Heavy Quarks in QGP: Nonperturbative Input from Lattice

Saumen Datta

Tata Institute of Fundamental Research, Mumbai

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Heavy quarks as probe of QGP

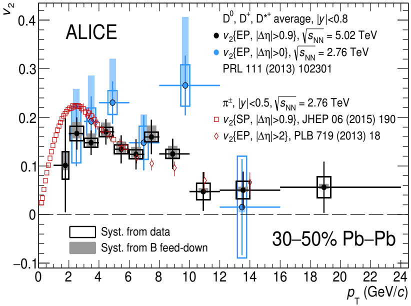
- Heavy quarks c, b provide interesting probes of the medium created in heavy ion collision experiments.
- These experiments lead to a thermalized deconfined medium with a small $\tau_{\rm th} < 1$ fm for the light quarks and gluons.
- The thermalization time of the heavy quarks is expected to be different.

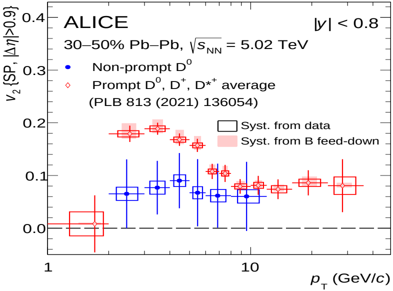
  Svetitsky, PRD 37(1988)2484; Alam, Raha & Sinha, PRL 73(1994)1805; ...

- $m_c, m_b \gg \Lambda_{QCD},\ T_{\rm fireball}$
  production: initial hard scatterings
  $p \sim \sqrt{m_Q T} \gg T$ even for thermalized quarks.
  Scattering with medium particles: momentum exchange $\sim T$
  Takes $\mathcal{O}(m_Q/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
- $\tau_{\rm th} \sim \frac{m_Q}{T}\, \tau_{\rm light}$ large: late/incomplete thermalization?
- R_{AA}, flow of heavy-light mesons: interesting probes.

heavy-light mesons in QGP

Moderate momentum charm quarks flow with the medium flow, almost like the light quarks.





ALICE Coll., PRL 120 (2018) 102301; arXiv: 2307.14084

Heavy quarks in plasma

- Heavy quarks: many (nearly) uncorrelated collisions before momentum change of $\mathcal{O}(1)$.
- A Langevin framework is suitable.
- For a low momentum heavy quark,

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D(p) p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle = \kappa_{ij}(p)\, \delta(t-t')$$

Svetitsky '88; Mustafa, Pal, Srivastava, '97;
Moore & Teaney '05; Rapp & van Hees '05

- Leads to the Fokker-Planck equation

$$\frac{\partial f_Q(p,t)}{\partial t} = -\frac{\partial}{\partial p_i}\left[p_i\, \eta_D(p)\, f_Q(p,t)\right] + \frac{\partial^2}{\partial p_i\, \partial p_j}\left[\kappa_{ij}(p)\, f_Q(p,t)\right]$$

- For low momenta, just one coefficient κ.
Standard nonrelativistic relations connect κ to η_D and D_s, the position space diffusion coefficient.

$$\eta_D = \frac{\kappa}{2\,M\,T}, \qquad \langle x^2(t) \rangle = 6 D_s t, \qquad D_s = \frac{2T^2}{\kappa}$$

Calculation of κ

- A field theoretic definition of κ can be given from the force-force correlator:

$$3\kappa = \frac{1}{\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \{F_i(t,x)\,,\, F_i(0,0)\} \right\rangle$$

- Expanding the force term in a series in $1/M$:

$$F^i = M \frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{[D^i, D^2 + c_b g \sigma \cdot B]}{2M} + ... \right\} \phi$$

- In leading order in $\mathcal{O}\left(\frac{1}{M}\right)$ one gets only the gE force.

$$3\kappa = \frac{1}{3} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt\, e^{i\omega t} \left\langle \mathrm{Tr}\, W(t,-\infty)^\dagger\, E_i(t)\, W(t,0)\, E_i(0)\, W(0,-\infty) \right\rangle$$

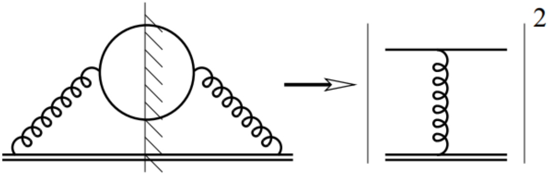
J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53;
A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

Calculation of κ_E

- At leading order of perturbation theory, this simplifies to

$$3\kappa = \frac{g^2}{3} \int \frac{d^3p}{(2\pi)^3} \langle \mathrm{Tr}\, E_i\, E_i \rangle(\omega = 0, \vec{p}) = g^2 \frac{4}{3} \int \frac{d^3p}{(2\pi)^3}\, p^2\, G^{>}_{A_0\, A_0}(0, \vec{p})$$

- Using $G^{>}_{00}(\omega, \vec{p}) = 2\,(1 + n_B(\omega))\,\rho_{00}(\omega, \vec{p})$ this can be connected to standard scattering with thermal particles.



Caron-Huot & Moore, PRL 100 (2008) 052301

- The t-channel $2 \to 2$ collision processes dominate for quarks at medium energy. Radiative processes become important at higher momenta.

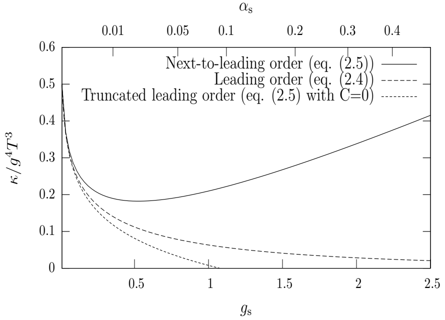
Svetitsky '88, Moore & Teaney 2005, ...
S. Das, J. Alam & P. Mohanty, PRC 82(2010)014908

κ_E in NLO

- The diagrams infrared singular: cured by the Debye screening of $A_0 A_0$ correlator. Calculable in HTL.

$$\frac{\kappa_E}{T^3} = \frac{2\,g^4}{27\,\pi}\left[N_c\left(\ln\frac{2T}{m_D}+C\right) + \frac{N_f}{2}\left(\ln\frac{4T}{m_D}+C\right)\right]$$

- Series in g rather than in α. NLO corrections very large.



Caron-Huot & Moore, PRL 100 (2008) 052301

κ_E from lattice

- The LO calculation gives a value of κ substantially smaller than required to interpret the R_{AA} and v_2 results.
- For phenomenology, one often uses the LO form, with some tuning to get κ in the right ballpark.

see Cao et al., PRC 99 (2019) 054907, for a summary.

- Nonperturbatively, one can calculate the Matsubara correlator on the lattice. This can be connected to the spectral function.

$$G_{EE}(\tau) = \int_0^\infty \frac{d\omega}{\pi} \rho_{EE}(\omega) \frac{\cosh \omega(\tau - 1/2T)}{\sinh \omega/2T}$$

- Then κ_E can be extracted from the infrared behavior of $\rho_{EE}(\omega)$:

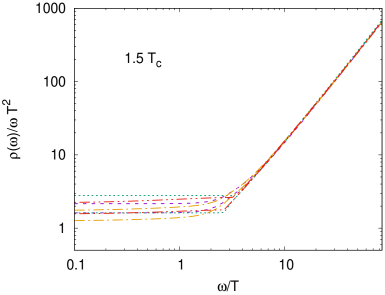
$$\rho_{IR} \underset{\omega \to 0}{\approx} \frac{\kappa_E \, \omega}{2T}$$

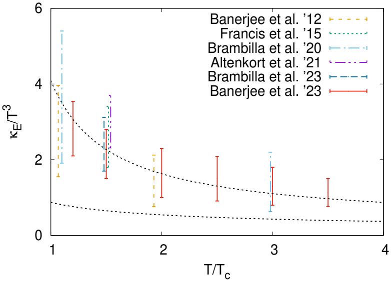
- Extraction of κ_E from $G_{EE}(t)$ requires modelling of $\rho_{EE}(\omega)$.

κ_E from lattice

- In the theory with only thermal gluons, κ_E has been calculated by various groups, and continuum results have been obtained in the temperature range $T_c < T \lesssim 3.5\,T_c$.

Banerjee, Datta, Gavai, Majumdar, PRD (2012), NP A 1038 (2023) 122721
Bielefeld Group(Kaczmarek, et al.), PRD (2015), PRD 103 (2021) 014511.
Brambilla, et al. (TUMQCD), PRD (2020), PRD 107 (2023) 054508



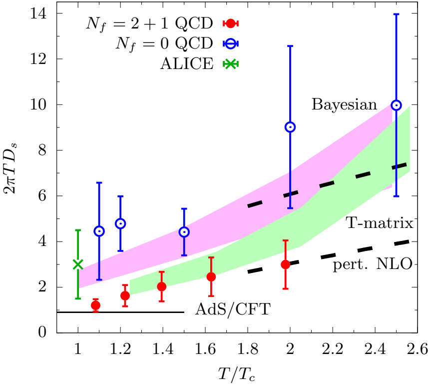


- Good agreement between different calculations, using very different methods and different models for $\rho_{EE}(\omega)$.
- NLO PT result is in agreement with the lattice results.

Unquenching effects

- First results with thermal quarks (with somewhat unphysical quarks, $m_\pi \approx 320$ MeV): large effect of unquenching.

Altenkort, et al., PRL 123(2023)231902



κ from lattice

- At $\mathcal{O}\left(\frac{1}{M}\right)$ the contribution from the BB correlator needs to be taken into account.

$$\kappa_Q \;\approx\; \kappa_E \;+\; \frac{2}{3}\,\langle v^2\rangle\,\kappa_B\,, \qquad \langle v^2\rangle \;=\; \frac{3T}{M_{\rm kin}}$$

Bouttefeux & Laine, JHEP 12 ('20) 150

- Unlike G_{EE}, G_{BB} correlator has an anomalous dimension. κ_B is obtained from the IR part of

$$c_b^2(\mu)\, G_{BB}^{\rm ren}(\mu; T)$$

The μ dependence of c_b^2 cancels that of $G_{BB}(\mu; T)$.

- The infrared singularity from BB correlator is not cured by Debye screening: κ_B is inherently nonperturbative.

$$\frac{\kappa_B}{T^3} \;\approx\; \frac{2\,g^4}{27\,\pi}\left[N_c\left(\ln\frac{2T}{m_G}+C\right) \;+\; \frac{N_f}{2}\left(\ln\frac{4T}{m_G}+\frac{1}{2}+C\right)\right]$$

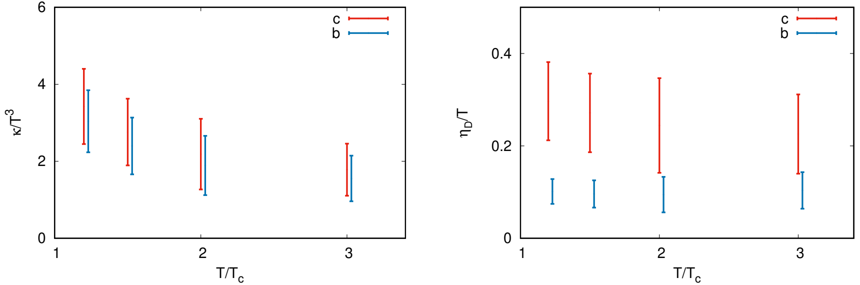
where $m_G \sim c' g^2 T$ is an inherently nonperturbative scale.

Laine, JHEP 06 (2021) 139

$1/m_Q$ correction

- The anomalous dimension of $G_{BB}(t)$ makes the extraction of κ_B somewhat more complicated.
- Z_B large: nonperturbative estimation required.
- A nonperturbative analysis done for the gluon plasma.

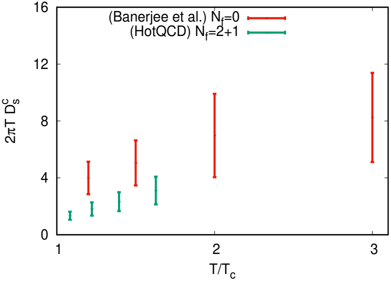
Banerjee, Datta & Laine, JHEP 08(2022) 128

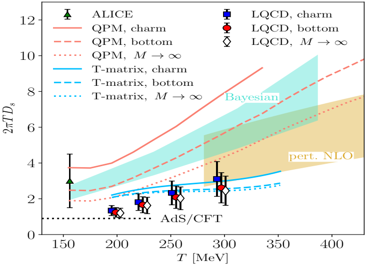


- Size of $1/m_Q$ correction moderate, $\sim$ 30-35% (charm), 15-20% (bottom) at 2 T_c.
- $\tau_b \approx 3\tau_c > 10$ fm indicates incomplete thermalization of b.

Effect of thermal quarks

- The first lattice results for κ_c from a calculation with thermal quarks ($m_\pi \sim 320$ MeV) came out recently.
  L. Altenkort, et al. (HotQCD), arXiv:2311.01525



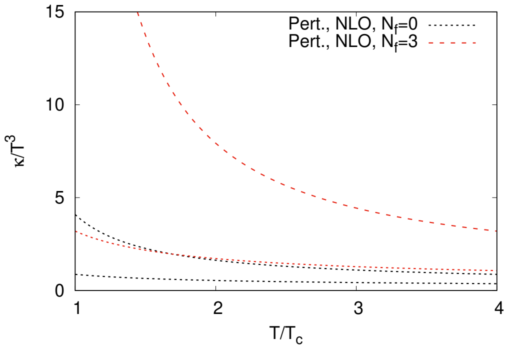


- Size of $1/m_Q$ correction smaller than was seen for gluon plasma.
- The renormalization done differently: would be reassuring to have a similar analysis of Z_B as in the gluonic case.
- Close to T_c unquenching effect substantial.

Summary of results

- Lattice calculations provide reliable nonperturbative results for static properties of the QGP at zero and small baryon chemical potential.
  But transport properties are very difficult to obtain.
- The heavy quark diffusion coefficient κ is one of the quantities where we have had good success.
- For a gluonic plasma, various systematics well-studied.
- Probably the only transport coefficient where a quite sophisticated lattice calculation available for the theory with thermal (if somewhat unphysical) quarks.
- Effect of thermal quarks large. Motivates further study of the systematics of the renormalization, the analysis, and the quark mass effect.

EXTRA SLIDE: Effect of thermal quarks in NLO PT



S. Datta, in:Dynamics of Hot QCD Matter, S.K. Das et al., IJMP E 31 (2022) 12.

EXTRA SLIDE: Matching to Lorentz Force

We need to calculate

$$M \frac{\partial J^i}{\partial t} \equiv \frac{M A_i^{QCD}}{N^{QCD}},$$

where A_i^{QCD} and N^{QCD} are the matrix elements of $\frac{\partial}{\partial t} \int d^3x \, J_i^{QCD}$ and $\int d^3x \, J_0^{QCD}$, respectively.

Matching of the QCD operators to the Lorentz force term has been worked out to NLO by Laine (*2103.14270*).

Taking the IR force term

$$F_i^{IR} = \theta^\dagger \left\{ Z_E \, E_i + Z_B \, (v \times B)_i \right\} \theta$$

Z_E, Z_B are fixed by demanding

$$\left. \frac{MA_i}{N} \right|_{QCD} = \left. \frac{F_i}{N} \right|_{IR} .$$

Calculating both sides to NLO, one gets (*2103.14270*)

$$Z_E = 1, \qquad Z_B = 1 + \frac{g^2 N_c}{16\pi^2} \left[\frac{1}{\epsilon} + 2 \log \frac{\mu_B \, e^{\gamma_E}}{4\pi T} - 2 \right]$$

EXTRA SLIDE: Z_B

- The $\frac{1}{\epsilon}$ term in Z_B cancels a singularity in the BB correlator.
- The scale dependence in the finite term should be cancelled by a Wilson coefficient, equivalent to evaluating the correlator at $\mu_B \approx 4\pi T e^{1-\gamma_E} \approx 19.2 T$.
- The nontrivial task is to connect the lattice correlator to $\overline{MS}$.
- A detailed NP renormalization program for the clover implementation of the B field was carried out by the ALPHA collaboration.

D. Guazzini, H. Meyer & R. Sommer, JHEP 10 (2007) 081.

- NP renormalization coefficient was calculated in Schrödinger functional scheme, and connected to $\overline{MS}$ at a a large scale.
- Then we run it down to 19.2 T.

EXTRA SLIDE: Analysis

- $\rho_{EE}^{IR}(\omega) \equiv \frac{\kappa\,\omega}{2T}$, $\rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$
- $$\mu_B = \max\left[\omega^{1-\frac{\gamma_0}{b_0}}\,(\pi T)^{\frac{\gamma_0}{b_0}},\ \pi T\right], \qquad \gamma_0 = \frac{3}{8\pi^2}\ \mathrm{(BB)},\ \ 0\ \mathrm{(EE)}$$
- Model $\rho(\omega)$ as
  (1) $\rho_1(\omega) \equiv \max\left(\rho_{EE}^{IR}(\omega),\ c\,\rho_{UV}(\omega)\right)$
- Physically better motivated: Francis et al. '15
  (2) $\rho_2(\omega) \equiv \sqrt{(\rho_{EE}^{IR}(\omega))^2 + (c\,\rho_{UV}(\omega))^2}$
- In our fits, we got $c \sim 1$, as expected.
- Also tried adding a Fourier mode

$$\rho_{1f,2f}(\omega) \equiv (1 + d \sin \pi y)\ \rho_{1,2}^{c=1}(\omega), \qquad y = \frac{\log\left(1 + \frac{\omega}{\pi T}\right)}{1 + \log\left(1 + \frac{\omega}{\pi T}\right)}$$

- d was found to be small, $\lesssim 0.12$.
- The whole analysis is done within a bootstrap framework.