Anomalous chiral symmetry in finite temperature QCD and its implications

Sayantan Sharma

The Institute of Mathematical Sciences

January 29, 2024

Meeting on the physics of ALICE, CBM and STAR, VECC

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \rightarrow SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.
  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous yet can affect the order of the chiral phase transition as $m_{u,d} \rightarrow 0$.
  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy $156.5(1.5)$ MeV.
  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous yet can affect the order of the chiral phase transition as $m_{u,d} \to 0$.
  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous yet can affect the order of the chiral phase transition as $m_{u,d} \to 0$.

  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous yet can affect the order of the chiral phase transition as $m_{u,d} \to 0$.

  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

Chiral symmetry in QCD

- Since $m_{u,d} \ll \Lambda_{QCD}$, 2+1 flavor QCD respects $U_L(2) \times U_R(2)$ chiral symmetry to a good extent.

- The non-singlet part of this chiral symmetry gets broken at low T, $SU_A(2) \times SU_V(2) \to SU_V(2)$

- This happens through a crossover transition at a temperature now known to unprecedented accuracy 156.5(1.5) MeV.

  [HotQCD collab. 18, F. Burger et. al. 18, Budapest-Wuppertal collab. 20]

- The singlet part $U_A(1)$ is anomalous yet can affect the order of the chiral phase transition as $m_{u,d} \to 0$.

  [Pisarski & Wilczek 84, Pelissetto & Vicari 13, G. Fejos, 22]

- Do singlet and non-singlet chiral symmetries gets restored simultaneously?

- Not an exact symmetry→ what observables to look for? Degeneracy of the 2-point (integrated) correlation functions [Shuryak, 94]

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- For $\rho(\lambda)$: near-zero modes need careful study. Suffer from lattice cut-off + finite volume effects.

  [HotQCD collaboration, 12, G. Cossu et. al, 13, 14, 15, V. Dick et. al. 15, Suzuki et. al., 18, 20]

- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]

- Measuring higher point correlation functions is relevant.

- Not an exact symmetry$\rightarrow$ what observables to look for? Degeneracy of the 2-point (integrated) correlation functions [Shuryak, 94]

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\to} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- For $\rho(\lambda)$: near-zero modes need careful study. Suffer from lattice cut-off + finite volume effects.

  [HotQCD collaboration, 12, G. Cossu et. al, 13, 14, 15, V. Dick et. al. 15, Suzuki et. al., 18, 20]

- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]

- Measuring higher point correlation functions is relevant.

- Not an exact symmetry$\rightarrow$ what observables to look for? Degeneracy of the 2-point (integrated) correlation functions [Shuryak, 94]

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2+m_f^2)^2}$$

- For $\rho(\lambda)$: near-zero modes need careful study. Suffer from lattice cut-off + finite volume effects.

  [HotQCD collaboration, 12, G. Cossu et. al, 13, 14, 15, V. Dick et. al. 15, Suzuki et. al., 18, 20]

- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]

- Measuring higher point correlation functions is relevant.

- Not an exact symmetry $\rightarrow$ what observables to look for? Degeneracy of the 2-point (integrated) correlation functions [Shuryak, 94]

$$\chi_\pi - \chi_\delta \overset{V\to\infty}{\rightarrow} \int_0^\infty d\lambda \frac{4m_f^2\ \rho(\lambda, m_f)}{(\lambda^2 + m_f^2)^2}$$

- For $\rho(\lambda)$: near-zero modes need careful study. Suffer from lattice cut-off + finite volume effects.

  [HotQCD collaboration, 12, G. Cossu et. al, 13, 14, 15, V. Dick et. al. 15, Suzuki et. al., 18, 20]

- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]

- Measuring higher point correlation functions is relevant.

Looking at spectral density of QCD at finite temperature

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.
- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Non-analyticities in the deep infrared part of the spectrum+analytic form of the bulk.

Looking at spectral density of QCD at finite temperature

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.
- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Non-analyticities in the deep infrared part of the spectrum+analytic form of the bulk.

Looking at spectral density of QCD at finite temperature

- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.
- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Non-analyticities in the deep infrared part of the spectrum+analytic form of the bulk.

Looking at spectral density of QCD at finite temperature

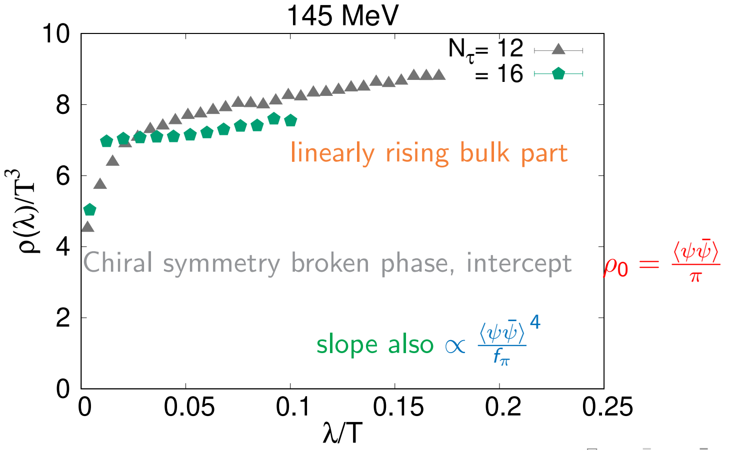
- Very little known. Only recently studied in detail [Aoki, Fukaya & Taniguchi, 12].
- Assuming $\rho(\lambda, m)$ to be analytic in λ, m^2, look at Ward identities of n-point function of scalar & pseudo-scalar currents when chiral symmetry is restored.
- Bulk part: $\rightarrow \rho(\lambda) \sim \lambda^3$ is a necessary cond. for $U_A(1)$ breaking invisible in upto 6 point correlators [Aoki, Fukaya & Taniguchi, 12]
- Non-analyticities in the deep infrared part of the spectrum+analytic form of the bulk.

Typical Spectral Density of QCD

- The eigenvalue density can be characterized as

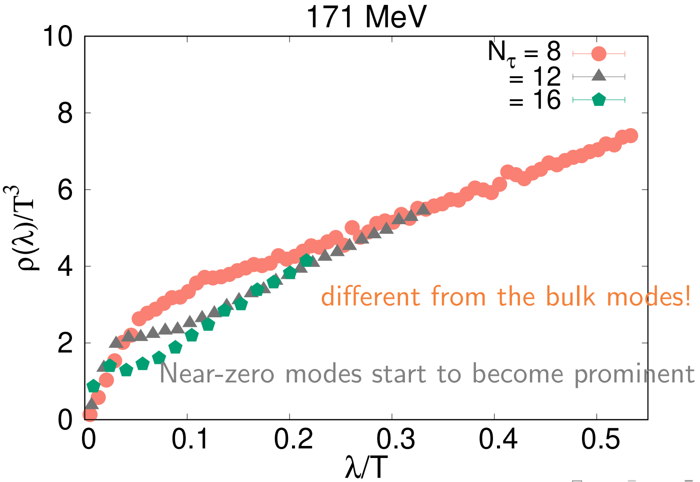
$$\frac{\rho(\lambda)}{T^3} = \frac{\rho_0}{T^3} + \frac{\lambda}{T}.\frac{c_1(T,m)}{T^2} + \frac{\lambda^2}{T^2}.\frac{c_2(T,m)}{T} + \frac{\lambda^3}{T^3} c_3(T,m) \ .$$

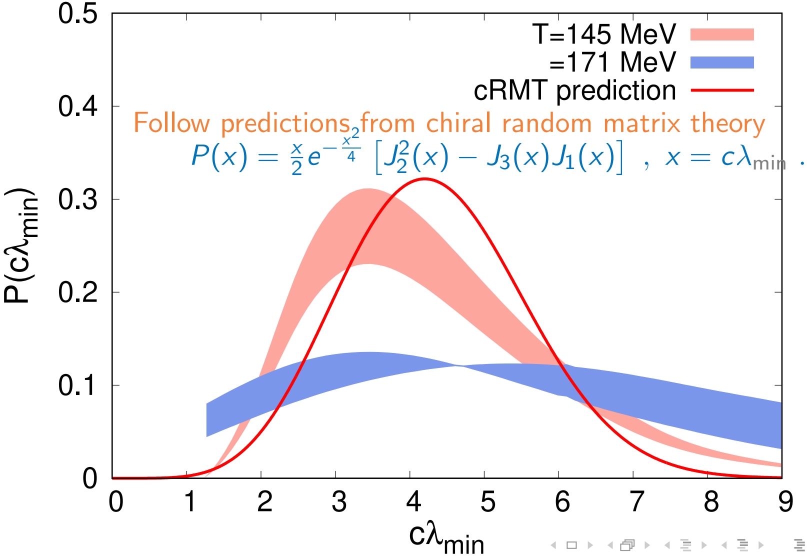
[Fig. from O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].



Spectral Density when chiral symmetry is restored

- The bulk modes show a linear rise characterized by $c(T, m) = 16.8(4)\,T^2 + \mathcal{O}(m^2/T^2)$. This is a new finding which has consequences for U A (1) breaking in the chiral limit.

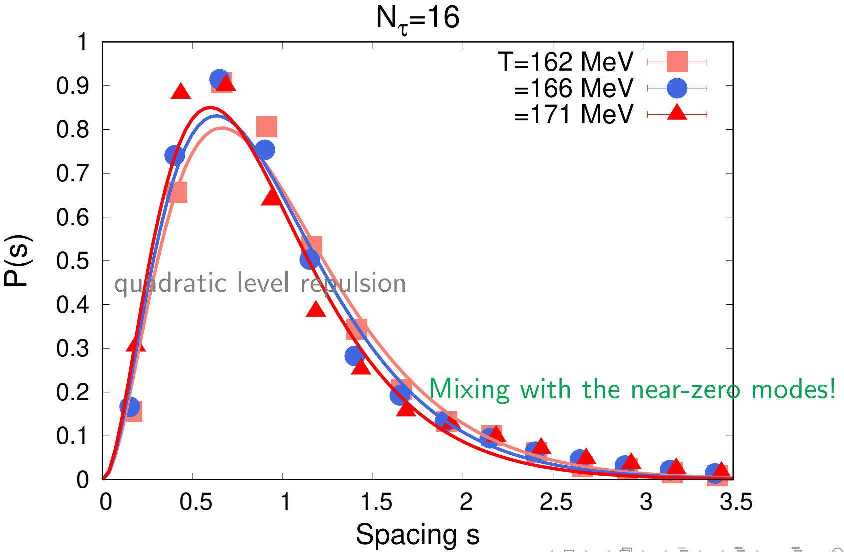




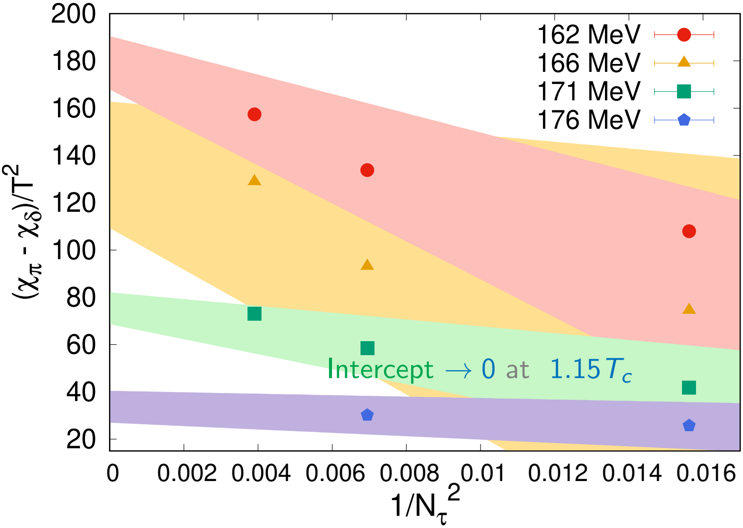
Follow predictions from chiral random matrix theory

$$P(x)=\frac{x}{2}e^{-\frac{x^2}{4}}\left[J_2^2(x)-J_3(x)J_1(x)\right]\ ,\ x=c\lambda_{min}\ .$$

Level-spacing distribution of the bulk modes



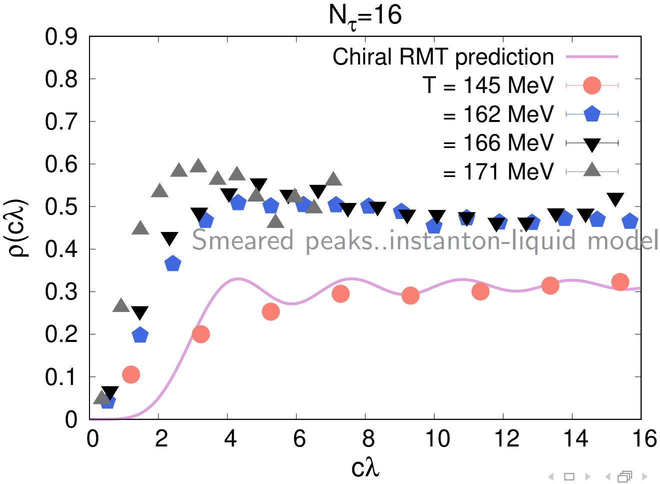
When is $U_A(1)$ effectively restored $\rightarrow$ $1.15\,T_c$



[Fig. from O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].

What more does the eigenspectra tell us?

- $T < T_c$: random matrix theory predicts eigenvalues of QCD $\rightarrow$ disordered phase [Fig. from O. Kaczmarek, Ravi Shanker, S. S., PRD 108, 094501, 2023].
- $T > T_c$: disorder decreases: interactions become short ranged.



Smeared peaks..instanton-liquid model at finite T?

How can we understand our results

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields called instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential creates bulk modes in the quark Dirac spectrum
- $T > T_c$: Interactions among instantons become short range $\rightarrow$ liquid-like
- $T = 1.15T_c$: The near-zero and the bulk modes disentangle. the axial part of chiral symmetry is also restored
- Studies also observe jump in the electrical conductivity at the same T [A. Amato et. al., 14]. Same is observed in interacting many-electron system in a disordered potential [Altshuler et. al, 04]

How can we understand our results

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields called instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential creates bulk modes in the quark Dirac spectrum
- $T > T_c$: Interactions among instantons become short range $\rightarrow$ liquid-like
- $T = 1.15T_c$: The near-zero and the bulk modes disentangle. the axial part of chiral symmetry is also restored
- Studies also observe jump in the electrical conductivity at the same T [A. Amato et. al., 14]. Same is observed in interacting many-electron system in a disordered potential [Altshuler et. al, 04]

How can we understand our results

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields called instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential creates bulk modes in the quark Dirac spectrum
- $T > T_c$: Interactions among instantons become short range $\rightarrow$ liquid-like
- $T = 1.15T_c$: The near-zero and the bulk modes disentangle. the axial part of chiral symmetry is also restored
- Studies also observe jump in the electrical conductivity at the same T [A. Amato et. al., 14]. Same is observed in interacting many-electron system in a disordered potential [Altshuler et. al, 04]

How can we understand our results

- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields called instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential creates bulk modes in the quark Dirac spectrum
- $T > T_c$: Interactions among instantons become short range $\rightarrow$ liquid-like
- $T = 1.15 T_c$: The near-zero and the bulk modes disentangle. the axial part of chiral symmetry is also restored
- Studies also observe jump in the electrical conductivity at the same T [A. Amato et. al., 14]. Same is observed in interacting many-electron system in a disordered potential [Altshuler et. al, 04]

How can we understand our results

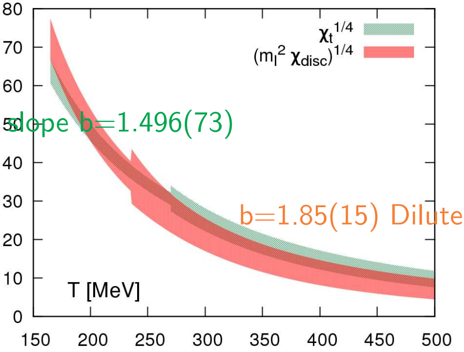
- One can visualize quarks as many-body states moving in the background of lowest energy topological states of gauge fields called instantons
- $T < T_c$: Instantons strongly interacting $\rightarrow$ disordered potential creates bulk modes in the quark Dirac spectrum
- $T > T_c$: Interactions among instantons become short range $\rightarrow$ liquid-like
- $T = 1.15\,T_c$: The near-zero and the bulk modes disentangle. the axial part of chiral symmetry is also restored
- Studies also observe jump in the electrical conductivity at the same T [A. Amato et. al., 14]. Same is observed in interacting many-electron system in a disordered potential [Altshuler et. al, 04]

Topological origin of the $U_A(1)$?

- The topological susceptibility is related to $U_A(1)$ breaking through $\chi_t = \ m^2 \ \chi_{disc} = m^2(\chi_\pi - \chi_\delta)/4$.
- Characterizing, $\chi_t^{1/4}(T) \sim (T_c/T)^b$ [Petreczky, Schadler, S.S. 16].

[See also C. Bonati et. al., 15, 18, Sz. Borsanyi et. al., 16, F. Burger et. al, 18]

Abrupt change in slope b=1.496(73)



b=1.85(15) Dilute instanton gas

Summary

- We are now able to understand the thermodynamics of chiral symmetry restoration, including its anomalous part from the fundamental theory of strong interactions, QCD.
- It involves subtle interplay of many-body interactions among quarks and disorder due to the gauge fields.
- The precise microscopic origin is not yet understood.
- Interesting if one can observe this from the decay of $\eta' \to \gamma\gamma(2.3\%)$, $\eta' \to \rho^0\gamma(30\%)$ meson in the CBM experiment.

Summary

- We are now able to understand the thermodynamics of chiral symmetry restoration, including its anomalous part from the fundamental theory of strong interactions, QCD.
- It involves subtle interplay of many-body interactions among quarks and disorder due to the gauge fields.
- The precise microscopic origin is not yet understood.
- Interesting if one can observe this from the decay of $\eta^{'} \to \gamma\gamma(2.3\%)$, $\eta^{'} \to \rho^{0}\gamma(30\%)$ meson in the CBM experiment.

Summary

- We are now able to understand the thermodynamics of chiral symmetry restoration, including its anomalous part from the fundamental theory of strong interactions, QCD.
- It involves subtle interplay of many-body interactions among quarks and disorder due to the gauge fields.
- The precise microscopic origin is not yet understood.
- Interesting if one can observe this from the decay of $\eta^{'} \rightarrow \gamma\gamma(2.3\%)$, $\eta^{'} \rightarrow \rho^0\gamma(30\%)$ meson in the CBM experiment.

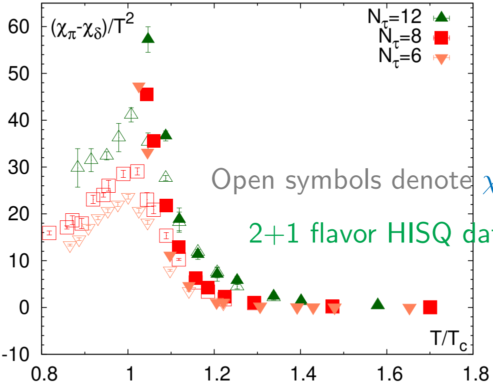
Summary

- We are now able to understand the thermodynamics of chiral symmetry restoration, including its anomalous part from the fundamental theory of strong interactions, QCD.
- It involves subtle interplay of many-body interactions among quarks and disorder due to the gauge fields.
- The precise microscopic origin is not yet understood.
- Interesting if one can observe this from the decay of $\eta^{'} \to \gamma\gamma(2.3\%)$, $\eta^{'} \to \rho^{0}\gamma(30\%)$ meson in the CBM experiment.

- When chiral symmetry is restored

[L. Giusti, G. C. Rossi, M. Testa, 04, HotQCD 1205.3535]

$$\chi_\pi - \chi_\delta = 2\ \chi_{5,disc} = 2\ \chi_{disc}\ .$$



Open symbols denote χ_{disc}

2+1 flavor HISQ data $m_\pi = 160$ MeV

[Fig. from Petreczky, Schadler, S.S. 16].