Transport coefficients of hot and dense matter in relativistic heavy-ion collisions

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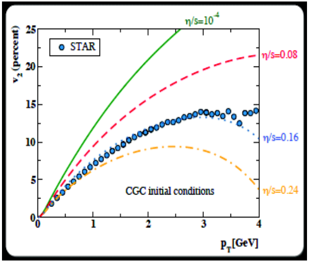
Evidence of dissipative phenomena in the matter created at relativistic heavy-ion collision



M. Luzum and P. Romatschke, PRC 78, 034915 (2008)

- 1982: Analytic fluid modelling of heavy-ion collision by Bjorken.
- 1990s/early 2000s: Relativistic ideal fluid modelling of heavy-ion collisions - non-viscous hydro simulations.
- 2005: Break-through in string theory, estimating shear viscosity for strongly coupled system.
- 2001-2003: Work on relativistic evolution for viscous fluids by Muronga, Teaney etc.
- 2008: The viscous hydrodynamics using small but finite $\eta/s(= 2 \times (1/4\pi))$ explains quantitatively STAR data.

The matter created in heavy ion collision behaves as a strongly interacting liquid

Transport properties – Irreversible phenomena taking place in non-equilibrium systems

Macroscopic Definition

- Tends to smooth out the non-uniformities created by thermodynamic forces

Thermodynamic flows = C Thermodynamic forces

- Constant of proportionality - **Transport coefficients**
- Creates spatial non-uniformities of the macroscopic thermodynamic state variables

Coarse-grained Definition

$$\text{Thermodynamic flows} = \int (\text{phase space part}) \times (\text{irreducible tensor of transported quantity})$$
$$\times \text{ non-equilibrium part of the single particle distribution function}$$

- Needed to be solved from the **transport equation of the system**

Formal framework - Theoretical estimation of transport coefficients

Identification of thermodynamic fields

Particle four flow:
$$N^\mu(x) = N_0^\mu + \delta N^\mu = n u^\mu + V^\mu = \int \frac{d^3p}{(2\pi)^3 p^0} p^\mu (f^{(0)} + \delta f)$$

Energy-momentum tensor:
$$T^{\mu\nu}(x) = T_0^{\mu\nu} + \delta T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \{W^\mu u^\nu + W^\nu u^\mu\} + \pi^{\langle\mu\nu\rangle} = \int \frac{d^3p}{(2\pi)^3 p^0} p^\mu p^\nu (f^{(0)} + \delta f)$$

Equilibrium quantities

1. **Particle number density:** $n(x) = u_\mu N_0^\mu$
2. **Energy density:** $\epsilon = u_\mu u_\nu T_0^{\mu\nu}$
3. **Pressure:** $P(x) = -\frac{1}{3}\Delta_{\mu\nu} T_0^{\mu\nu}$

Dissipative quantities

1. **Bulk viscous pressure :** $\Pi(x) = -\frac{1}{3}\Delta_{\mu\nu}\delta T^{\mu\nu}$
2. **Mean-particle current :** $V^\mu = \Delta^{\mu\nu}\delta N_\nu$
3. **Energy flow or momentum density :** $W^\alpha = \Delta_\mu^\alpha u_\nu \delta T_{\mu\nu}$
4. **Shear viscous pressure :** $\pi^{\mu\nu}(x) = \Delta_{\alpha\beta}^{\mu\nu}\delta T^{\alpha\beta}$

Formal framework - Theoretical estimation of transport coefficients

The relativistic Boltzmann transport equation

$$p^\mu \partial_\mu f(x,p) = C[f] \ ,$$

Single particle distribution function

Collision term - dynamical input

Collision term for a binary elastic collision $p + k \to p' + k'$

$$C[f] = \int \frac{d^3k}{(2\pi)^3 k^0} \frac{d^3p'}{(2\pi)^3 p'^0} \frac{d^3k'}{(2\pi)^3 k'^0} [f(x,p')f(x,k')\{1 \pm f(x,p)\}\{1 \pm f(x,k)\} - f(x,p)f(x,k)\{1 \pm f(x,p')\}\{1 \pm f(x,k')\}]W$$

Reaction rate - dynamical input via differential scattering cross section

$$W = \frac{s}{2}\frac{d\sigma}{d\Omega}(2\pi)^6 \delta^4(p + k - p' - k')$$

Formal framework - Theoretical estimation of transport coefficients in order by order gradient expansion

Chapman-Enskog method in a nutshell

$$p^\mu u_\mu D f = -p^\mu \nabla_\mu f + C[f]$$

$$f = f^{(0)} + \epsilon f^{(1)} + \epsilon^2 f^{(2)} + \cdots$$

Associated lengthscale - dimension of spatial non-uniformities $\sim L$

Associated lengthscale - mean free path $\sim \lambda$

Equilibrium distribution function

Expansion parameter $\sim \frac{\lambda}{L}$

First-order Chapman-Enskog approximation

$$p^\mu \partial_\mu f^{(0)}(x,p) = -\mathcal{R}[\phi]$$

Next to leading order particle distribution function: $f(x,p) = f^{(0)} + f^{(0)}(1 \pm f^{(0)})\phi$

Linearized collision term : $\mathcal{R}[\phi] = \int (\text{phase-space part}) \times [\phi(x,p) + \phi(x,k) - \phi(x,p') - \phi(x,k')]W$

Formal framework - Theoretical estimation of transport coefficients in order by order gradient expansion

Decomposition of space-time derivative

$$\partial^\mu = u^\mu D + \nabla_\mu$$

Covariant time-derivative $\equiv u^\nu \partial_\nu$

Spatial gradient $\equiv \Delta_{\mu\nu}\partial^\nu$

Linearized transport equation

$$\left[\left(\frac{p\cdot u}{T^2}\right)^2 \frac{DT}{T} + \frac{(p\cdot u)}{T} D\left(\frac{\mu}{T}\right) - \left(\frac{p\cdot u}{T}\right)\frac{p^\nu}{T} Du_\nu\right] + \left[\frac{p_\mu}{T}\left\{\frac{p\cdot u}{T}\left(\frac{\nabla^\mu T}{T}\right) + \nabla^\mu\left(\frac{\mu}{T}\right) - \frac{p_\nu}{T}\nabla^\mu u^\nu\right\}\right] = -\frac{1}{T}\mathcal{R}[\phi]$$

Time derivatives need to be replaced by thermodynamic identities

Conservation laws

$$\partial_\mu N^\mu = 0 \quad , \quad \partial_\nu T^{\mu\nu} = 0$$

1. **Equation of continuity :** $Dn = -n\partial_\mu u^\mu - \nabla_\mu V^\mu + V_\mu Du^\mu$
2. **Equation of energy :** $D\epsilon = -hn\partial_\mu u^\mu + \Pi^{\mu\nu}\nabla_\nu u_\mu - \nabla_\mu W^\mu + 2W_\mu Du^\mu$
3. **Equation of motion :** $hnDu^\mu = \nabla^\mu P - \Delta^\mu_\nu \nabla_\sigma \Pi^{\nu\sigma} + (nh)^{-1}\Pi^{\mu\sigma}\nabla_\sigma P - \Delta^\mu_\nu DW^\nu - W^\mu(\partial\cdot u) - W^\nu\nabla_\nu u^\mu$

Formal framework - Theoretical estimation of transport coefficients in order by order gradient expansion

Linearized transport equation in terms of thermodynamic forces

$$\left[\frac{1}{3}|\vec{p}|^2-\tilde{E}_p^2\left(\frac{\partial P}{\partial\epsilon}\right)_n-\tilde{E}_p\frac{1}{T}\left(\frac{\partial P}{\partial n}\right)_\epsilon\right](\partial\cdot u)-p^{\langle\mu\rangle}\left[\tilde{E}_p\left(\frac{\nabla_\mu T}{T}-\frac{\nabla_\mu P}{\epsilon+P}\right)+\nabla_\mu\left(\frac{\mu}{T}\right)\right]-p^{\langle\mu}p^{\nu\rangle}\nabla_{\langle\mu}u_{\nu\rangle}=-\frac{1}{T}\mathcal{R}[\phi]$$

- **Estimating bulk viscous coefficient :**

$$\Pi=-T^2\int\frac{d^3p}{(2\pi)^3p^0}\left[\frac{1}{3}|\vec{p}|^2-\tilde{E}_p^2\left(\frac{\partial P}{\partial\epsilon}\right)_n-\tilde{E}_p\frac{1}{T}\left(\frac{\partial P}{\partial n}\right)_\epsilon\right]f^{(0)}(1\pm f^{(0)})\phi=-\zeta(\partial\cdot u)$$

- **Estimating diffusion coefficient :**

$$q^\mu=T^2\int\frac{d^3p}{(2\pi)^3p^0}\tilde{p}^{\langle\mu\rangle}\left(\tilde{E}_p-\frac{\epsilon+P}{nT}\right)f^{(0)}(1\pm f^{(0)})\phi=-\mathbf{K_n}\left(\frac{\epsilon+p}{nT}\right)\nabla^\mu\left(\frac{\mu}{T}\right)$$

- **Estimating shear viscous coefficient :**

$$\pi^{\mu\nu}=T^2\int\frac{d^3p}{(2\pi)^3p^0}\tilde{p}^{\langle\mu}\tilde{p}^{\nu\rangle}f^{(0)}(1\pm f^{(0)})\phi=2\eta(\nabla_{\langle\mu}u_{\nu\rangle})$$

Formal framework - Theoretical estimation of transport coefficients in order by order gradient expansion

- **Bulk viscous coefficient :**

$$\frac{\zeta}{T^3} = \frac{\left\{\frac{z^2}{3}I_2 + \frac{1}{T}\left(\frac{\partial P}{\partial n}\right)_{\epsilon} I_3 + \left(\left(\frac{\partial P}{\partial \epsilon}\right)_n - \frac{1}{3}I_4\right)\right\}^2}{\left[\tilde{E}_p^2,\ \tilde{E}_p^2\right]}$$

- **Thermal conductivity :**

$$\frac{\lambda}{T^2} = -\frac{\left\{J_2 - \left(\frac{\epsilon+P}{nT}\right) J_1\right\}^2}{\left[\tilde{E}_p \tilde{p}^{\langle\mu\rangle},\ \tilde{E}_p \tilde{p}_{\langle\mu\rangle}\right]}$$

- **Shear viscous coefficient :**

$$\frac{2\eta}{T^3} = \frac{\{K_0\}^2}{\left[\tilde{p}^{\langle\mu}\tilde{p}^{\nu\rangle},\ \tilde{p}_{\langle\mu}\tilde{p}_{\nu\rangle}\right]}$$

$$[F, G] = \int_{p,k,p',k'} \{F(p) + F(k) - F(p') - F(k')\} \{G(p) + G(k) - G(p') - G(k')\}\ W$$

Transport coefficients are inversely proportional to the interaction cross section

Formal framework - Theoretical estimation of transport coefficients in moment method

Grad's moment method in a nutshell

$$p^\mu \partial_\mu f^{(0)} + f^{(0)}(1 \pm f^{(0)}) p^\mu \partial_\mu \phi + \phi p^\mu \partial_\mu f^{(0)} = -\mathcal{L}[\phi]$$

Additional higher order terms

Grad's 14 moment method

1. ϕ is expressed as a sum of scalar products of tensors formed from $\tilde{p}^\mu = p^\mu / T$ and tensor functions of x_μ
2. The coefficients further expanded in a power series of $\tilde{E}_p = (p \cdot u)/T$ up to the first non-vanishing contribution to the irreversible flows

Constructing the deviation function ϕ

$$\phi = A(x, \tilde{E}_p) + B^\mu(x, \tilde{E}_p)\tilde{p}_{\langle\mu\rangle} + C^{\mu\nu}(x, \tilde{E}_p)\tilde{p}_{\langle\mu}\tilde{p}_{\nu\rangle}$$

$A = \sum_{s=0}^{2} A^s(x)\tilde{E}_p^s$

$B^\mu = \sum_{s=0}^{1} \{B^s(x)\}^\mu \tilde{E}_p^s$

$C^{\mu\nu} = \{C^0(x)\}^{\mu\nu}$

Formal framework - Theoretical estimation of transport coefficients in moment method

The unknown coefficients are estimated in terms of the dissipative flows

$$A^1, A^2, A^3 \sim \Pi \qquad \{B^0\}^\mu, \{B^1\}^\mu \sim q^\mu \qquad \{C^0\}^{\mu\nu} \sim \pi^{\mu\nu}$$

ϕ **is expressed as a linear combination of the thermodynamic irreversible dissipative flows**

1. Zeroth moment of transport equation gives conservation of particle four-flow - $\partial_\mu N^\mu = 0$
2. First moment of transport equation gives conservation of energy-momentum - $\partial_\mu T^{\mu\nu} = 0$
3. Second moment of transport equation gives hydrodynamic evolution equations of the form

$$\rho^{\mu_1\mu_2\cdots} + \tau_\rho D\rho^{\mu_1\mu_2\cdots} = first \;\; order \;\; terms$$

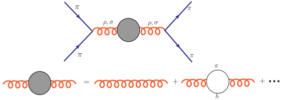
Transport coefficients in hot hadronic matter

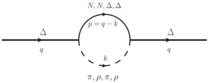
Introducing medium effect in hadronic cross section

$$D_{\mu\nu} = D^{(0)}_{\mu\nu} + D^{(0)}_{\mu\rho}\Pi^{\rho\lambda}D_{\lambda\nu}$$

Vacuum propagator

One loop self energy function





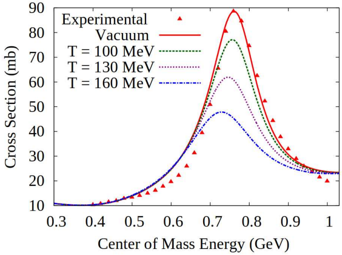


Figure: $\pi - \pi$ cross section

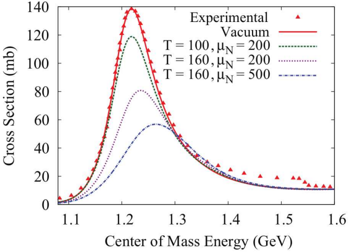
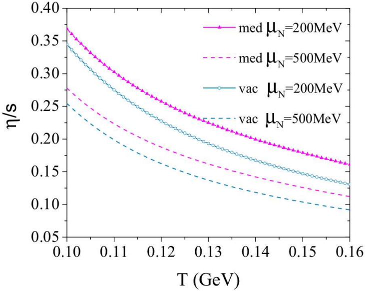


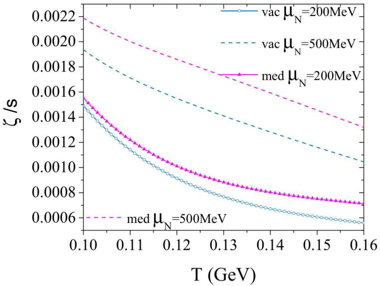
Figure: $\pi - N$ cross section

Transport coefficients in hot hadronic matter

Shear viscosity



Bulk viscosity



Ref : Gangopadhyaya, Ghosh, Sarkar and Mitra, PRC 94, 044914 (2016)

- Considerable change in the value of the η/s and ζ/s due to the introduction of the medium effect.
- The decrease of the in-medium cross section with increasing T and μ_N results in an increase the viscous coefficients.

Transport coefficients quasi particle theory

Properties of elementary particles becomes "dressed" by the medium interactions

Quasi-parton equilibrium distribution : $f_k = z_k(T) \exp\left[\frac{E_{p_k}}{T} - \frac{\mu_{B_k}}{T}\right] / \left(1 \mp z_k \exp\left[\frac{E_{p_k}}{T} - \frac{\mu_{B_k}}{T}\right]\right)$

Energy dispersion : $\omega_{p_k} = E_{p_k} + \Delta_k, \quad \Delta_k = T^2 \partial_T ln(z_k)$ Ref :V. Chandra et al. PRD **84** (2011) 074013

EOS : (2+1)-flavor lattice QCD simulations at physical quark masses
Ref : A. Bazavov et al. [HotQCD Collaboration], Phys. Rev. D 90 (2014) 094503

Covariant theory for EQPM model

$$\frac{1}{\omega_{p_k}} p_k^\mu \partial_\mu f_k(x, p_k) + F^i \frac{\partial f_k}{\partial p_k^i} = \sum_{l=1}^{N} C_{kl}[f_k, f_l] , \qquad p_k^\mu = (\omega_{p_k}, |\vec{p_k}|), \qquad [k = 1, ..., N]$$

mean field force $= -\partial_\mu \{\Delta_k \; u^\mu u^i\}$

Particle four flow: $N^\mu(x) = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p_k}|}{(2\pi)^3 \omega_{p_k}} p_k^\mu f_k + \sum_{k=1}^N \nu_k \Delta_k \int \frac{d^3|\vec{p_k}|}{(2\pi)^3 \omega_{p_k}} \frac{\langle p_k^\mu \rangle}{|\vec{p_k}|} f_k$

Energy-momentum tensor: $T^{\mu\nu}(x) = \sum_{k=1}^N \nu_k \int \frac{d^3|\vec{p_k}|}{(2\pi)^3 \omega_{p_k}} {p_k}^\mu {p_k}^\nu f_k + \sum_{k=1}^N \nu_k \Delta_k \int \frac{d^3|\vec{p_k}|}{(2\pi)^3 \omega_{p_k}} \frac{\langle\langle {p_k}^\mu {p_k}^\nu \rangle\rangle}{|\vec{p_k}|} f_k$

Ref : S. Mitra and V. Chandra, Phys. Rev. D 97 (2018) no.3, 034032

Transport coefficients quasi particle theory

Second order hydrodynamic equation satisfied by shear viscous flow

$$\pi^{\alpha\beta} = 2\eta\nabla^{\langle\alpha}u^{\beta\rangle} - \tau_\pi\left\{\Delta^{\alpha\beta}_{\mu\nu}D\pi^{\mu\nu} - 2\pi_\rho^{\langle\alpha}\omega^{\beta\rangle\rho} + \delta_{\pi\pi}\pi^{\alpha\beta}(\partial\cdot u) + \tau_{\pi\pi}\pi_\rho^{\langle\alpha}\langle\nabla^{\beta\rangle}u^\rho\rangle\right\}$$

Hydro equations for a 1+1 boost invariant system

Pressure anisotropy : $P_L/P_T = (P-\phi)/(P+\frac{\phi}{2})$

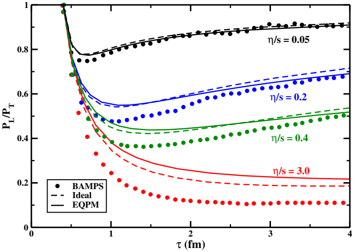
Hydrodynamic evolution equations

$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon+P}{\tau} + \frac{\phi}{\tau}$$

$$\frac{d\phi}{d\tau} = -\frac{\phi}{\tau_\pi} + \frac{2}{3}\frac{1}{\tau}\frac{2\eta}{\tau_\pi} - \lambda\frac{\phi}{\tau}$$

$\phi = -\tau^2\Pi^{\eta_s\eta_s}$

$\lambda = \frac{1}{3}\tau_{\pi\pi} + \delta_{\pi\pi}$



Ref : S. Mitra, PRD 100 (2019) no.1, 014012

Field redefinition of relativistic hydrodynamic theory

Relativistic Transport Equation of Single particle Distribution Function

Homogeneous solution	**Inhomogeneous solution**
Fully arbitrary	Controlled by system interaction
Decided by the matching condition owing to the frame choice	Extracted solely from the underlying microscopic theory

Macroscopic Hydrodynamic Equation of Conserved Quantities

Special cases :

- Landau-Lifshitz and Eckart frames both set the energy and particle number density to their equilibrium values - Energy density and particle number density correction is zero.
- Landau-Lifshitz frame sets the hydrodynamic velocity as the eigen vector of energy momentum tensor - Energy flux or momentum density is zero.
- Eckart frame expresses the hydrodynamic velocity as the particle-4-flow - Particle current is zero.

Field redefinition of relativistic hydrodynamic theory

Relativistic transport equation

$$p^\mu \partial_\mu f(x,p) = C[f] = -\mathcal{L}[\phi]$$

$f = f^{(0)} + f^{(0)}(1 \pm f^{(0)})\phi$

collision term

Linearized collision term and its properties:

$$\mathcal{L}[\phi] = \int \frac{d^3p_1}{(2\pi)^3 p_1^0} \frac{d^3p'}{(2\pi)^3 p'^0} \frac{d^3p'_1}{(2\pi)^3 p_1'^0} f^{(0)} f_1^{(0)} (1 \pm f'^{(0)})(1 \pm f_1'^{(0)}) \{\phi + \phi_1 - \phi' - \phi'_1\} W(pp_1 \mid p'p'_1)$$

- $\mathcal{L}[p^\mu] = 0, \;\; \mathcal{L}[1] = 0$: Energy-momentum and particle number conservation.
- $\int d\Gamma_p \psi \mathcal{L}[\phi] = \int d\Gamma_p \phi \mathcal{L}[\psi]$ with $\psi = \psi(x, p^\mu)$: Self adjoint properties.
- $\int d\Gamma_p \mathcal{L}[\phi] = 0$ and $\int d\Gamma_p p^\mu \mathcal{L}[\phi] = 0$: Summational invariant property.
- $\int d\Gamma_p \phi \mathcal{L}[\phi] \geq 0$: Non-negative entropy production rate.

$\phi \sim 1, p^\mu \longrightarrow$ Homogeneous solution - Fully arbitrary

$\phi \sim p^\mu p^\nu$ or beyond $\longrightarrow$ Inhomogeneous solution - Controlled by system interaction

Field redefinition of relativistic hydrodynamic theory

r^{th}order of gradient expansion : $\sum_l Q_l^{(r)} X_l^{(r)} + \sum_m R_m^{(r)\mu} Y_{m\mu}^{(r)} + \sum_n S_n^{(r)\mu\nu} Z_{n\mu\nu}^{(r)} = -\mathcal{L}[\phi^{(r)}]$

$X_l^{(r)}, Y_{m\mu}^{(r)}$ and $Z_{n\mu\nu}^{(r)} \rightarrow$ scalar, vector and rank-2 tensor thermodynamic forces of gradient expansion order r

General solution for ϕ^r as a linear combination of the thermodynamic forces

$$\phi^{(r)} = \sum_l A_l^r X_l^{(r)} + \sum_m B_m^{r\mu} Y_{m\mu}^{(r)} + \sum_n C_n^{r\mu\nu} Z_{n\mu\nu}^{(r)}$$

$A_l^r, B_m^{r\mu}$ and $C_n^{r\mu\nu}$ are unknown coefficients needed to be estimated from the transport equation

Solution from relativistic transport equation

$$Q_l^{(r)} = -\mathcal{L}[A_l^r]\ , R_l^{(r)\mu} = -\mathcal{L}[B_l^{r\mu}]\ , S_l^{(r)\mu\nu} = -\mathcal{L}[C_l^{r\mu\nu}]$$

$A_l^{(r)} = \sum_{s=0}^{\infty} A_l^{r,s}(z,x)\tilde{E}_p^s$

$B_m^{(r)\mu} = \sum_{s=0}^{\infty} B_m^{r,s}(z,x)\tilde{E}_p^s \tilde{p}^{\langle\mu\rangle}$

$C_n^{(r)\mu\nu} = \sum_{s=0}^{\infty} C_n^{r,s}(z,x)\tilde{E}_p^s \tilde{p}^{\langle\mu}\tilde{p}^{\nu\rangle}$

Field redefinition of relativistic hydrodynamic theory

Interaction solutions $\rightarrow (A_l^r)^2, (B_l^r)^1, (C_l^r)^0$ **:**

Can be extracted from transport equation with the form $\sim 1/[\phi,\phi]$ Bracket quantity

$\rightarrow [\phi,\phi] = \int d\Gamma_p \phi \mathcal{L}[\phi] \rightarrow$ always non-negative, contains interaction

Homogeneous solutions $\rightarrow (A_l^r)^0, (A_l^r)^1$, $(B_l^r)^0$ **:**

Can not be extracted from transport equation, needed to be fixed from matching conditions

Field corrections

1. **Particle number density correction:** $\delta n = \int \frac{d^3p}{(2\pi)^3 p^0}(p\cdot u)\delta f$
2. **Energy density correction:** $\delta\epsilon = \int \frac{d^3p}{(2\pi)^3 p^0}(p\cdot u)^2\delta f$
3. **Pressure correction:** $\delta P = -\frac{1}{3}\int \frac{d^3p}{(2\pi)^3 p^0}|\vec{p}|^2\delta f$
4. **Corrections to mean-particle velocity :** $n_0\delta u_N^\mu = V^\mu = \int \frac{d^3p}{(2\pi)^3 p^0} p^{\langle\mu\rangle}\delta f$
5. **Corrections to energy flow or momentum density :** $(\epsilon_0 + P_0)\delta u_E^\alpha = W^\alpha = \int \frac{d^3p}{(2\pi)^3 p^0}(p\cdot u)p^{\langle\mu\rangle}\delta f$

The most general field expressions including corrections

$$N^\mu = (n_0 + \delta n)u^\mu + V^\mu$$
$$T^{\mu\nu} = (\epsilon_0 + \delta\epsilon)u^\mu u^\nu - (P_0 + \delta P)\Delta^{\mu\nu} + (W^\mu u^\nu + W^\nu u^\mu) + \pi^{\mu\nu}$$

RTA is a simple method to linearize the collision term

Relativistic transport equation

$$p^\mu \partial_\mu f(x,p) = -\frac{\tau_p}{\tau_R} f^{(0)}(1 \pm f^{(0)})\phi, \qquad \tau_R(x,p) = \tau_R^0(x)\tau_p^n$$

relaxation time of single particle distribution

momentum independent part

momentum dependent part

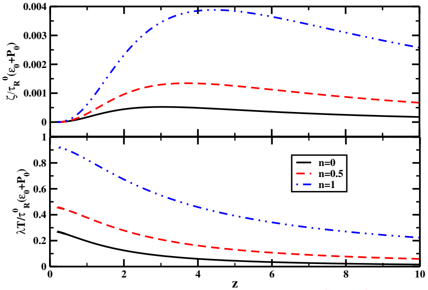
Motivation behind ···

- Microscopic interaction theory relevant for a medium relates to momentum dependence of relaxation time, taking a constant one undermines the momentum transfer of underlying interactions.
- Momentum independent RTA gives rise to identical relaxation times for microscopic particle distributions and macroscopic fields like viscous flow, while the later is expected to have a slower relaxation rate.

First order field corrections

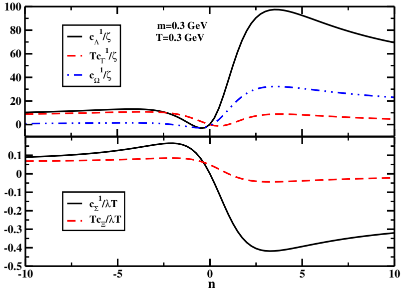
$$\delta\epsilon^{(1)} = c^1_\Lambda(\partial\cdot u)\ ,\quad \delta n^{(1)} = c^1_\Gamma(\partial\cdot u)\ ,\quad \delta P^{(1)} = c^1_\Omega(\partial\cdot u)$$
$$W^{(1)\alpha} = -c_\Sigma\hat{h}(\nabla^\alpha T/T - Du^\alpha)\ ,\quad V^{(1)\alpha} = c_\Xi(\nabla^\alpha\tilde{\mu})$$

Physical transport coefficients



Ref : S. Mitra, PRC 105, 014902 (2022)

Correction coefficients



Field redefinition of relativistic hydrodynamic theory

Number of independent transport coefficients are conserved

$$\textbf{Scalar coefficients :}\ c^1_\Omega - c^1_\Lambda \Big(\frac{\partial P_0}{\partial \epsilon_0}\Big)_{n_0} - c^1_\Gamma \Big(\frac{\partial P_0}{\partial n_0}\Big)_{\epsilon_0} = -\zeta$$

$$\textbf{Vector coefficients :}\ c^1_\Sigma - \frac{(\epsilon_0 + P_0)}{n_0} c^1_\Xi = -\frac{\lambda T}{\hat{h}}$$

Field corrections combine to produce dissipative flux

$$\textbf{Scalar correction :}\ \delta P^{(1)} - \Big(\frac{\partial P_0}{\partial \epsilon_0}\Big)_{n_0} \delta\epsilon^{(1)} - \Big(\frac{\partial P_0}{\partial n_0}\Big)_{\epsilon_0} \delta n^{(1)} = \Pi^{(1)}$$

$$\textbf{Vector correction :}\ W^{(1)\mu} - \hat{h} T V^{(1)\mu} = q^{(1)\mu}$$

- For $n = 0$, energy correction, particle number correction and energy flux vanishes leaving the entire scalar dissipation to pressure correction and vector dissipation to particle flux.
- Dissipative corrections is determined by how the medium interaction distributes the dissipative fluxes among the scalar and vector fields.

Field redefinition of relativistic hydrodynamic theory

Validation of MDRTA

- **Conservation of particle four-flow and energy-momentum tensor hold**

$$\partial_\mu N^\mu = \int \frac{d^3p}{(2\pi)^3 p^0} p^\mu \partial_\mu f = -\frac{T}{\tau_R^0}\int \frac{d^3p}{(2\pi)^3 p^0}\tau_p^{1-n} f^{(0)}(1\pm f^{(0)})\phi^{(1)} = 0$$

$$\partial_\mu T^{\mu\nu} = \int \frac{d^3p}{(2\pi)^3 p^0} p^\nu p^\mu \partial_\mu f = -\frac{T^2}{\tau_R^0}\int \frac{d^3p}{(2\pi)^3 p^0}\{u^\nu \tau_p^{2-n} + \tilde{p}^\nu \tau_p^{1-n}\} f^{(0)}(1\pm f^{(0)})\phi^{(1)} = 0$$

- **Non-negative entropy production rate**

$$T\partial_\mu S^\mu = \zeta(\partial\cdot u)^2 + 2\eta\sigma^{\mu\nu}\sigma_{\mu\nu} + \frac{\lambda T}{\hat{h}^2}(\vec{\nabla}\tilde{\mu})^2$$

1. This is non-negative as long as $\zeta, \lambda, \eta \geq 0$.
2. Individual values of dissipative correction coefficients do not affect the positive entropy production rate.

Field redefinition of relativistic hydrodynamic theory

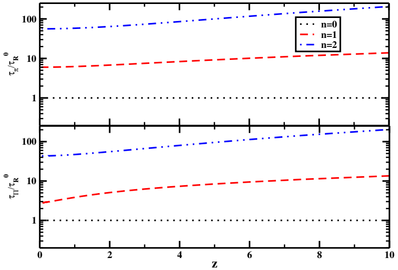
Second order field correction

Bulk viscous flow : $\Pi = -\zeta\partial\cdot u - \tau_\Pi D\Pi + c_\Pi^\sigma \pi^{\mu\nu}\sigma_{\mu\nu} + c_\Pi^\theta \Pi(\partial\cdot u)$

Shear viscous flow :

$$\pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} - \tau_\pi D\pi^{\langle\mu\nu\rangle} + c_\pi^\omega \pi_\rho^{\langle\mu}\omega^{\nu\rangle\rho} + c_\pi^\sigma \pi_\rho^{\langle\mu}\sigma^{\nu\rangle\rho} + c_\pi^\theta \pi^{\mu\nu}(\partial\cdot u) + c_\pi^\zeta \Pi\sigma^{\mu\nu}$$

Relaxation time of viscous flow with MDRTA

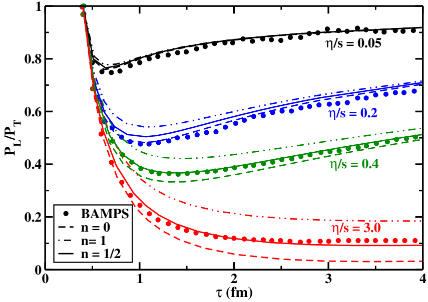


Ref : S. Mitra, PRC 105, 014902 (2022)

- $\tau_\pi = \tau_\Pi = \tau_R^0$ **holds only for** $n = 0$ **(momentum independent case)**, for all other n, the three time scales are evidently separate.
- With increasing n both τ_Π and τ_π become larger with respect to τ_R^0, which is expected for the macroscopic time scale.

Phenomenological implication

Time evolution of pressure anisotropy $P_L/P_T = (P - \pi)/(P + \pi/2)$



$$\frac{d\epsilon}{d\tau} = -\frac{\epsilon + P}{\tau} + \frac{\pi}{\tau}$$

$$\frac{d\pi}{d\tau} = -\frac{\pi}{\tau_\pi} + \beta_\pi \frac{4}{3\tau} - \frac{(4+\lambda)}{3}\frac{\pi}{\tau}$$

$$\beta_\pi = \frac{\eta}{\tau_\pi}, \quad \tau_\pi = \frac{\eta/s}{T}\frac{5!(2n+4)!}{((n+4)!)^2}, \quad \lambda = \frac{2}{7}(2n+5)$$

n	τ_π	λ
0	$5\eta/4P$	10/7
1	$3\eta/2P$	2
1/2	$1.31\eta/P$	12/7

Ref : S. Mitra, PRC 103, 014905 (2021)

- $n = 1/2$ case provides a faithful representation of BAMPS data.
- The reasonable agreement of numerical data with fractional power of n argues that most interaction theories relevant for QGP lie between two extreme limits - linear ($n = 0$) and quadratic ansatz ($n = 1$).

Conclusion and outlook

Summary$\cdots$

- Transport coefficients are the bridge between the macroscopic hydrodynamics and microscopic coarse-grained theory. Relativistic kinetic equation has been adopted here to estimate their values.
- Two parallel methods - gradient expansion method and moment method have been discussed to extract the values of transport coefficients.
- Transport phenomena in both hot hadronic and interacting QCD matter has been discussed.
- Momentum dependent relaxation time approximation has been used to redefine the thermodynamic fields in order to include the out of equilibrium dissipative effects up to second order in gradient correction.