The Production Rate of Dilepton from Magnetized Hot Hadronic Matter

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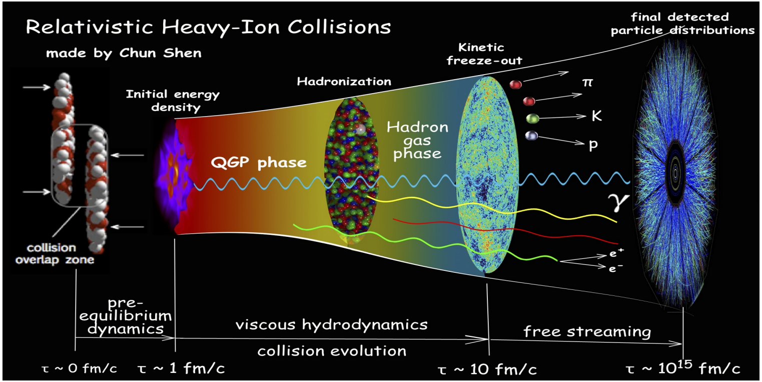
Overview of the Presentation

Introduction

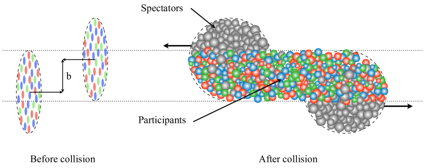


Chun Shen

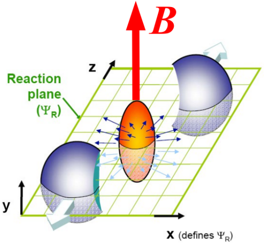
Dilepton as a probe:

- Dileptons are emitted during all the stages of the expanding fireball.
- Provide unaltered information about their production channels.

- Possibility to generate very strong magnetic field in non-central or asymmetric HIC experiment. ⇒ K. Tuchin



arXiv:2111.01607



arXiv:1603.08226v1

- **LHC :** $eB \approx 10^{14}$ T; **RHIC :** $eB \approx 10^{13}$ T

Interaction Lagrangain:

$$\mathcal{L}_{\rm int}(x) = \left\{ j^l_\mu(x) + J^h_\mu(x) \right\} A^\mu(x)$$

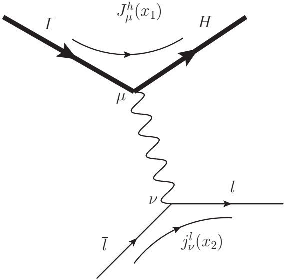
$$\text{with} \quad j^l_\mu(x) = e\bar{\psi}(x)\gamma_\mu\psi(x) \quad \text{and} \quad J^h_\mu(x) = F_\rho m_\rho \rho_\mu(x)$$

The dilepton production rate:

$$\frac{dN}{d^4x d^4q} = \frac{\alpha^2}{\pi^3 q^2} f_{\rm BE}(q_0) L(q^2) F_\rho^2 m_\rho^2 \mathcal{A}(q;T)$$

$$L(q^2) = \left(1 + \frac{2m_l^2}{q^2}\right)\sqrt{1 - \frac{4m_l^2}{q^2}}$$

$$\mathcal{A}(q;T) = -\frac{1}{3} g^{\mu\nu} \,\text{Im}\, \overline{D}_{\mu\nu}$$



Spectral Function

- **Dyson-Schwinger equation to obtain ρ^0 propagator $\overline{D}^{\mu\nu}$ is**

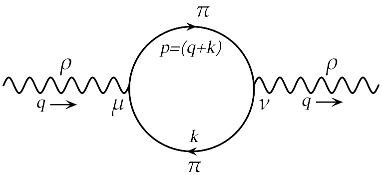
$$\overline{D}^{\mu\nu} = \overline{D}^{\mu\nu}_{(0)} - \overline{D}^{\mu\alpha}_{(0)} \Pi_{\alpha\beta} \overline{D}^{\beta\nu}$$

where

$$\overline{D}^{\mu\nu}_{(0)}(q) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{m_\rho^2} \right) \frac{-1}{q^2 - m_\rho^2 + i\epsilon}$$

Interaction lagrangian for ρ^0 self-energy

$$\mathcal{L}_{\text{int}} = -g_{\rho\pi\pi}\,(\partial_\mu \rho_\nu)\cdot(\partial^\mu \pi \times \partial^\nu \pi)$$



The one-loop thermomagnetic self-energy

$$\mathbf{\Pi}^{\mu\nu}_{11}(q,T,B) = i\int \frac{d^4k}{(2\pi)^4} N^{\mu\nu}(q,k) D^B_{11}(k) D^B_{11}(p=q+k)$$

where

$$N^{\mu\nu}(q,k) = g^2_{\rho\pi\pi}\left[q^4 k^\mu k^\nu + (q\cdot k)^2 q^\mu q^\nu - q^2 (q\cdot k)\left(q^\mu k^\nu + q^\nu k^\mu\right)\right]$$

Spectral function: Rho meson thermomagnetic self-energy

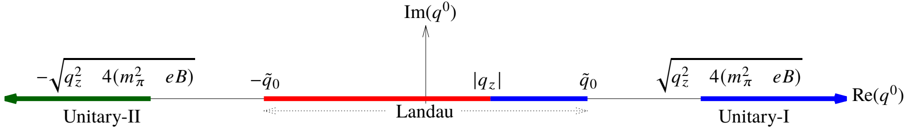
In thermomagnetic background, charged pion propagator is

$$D_{11}^{B}(k) = \sum_{l=0}^{\infty} 2(-1)^{l} e^{-\alpha_k} L_l(2\alpha_k) \left[\frac{-1}{k_{\parallel}^{2} - m_l^{2} + i\epsilon} + 2\pi i \eta(k \cdot u) \delta(k_{\parallel}^{2} - m_l^{2}) \right]$$

Landau level dependent pion mass,

$$m_l = \sqrt{m_\pi^2 + (2l+1)eB}$$

Spectral function: Analytic structure ρ^0 self-energy at $q_\perp = 0$



Unitary-I: $\sqrt{q_z^2 + 4(m_\pi^2 + eB)} < q^0 < \infty,$ **and**

Unitary-II: $-\infty < q^0 < -\sqrt{q_z^2 + 4(m_\pi^2 + eB)}$

Landau cuts: $|q^0| < \tilde{q}_0 = \sqrt{q_z^2 + \left(\sqrt{m_\pi^2 + eB} - \sqrt{m_\pi^2 + 3eB}\right)^2}$

Spectral function: Lorentz structure of self-energy

Finite Temperature and Non-zero Magnetic field

- Two independent four-vector b^μ (due to magnetic field) and u^μ (medium four-velocity) along with q^μ and $g^{\mu\nu}$. Set of 7 independent symmetric tensors:

$$g^{\mu\nu}, q^\mu q^\nu, u^\mu u^\nu, b^\mu b^\nu, q^\mu b^\nu, q^\mu u^\nu + q^\nu u^\mu, u^\mu b^\nu + b^\mu u^\nu$$

- The Ward-Takahashi Identity $q_\mu \Pi^{\mu\nu}(q) = 0$ further reduce the number of independent basis tensors to four.
- The choice of orthogonal basis tensors is

$$P_A^{\mu\nu} = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2} - \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2} - \frac{\tilde{b}^\mu \tilde{b}^\nu}{\tilde{b}^2} \right), \quad P_B^{\mu\nu} = \frac{\tilde{b}^\mu \tilde{b}^\nu}{\tilde{u}^2},$$

$$P_C^{\mu\nu} = \frac{1}{\sqrt{\tilde{u}^2 \tilde{u}^2}} \left(\tilde{u}^\mu \tilde{b}^\nu + \tilde{u}^\nu \tilde{b}^\mu \right), \quad P_L^{\mu\nu} = \frac{\tilde{u}^\mu \tilde{u}^\nu}{\tilde{u}^2}$$

with
$$\tilde{b}^\mu = b^\mu - \frac{q\cdot b}{q^2} q^\mu - \frac{b\cdot\tilde{u}}{\tilde{u}^2} \tilde{u}^\mu$$

Spectral function: Lorentz structure of self-energy

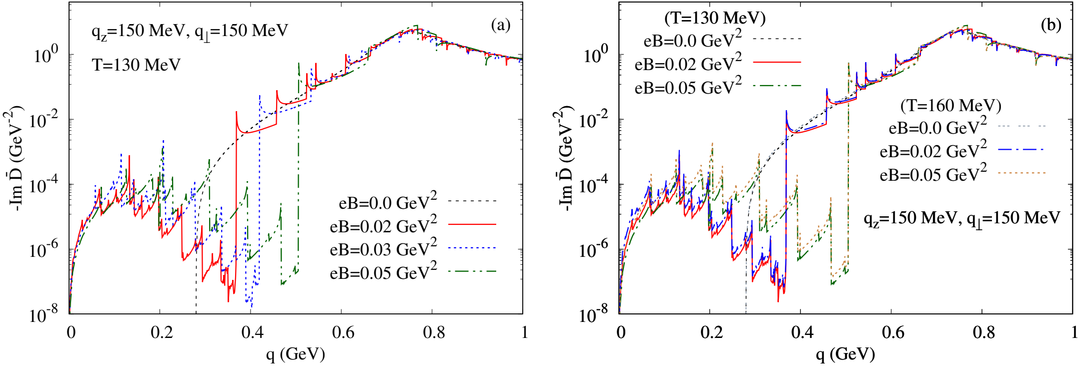
- **Lorentz decomposition of the self-energy**

$$\Pi^{\mu\nu}(T,B)=\Pi_A P_A^{\mu\nu}+\Pi_B P_B^{\mu\nu}+\Pi_C P_C^{\mu\nu}+\Pi_L P_L^{\mu\nu}$$

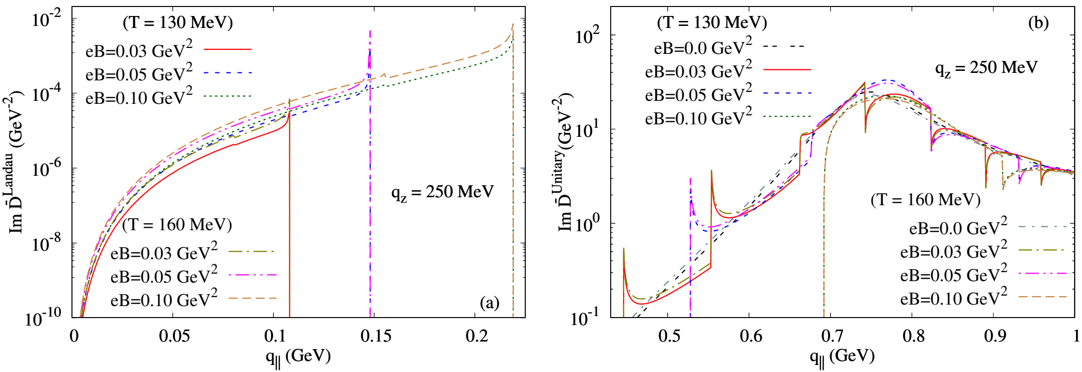
- **By solve Dyson-Schwinger equation, the complete thermomagnetic ρ^0 propagator is**

$$\overline{D}^{\mu\nu}(T,B)=\frac{P_A^{\mu\nu}}{q^2-m_\rho^2+\Pi_A}+\frac{(q^2-m_\rho^2+\Pi_L)P_B^{\mu\nu}}{\left(q^2-m_\rho^2+\Pi_B\right)\left(q^2-m_\rho^2+\Pi_L\right)-\Pi_C^2}-\frac{\Pi_C P_C^{\mu\nu}}{\left(q^2-m_\rho^2+\Pi_L\right)\left(q^2-m_\rho^2+\Pi_B\right)-\Pi_C^2}$$
$$+\frac{(q^2-m_\rho^2+\Pi_B)P_L^{\mu\nu}}{\left(q^2-m_\rho^2+\Pi_B\right)\left(q^2-m_\rho^2+\Pi_L\right)-\Pi_C^2}-\frac{q^\mu q^\nu}{q^2 m_\rho^2}$$

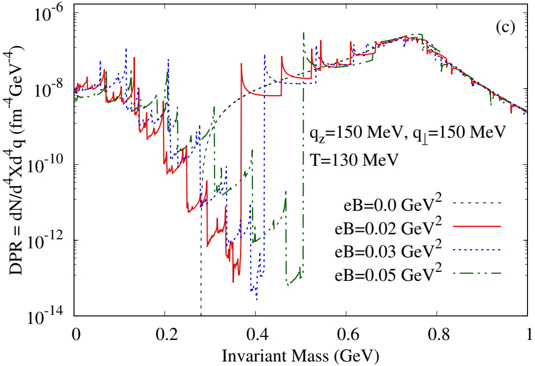
Numerical Result: Spectral funtion, $q_\perp \neq 0$

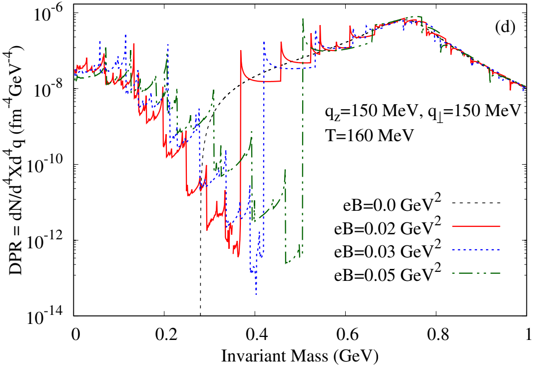


Numerical Result: Spectral funtion, $q_\perp = 0$

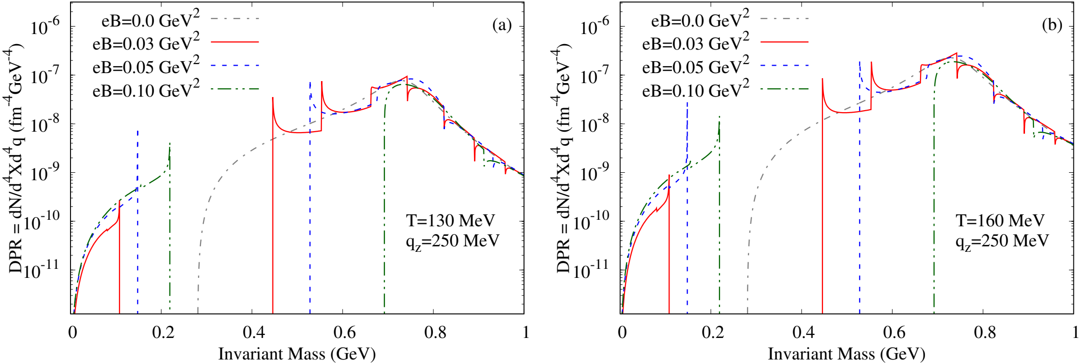


Numerical Result: Dilepton production, $q_\perp \neq 0$





Numerical Result: Dilepton production, $q_\perp = 0$



Summary

- We have presented an analysis of dilepton production from magnetized hot hadronic medium. The principal component is the in-medium thermomagnetic spectral function of ρ^0.
- The spectral function has been obtained by solving Dyson Schwinger equation.
- The thermomagnetic ρ^0 self-energy is calculated in employing the RTF of TFT.
- The analytic structure is investigated in the complex energy plane.
- A non-trivial Landau cut has been found in the physical kinematic domain in addition to the usual contribution coming from the Unitary cut.

Collaborators:

- Nilanjan Chaudhuri, VECC, Kolkata
- Dr. Snigdha Ghosh, Government General Degree College Kharagpur-II
- Prof. Pradip Roy, SINP, Kolkata
- Prof. Sourav Sarkar, VECC, Kolkata

Thank You

BACKUP

Spectral function: Rho meson thermomagnetic self-energy

$$
\begin{aligned}
\mathrm{Im}\,\overline{\mathbf{\Pi}}^{\mu\nu}(q,T,B) = &-\tanh\left(\frac{|q^0|}{2T}\right)\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{1}{4\lambda^{1/2}(q_{\parallel}^2,m_l^2,m_n^2)}\sum_{k_z\in\{k_z^{\pm}\}} \\
&\Big[(1+f_k^l+f_p^n+2f_k^lf_p^n)\times\Big\{N_{nl}^{\mu\nu}(q,k^0=-\omega_k^l,k_z)\Theta\Big(q^0-\sqrt{q_z^2+(m_l+m_n)^2}\Big) \\
&+N_{nl}^{\mu\nu}(q,k^0=\omega_k^l,k_z)\Theta\Big(-q^0-\sqrt{q_z^2+(m_l+m_n)^2}\Big)\Big\}+(f_k^l+f_p^n+2f_k^lf_p^n) \\
&\times\Big\{N_{nl}^{\mu\nu}(q,k^0=-\omega_k^l,k_z)\Theta\left(q^0-\min(q_z,E^{\pm})\right)\Theta\Big(-q^0+\max(q_z,E^{\pm})\Big) \\
&+N_{nl}^{\mu\nu}(q,k^0=\omega_k^l,k_z)\Theta\left(-q^0-\min(q_z,E^{\pm})\right)\Theta\Big(q^0+\max(q_z,E^{\pm})\Big)\Big\}\Big]
\end{aligned}
$$

Where

$$
k_z^{\pm}=\frac{1}{2q_{\parallel}^2}\left[-yq_z\pm|q^0|\lambda^{1/2}(q_{\parallel}^2,m_l^2,m_n^2)\right];\quad E^{\pm}=\frac{m_l-m_n}{|m_l\pm m_n|}\sqrt{q_z^2+(m_l\pm m_n)^2};
$$

$$
y=\left(q_{\parallel}^2+m_l^2-m_n^2\right)
$$

Spectral function: Rho meson thermomagnetic self-energy

$$
\begin{aligned}
N^{\mu\nu}_{nl}(q, k_\parallel) &= \int \frac{d^2k_\perp}{(2\pi)^2}\widetilde{N}^{\mu\nu}_{nl}(q, k_\parallel, k_\perp) \\
\widetilde{N}^{\mu\nu}_{nl}(q, k_\parallel, k_\perp) &= 4(-1)^{n+l}e^{-\alpha_k-\alpha_p}L_l(2\alpha_k)L_n(2\alpha_p)N^{\mu\nu}
\end{aligned}
$$

- For the case $q_\perp = 0$, l will lie between $(n-1)$ to $(n+1)$ and

$$
\bar{N}^{\mu\nu}_{nl}(q_\parallel, k_\parallel) = 4g^2_{\rho\pi\pi}(-1)^{n+1}\frac{eB}{8\pi}\left[\left\{k^\mu_\parallel k^\nu_\parallel q^4_\parallel + q^\mu_\parallel q^\nu_\parallel\left(q_\parallel.k_\parallel\right)^2 - \left(k^\mu_\parallel q^\nu_\parallel + k^\nu_\parallel q^\mu_\parallel\right)\right.\right.
$$
$$
\left.\left.q^2_\parallel\left(q_\parallel.k_\parallel\right)\right\}\delta^n_l + q^4_\parallel g^{\mu\nu}_\perp\,\frac{eB}{4}\left\{n\delta^{n-1}_l - (2n+1)\,\delta^n_l + (n+1)\,\delta^{n+1}_l\right\}\right]
$$

Spectral function: Lorentz structure of self-energy

- **Lorentz decomposition of the self-energy**

$$\Pi^{\mu\nu}(T,B) = \Pi_A P_A^{\mu\nu} + \Pi_B P_B^{\mu\nu} + \Pi_C P_C^{\mu\nu} + \Pi_L P_L^{\mu\nu}$$

- **The form factors comes out to be**

$$\Pi_L = \frac{1}{\tilde{u}^2} u_\mu u_\nu \Pi^{\mu\nu}, \quad \Pi_A = (g_{\mu\nu}\Pi^{\mu\nu} - \Pi_L - \Pi_B)$$

$$\Pi_C = \frac{1}{\sqrt{\tilde{u}^2 \tilde{b}^2}} \Big\{ u_\mu b_\nu \Pi^{\mu\nu} - (b\cdot\tilde{u})\Pi_L \Big\}$$

$$\Pi_B = \frac{1}{\tilde{b}^2} \Big\{ b_\mu b_\nu \Pi^{\mu\nu} + \frac{(b\cdot\tilde{u})^2}{\tilde{u}^2}\Pi_L - 2\frac{b\cdot\tilde{u}}{\tilde{u}^2} u_\mu b_\nu \Pi^{\mu\nu} \Big\}$$