and it goes as $1/\omega\tau$

At high temperature, relaxation time is small and it goes as $\omega\tau$





It increases with chemical potential because no of particles increases compared to anti particles.

It vanishes at zero chemical potential due to equal no of particles and anti Particles and they move in same direction.

CONCLUSIONS

- Electrical and Hall conductivity have been measured for QGP for two different quasi particle models.

- Electrical conductivity for QPM 2 is one order magnitude larger than QPM 1 due to large relaxation time in QPM 2.

- Hall conductivity increases with magnetic field for QPM 1.

- It has a non monotonic behavior with magnetic field for QPM 2.