Heavy quark diffusion coefficient from lattice QCD

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Based on: D. Banerjee, S. Datta, M. Laine, 2204.14075,
D. Banerjee, S. Datta, R. Gavai, P. Majumdar, 2206.14571

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Heavy-light mesons in deconfined plasma

- R_{AA} and v_2 of heavy mesons, in particular D mesons, indicate thermalization of charm: small $t_{\rm kin}$

ALICE, JHEP 01 (2022) 174;
S. Das, Plenary talk this morning

- For thermal heavy quark, $M \gg T, \quad p \gtrsim \sqrt{MT}$
- Takes $\mathcal{O}(M/T)$ hard collisions to change momentum by $\mathcal{O}(1)$
- A Langevin framework can be used.
- For a low momentum heavy quark,

$$\frac{dp_i}{dt} \;=\; \xi_i(t) - \eta_D p_i, \qquad \langle \xi_i(t)\, \xi_j(t') \rangle \;=\; \kappa\, \delta(t-t')$$

Svetitsky '88; Mustafa, Pal, Srivastava, '97; Moore & Teaney '05; Rapp & van Hees '05

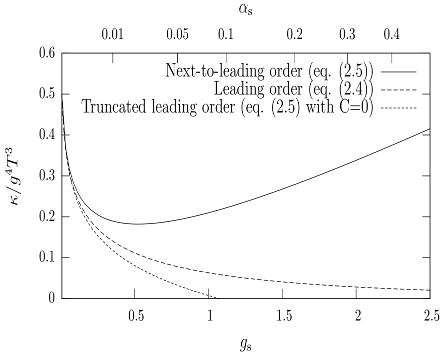
See Ruggieri, Pooja, Jaiprakash, Das (2203.06712) for non-Markov noise

- Standard nonrelativistic relations connect κ to η_D and D_s, the position space diffusion coefficient.

$$\eta_D \;=\; \frac{\kappa}{2\,M\,T}, \qquad \langle x^2(t) \rangle \;=\; 6 D_s t, \qquad D_s \;=\; \frac{2T^2}{\kappa}$$

κ in kinetic theory

- One can calculate κ from the $2 \rightarrow 2$ collision processes.
  Svetitsky '88, Moore & Teaney 2005, ...
- The LO calculation gives a value of κ substantially smaller than required to interpret the R_{AA} and v_2 results.
- NLO corrections very large.



Caron-Huot & Moore, PRL 100 (2008) 052301

Calculation of κ

- Phenomenological models: gives very different behaviors, even if parameters tuned to get κ in the right ballpark.
  Cao et al., PRC 99 (2019) 054907; S. Das, plenary talk
- A field theoretic definition of κ can be given from the force-force correlator:

$$\kappa \; = \; \frac{1}{3\chi} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt \, e^{i\omega t} \int d^3x \left\langle \frac{1}{2} \left\{ F_i(t,x) \, , \; F_i(0,0) \right\} \right\rangle$$

J. Casalderrey-Solana & D. Teaney, PRD 74 (2006) 085012;
S. Caron-Huot, M. Laine & G. Moore, JHEP 04 (2009) 53;
A. Bouttefeux & M. Laine, JHEP 12 (2020) 150

- Nonperturbatively, we can calculate the Matsubara correlator on the lattice. This can be connected to the real-time correlator through the spectral function.

$$G_{FF}(\tau) \; = \; \int_0^{\infty} \frac{d\omega}{\pi} \; \rho_{FF}(\omega) \; \frac{\cosh \, \omega(\tau - 1/2T)}{\sinh \omega/2T}$$

- Then κ can be extracted from the infrared behavior of $\rho_{FF}(\omega)$:

$$\rho_{IR} \underset{\omega \to 0}{\approx} \frac{\kappa \, \omega}{2T}$$

κ_E from lattice

- The force term can be expanded in a series in $1/M$:

$$F^i = M\frac{dJ^i}{dt} = \phi^\dagger \left\{ -gE^i + \frac{[D^i, D^2 + g\sigma \cdot B]}{2M} + \frac{g\,[D_0, \sigma \times E]^i}{4M} + ... \right\} \phi$$

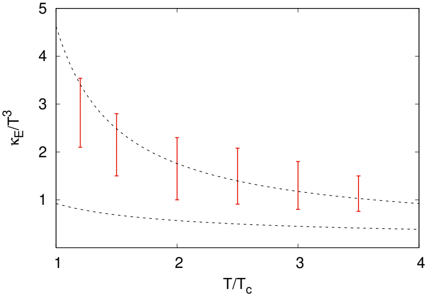
- In the static limit $m_Q \to \infty$, $F^i = \phi^\dagger \left(-gE^i\right) \phi$.
- The *EE* correlator has been investigated for a gluonic plasma by multiple groups, and κ_E extracted.

  Banerjee, Datta, Gavai, Majumdar, PRD (2012)
  Bielefeld Group(Kaczmarek, et al.), PRD (2015), PRD 103 (2021) 014511.
  Brambilla, et al. (TUMQCD), PRD (2020) and arXiv:2206.02861
- Results on κ_E in the temperature range $T_c < T \lesssim 3.5\,T_c$.

  Banerjee, Datta, Gavai, Majumdar, 2206.14571
- We have used the multilevel method to enhance the signal, and used the perturbative renormalization.
- Besides, Gradient flow has been used by others for both opertor renormalization and signal enhancement.
- Analysis techniques used: modelling of the ultraviolet, Brackus-Gilbert inversion, Bayesian techniques (MEM), ...

κ_E from lattice

Structure of spectral function expected to be simple: we model $\rho_{UV}(\omega)$ using perturbation theory.



Banerjee, Datta, Gavai & Majumdar, 2206.14571

- In agreement with other lattice determinations, where available.
- Matches well with NLO pert. theory (Caron-Huot & Moore). Higher order corrections small?

$1/m_Q$ correction

- The $1/m_Q$ correction has only recently been estimated.
- Under certain assumptions, (Bouttefeux & Laine, JHEP 12 ('20) 150)

$$\kappa_Q \;\approx\; \kappa_E \;+\; \frac{2}{3}\,\langle v^2\rangle\,\kappa_B\,, \qquad \langle v^2\rangle \;=\; \frac{3T}{M_{\rm kin}}$$

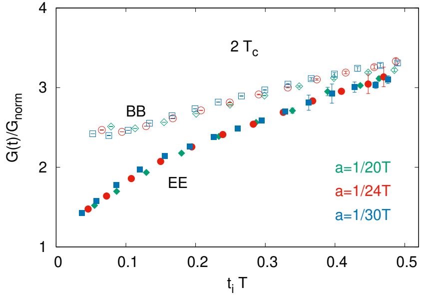
  where κ_B is the equivalent of κ_E from the $B-B$ correlator.
- The BB correlator has an anomalous dimension. This needs to be multiplied with a Wilson coefficient, to cancel the scale dependence.
- This becomes equivalent to evaluating $Z_B(\mu = 19.2\,T)$.

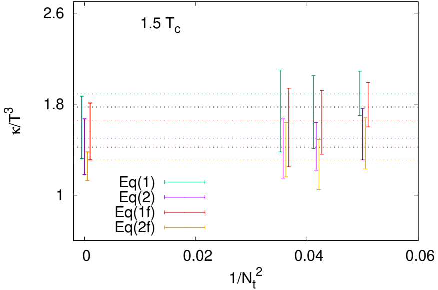
  M. Laine, JHEP 06 ('21) 139
- We used the clover discretization of the B field.
- Matching to the $\overline{MS}$ operator can be done nonperturbatively using the results of the ALPHA collaboration.

  Guazzini, Meyer & Sommer, JHEP 10 (2007) 081.

$1/m_Q$ correction

- κ_B obtained from the continum extrapolated correlator. Compared with the continuum limit of κ_B from discrete correlators.
- Analysis similar to the *EE* correlator, but subtle differences arise because of the anomalous dimension.



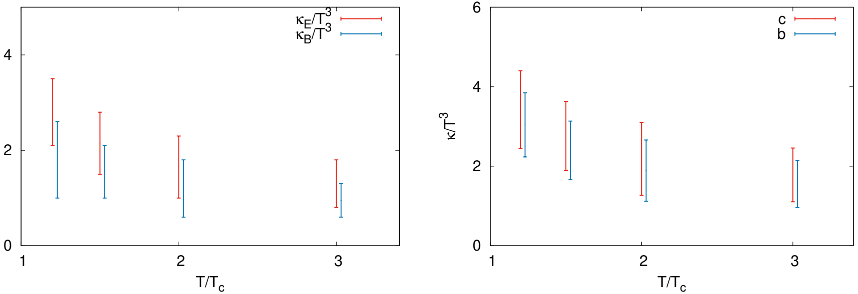


Banerjee, Datta & Laine, JHEP 2204.14075

κ_B from BB correlator

- $\langle v^2 \rangle$ is obtained from the constant part of $\frac{\langle J^i J^i \rangle}{\langle J^0 J^0 \rangle}$.
- We evaluate them from the NLO calculation of the vector correlator.

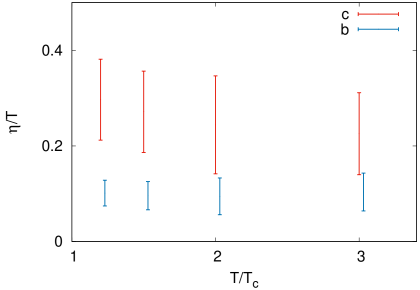
Burnier & Laine, JHEP 11 ('12) 086



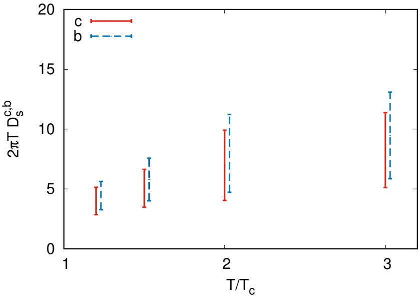
- $\langle v^2 \rangle$ can be large, $\sim$ 0.67 (charm) and 0.3 (bottom) at 2 T_c.
- But Correction to κ is moderate, $\sim$ 30-35% (charm), 15-20% (bottom) at 2 T_c.

η_D, D_s

$\eta_D \approx \frac{\kappa \langle v^2 \rangle}{6T^2}$



$D_s \approx \frac{2T^2}{\kappa}$



Indicates $\tau_b \approx 3\tau_c \gtrsim 10 fm$

η_D quite flat, especially for b.

Contradicts some models used.

$2\pi D_s T_c \sim 2.4 - 5$ for charm at T_c.

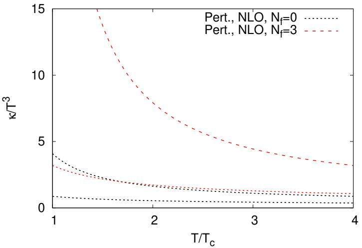
Slope $\sim$ 2.57(43) for charm, comparable to what was extracted from data.

Xu, et al., PRC 97 (2018) 014907.

Summary: what have we learnt?

- Nonperturbative estimate of κ for the gluonic plasma from force-force correlator.
- Static limit: results for $T \lesssim 3.5\,T_c$
  Results at very high T also available (Brambilla et al.)
  Agreement between different lattice calculations
  Agreement with NLO pert. theory.
- $1/M$ correction.
  Expansion works: $\sim 20\%$ correction for bottom at 2 T_c.
- Temperature dependence of κ_b, κ_c.
- Slope of D_c comparable with that extracted from data.

- *Unquenching effect.*



Large unquenching effect in preliminary lattice results .

L. Altenkort, INT Workshop, Oct. 2022
https://www.int.washington.edu/sites/default/files/schedule_session_files/Altenkort_L.pdf

EXTRA SLIDE: Matching to Lorentz Force

We need to calculate

$$M \frac{\partial J^i}{\partial t} \equiv \frac{M A_i^{QCD}}{N^{QCD}},$$

where A_i^{QCD} and N^{QCD} are the matrix elements of $\frac{\partial}{\partial t} \int d^3x \, J_i^{QCD}$ and $\int d^3x \, J_0^{QCD}$, respectively.

Matching of the QCD operators to the Lorentz force term has been worked out to NLO by Laine (*2103.14270*).

Taking the IR force term

$$F_i^{IR} = \theta^\dagger \left\{ Z_E \, E_i + Z_B \, (v \times B)_i \right\} \theta$$

Z_E, Z_B are fixed by demanding

$$\left. \frac{MA_i}{N} \right|_{QCD} = \left. \frac{F_i}{N} \right|_{IR} .$$

Calculating both sides to NLO, one gets (*2103.14270*)

$$Z_E = 1, \qquad Z_B = 1 + \frac{g^2 N_c}{16\pi^2} \left[\frac{1}{\epsilon} + 2 \log \frac{\mu_B \, e^{\gamma_E}}{4\pi T} - 2 \right]$$

EXTRA SLIDE: Z_B

- The $\frac{1}{\epsilon}$ term in Z_B cancels a singularity in the BB correlator.
- The scale dependence in the finite term should be cancelled by a Wilson coefficient, equivalent to evaluating the correlator at $\mu_B \approx 4\pi T e^{1-\gamma_E} \approx 19.2 T$.

M. Laine, 2103.14270

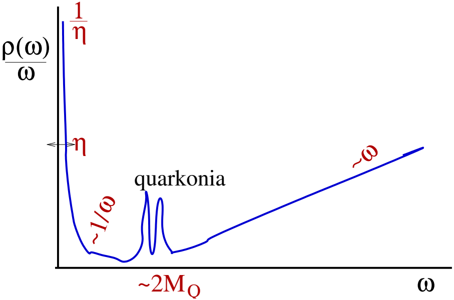
- The nontrivial task is to connect the lattice correlator to $\overline{MS}$.
- A detailed NP renormalization program for the clover implementation of the B field was carried out by the ALPHA collaboration.

D. Guazzini, H. Meyer & R. Sommer, JHEP 10 (2007) 081.

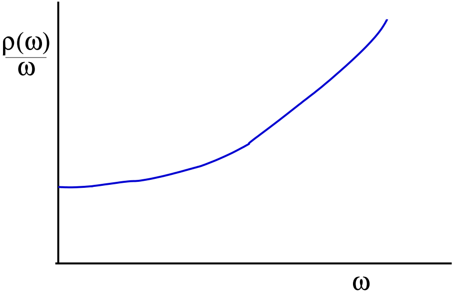
- NP renormalization coefficient was calculated in Schrödinger functional scheme, and connected to $\overline{MS}$ at a a large scale.
- Then we run it down to 19.2 T.

EXTRA SLIDE: Form of spectral function

Structure of current-current correlator complicated.



Force-force correlator expected to be considerably simpler.



EXTRA SLIDE: Analysis

- $\rho_{EE}^{IR}(\omega) \equiv \frac{\kappa\,\omega}{2T}, \qquad \rho_{UV}(\omega) \equiv \frac{g^2(\mu) C_f \omega^3}{6\pi}$
- $$\mu_B = \max\left[\omega^{1-\frac{\gamma_0}{b_0}} (\pi T)^{\frac{\gamma_0}{b_0}},\ \pi T\right], \qquad \gamma_0 = \frac{3}{8\pi^2}\ (\mathrm{BB}),\ \ 0\ (\mathrm{EE})$$
- Model $\rho(\omega)$ as
  (1) $\rho_1(\omega) \equiv \max\left(\rho_{EE}^{IR}(\omega),\ c\rho_{UV}(\omega)\right)$
- Physically better motivated: Francis et al. '15
  (2) $\rho_2(\omega) \equiv \sqrt{(\rho_{EE}^{IR}(\omega))^2 + (c\rho_{UV}(\omega))^2}$
- In our fits, we got $c \sim 1$, as expected.
- Also tried adding a Fourier mode (Francis et al. '15)

$$\rho_{1f,2f}(\omega) \equiv (1 + d \sin \pi y)\ \rho_{1,2}^{c=1}(\omega), \qquad y = \frac{\log\left(1 + \frac{\omega}{\pi T}\right)}{1 + \log\left(1 + \frac{\omega}{\pi T}\right)}$$

- d was found to be small, $\lesssim 0.12$.
- The whole analysis is done within a bootstrap framework.