Effects of Phase Transition in a Pulsar Core on Pulse Profile Modulation

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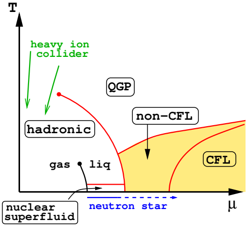
Ref. *MNRAS 513, 2794 (2022)*

Outline :

- Aim and Objective
- Our Earlier Studies (in Brief)
- Our Current Work : The Effect of Density Fluctuations on Pulse Profile Modulation
  - Formalism
  - Analytical Estimates of Various Quantities Associated with the Pulse Modulation
  - Numerical Algorithm for Determining the Pulse Modulation and Results
  - Observational Aspects and Conclusion

Aim and Objective:



- High baryon density regime of QCD provides an extremely rich landscape of various phases. [Alford et al.; RMP, 80 (2008)].
- These phases are expected to occur at very high values of baryon chemical potential, difficult to achieve in laboratory, may occur in the core of a neutron star.
- So, how do we probe the phases ?

Mapping Phase Transitions to Pulsar Observables

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Effects of phase transition induced density fluctuations on pulsar dynamics

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A B S T R A C T

We show that density fluctuations during phase transitions in pulsar cores may have non-trivial effects on pulsar timings, and may also possibly account for glitches and anti-glitches. These density fluctuations invariably lead to non-zero off-diagonal components of the moment of inertia, leading to transient wobbling of star. Thus, accurate measurements of pulsar timing and intensity modulations (from wobbling) may be used to identify the specific pattern of density fluctuations, hence the particular phase transition, occurring inside the pulsar core. Changes in quadrupole moment from rapidly evolving density fluctuations during the transition, with very short time scales, may provide a new source for gravitational waves.

Basic Idea

- Phase transition inside the core of a neutron star necessarily leads to density fluctuations.
- The density fluctuations become more prominent if the transition leads to formation of topological defects **(QCD domain walls, QCD strings, superfluid strings, etc.)**.
- Such density fluctuations should alter each component MI tensor; affecting star's rotation and may account for glitches and anti-glitches of pulsars.
- **The development of non-zero off-diagonal components in the MI tensor should cause the star to wobble, resulting in pulse profile modulation.**
- High precision measurements of pulsar timings and intensity modulations may identify sources of fluctuations; thus pinning down the specific phase transition occurring inside the star.

Effect of density fluctuations on various moments :

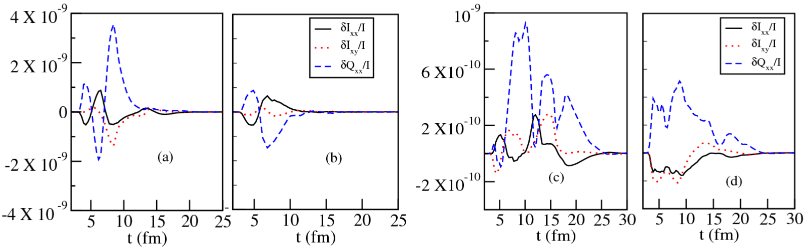
Bagchi, Das, Layek & Srivastava; Physics Letter B (2015)

	QCD Strings			QCD Walls			SF Strings		
$\frac{R_C}{\xi}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{Q_{xx}}{I}$	$\frac{\delta I_{xx}}{I}$	$\frac{\delta I_{xy}}{I}$	$\frac{\delta Q_{xx}}{I}$
5	5E-10	-3E-10	-1E-10	2E-8	-1E-8	-8E-10	2E-6	-1E-6	-4E-7
50	5E-12	-2E-12	2E-12	1E-10	-8E-11	-1E-11	2E-8	-7E-9	7E-9
200	1E-13	2E-14	-7E-14	5E-12	-4E-12	-6E-12	5E-10	6E-11	-2E-10
400	-3E-15	-5E-14	-9E-14	3E-12	-2E-12	3E-14	-1E-11	-2E-10	-3E-10

Fractional change of various moments of the pulsar caused by inhomogeneities due to defects.
The correlation length is taken as $\xi = 10$ fm with core size R_c.

Glitches/Anti-glitches :

Bagchi, Das, Layek & Srivastava; Physics Letter B (2015)

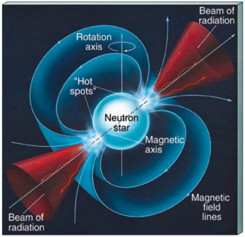


Fractional change in MI and QM during phase transitions.

- Left plots for lattice size (7.5 fm)^3 and right plots (15 fm)^3.
- (a) and (c) for a C-D phase transition with Z(3) walls and strings.
- (b) and (d) for a transition with only string as for the CFL phase.

Density Fluctuations $\Rightarrow$ Pulse Profile Modulation :

Bagchi, Layek, Sarkar & Srivastava; MNRAS 513, 2794(2022)



credit : Internet

- The (oblate) pulsar is assumed to be rotating about the symmetric axis (z-axis) with angular frequency ω.
- The MI components of unperturbed pulsar :

$$I_{ij}^0 = \begin{pmatrix} I_1^0 & 0 & 0 \\ 0 & I_2^0 & 0 \\ 0 & 0 & I_3^0 \equiv I_0 \end{pmatrix}$$

Formalism...

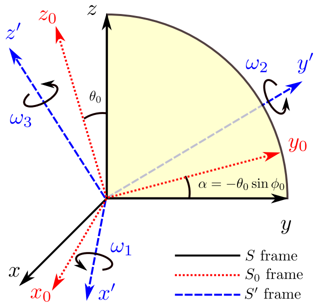
- For the perturbed pulsar :

$$I_{ij} = \begin{pmatrix} I_1 = I_1^0 + \delta I_{11} & \delta I_{12} & \delta I_{13} \\ \delta I_{12} & I_2 = I_2^0 + \delta I_{22} & \delta I_{23} \\ \delta I_{13} & \delta I_{23} & I_3 = I_3^0 + \delta I_{33} \end{pmatrix}$$

- δI_{ij} is assumed to be Gaussian of width $\sigma = \epsilon I_0$. The values of ϵ may lie in the range $10^{-14} - 10^{-6}$ depending on the nature of phase transition.

- The oblateness of the pulsar can be characterized by the parameter $\eta = (I_0 - I_1^0)/I_0$, the value of which may lie in the range $10^{-2} - 10^{-6}$.

Formalism...



- The pulsar's dynamics is governed by the Euler's equations,

$$I_1\dot{\omega}_1 - (I_2 - I_3)\omega_2\omega_3 = 0$$
$$I_2\dot{\omega}_2 - (I_3 - I_1)\omega_1\omega_3 = 0$$
$$I_3\dot{\omega}_3 - (I_1 - I_2)\omega_1\omega_2 = 0$$

I_1, I_2 and I_3 are the principal MI of the perturbed pulsar.

Formalism...

- For a very small density perturbations (i.e., $\omega_1, \omega_2 << \omega_3$),

$$I_3 \dot{\omega}_3 = 0 \; ; \Longrightarrow \quad \omega_3 = \text{constant}$$

$$\ddot{\omega}_1 + \Omega^2 \, \omega_1 = 0 \quad \text{and similarly for } \omega_2$$

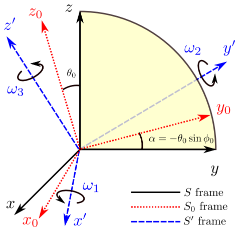
$\Omega = \omega_3[(I_3 - I_1)(I_3 - I_2)/(I_1 I_2)]^{1/2}$ is the precession frequency due to the perturbations. The solution for $\omega_{1,2}$ become oscillatory,

$$\omega_1(t) = \omega_1^0 \cos(\Omega \; t) - \frac{\omega_2^0}{k} \sin(\Omega \; t)$$

$$\omega_2(t) = k\omega_1^0 \sin(\Omega \; t) + \omega_2^0 \cos(\Omega \; t).$$

ω_1^0 and ω_2^0 are the angular velocities at $t = 0$ about x_0 and y_0, respectively and $k = [I_1(I_3 - I_1)/(I_2(I_3 - I_2))]^{1/2}$.

Initial Conditions :



- Before phase transition, $L_z = L$; $L_x = 0 = L_y$. Conserving angular momentum (no external torque),

$$L_{x_0}(t=0) = I_1\omega_1^0 = -L\theta_0 \cos\phi_0$$
$$L_{y_0}(t=0) = I_2\omega_2^0 = -L\theta_0 \sin\phi_0$$
$$L_{z_0}(t=0) = I_3\omega_3^0 = L.$$

- The angles θ_0, ϕ_0 of z_0-axis are determined by diagonalizing the perturbed MI matrix.

Initial Conditions...

- The angular frequencies can now be expressed in terms of θ_0 and ϕ_0 as (Taking $L/I_1 \simeq L/I_3 \simeq \omega$)

$$\omega_1(t) = \dot{\theta}_1 = -\omega\theta_0[\cos\phi_0 \cos(\Omega t) - \frac{\sin\phi_0}{k}\sin(\Omega t)]$$
$$\omega_2(t) = \dot{\theta}_2 = -\omega\theta_0[k\cos\phi_0 \sin(\Omega t) + \sin\phi_0 \cos(\Omega t)]$$
$$\omega_3(t) = \dot{\theta}_3 = \omega.$$

Analytical Estimates of Various Quantities Characterizing Modulation :

- Principal MI of the perturbed state, $I_{1,2} = I_0(1 - \eta + \epsilon_{1,2})$, $I_3 = I_0(1 + \epsilon_3)$; $\epsilon_i \simeq \epsilon$.
- Using $\epsilon, \ \eta << 1$ and $\epsilon << \eta$

$$\Omega = [(I_3 - I_1)(I_3 - I_2)/(I_1 I_2)]^{1/2}\omega \simeq \frac{\eta + \epsilon}{1 - \eta + \epsilon}\omega \simeq \eta\,\omega.$$

$$\theta_0 \simeq \sqrt{2}\,\left(\frac{\epsilon}{\eta}\right)$$

- The precession frequency Ω is completely determined by η. For a millisecond pulsar, the time period of precession (T_Ω) is about 1 sec for $\eta \simeq 10^{-3}$.

Analytical Estimates...

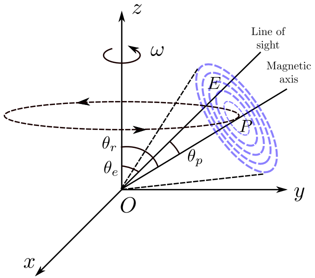
- Writing $\omega_1(t)$ and $\theta_1(t)$ as,

$$\omega_1(t) = -\omega\theta_0[\cos(\Omega t + \phi_0) + \frac{\epsilon}{2\eta}\sin\phi_0\sin(\Omega t)],$$

$$\theta_1(t) = -\frac{\omega\theta_0}{\Omega}[\sin(\Omega t + \phi_0) - \frac{\epsilon}{2\eta}\sin\phi_0\cos(\Omega t)]$$

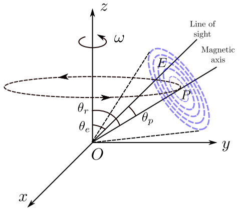
- Since $\theta_0 \simeq (\epsilon/\eta)$, the second term is of order $\sim (\epsilon/\eta)^2$.
- The amplitudes $\omega_m = \omega\theta_0$ and $\theta_m = (\omega/\Omega)\theta_0$ are of order $(\epsilon/\eta)\omega$ and (ϵ/η^2), respectively.
- Note : For $\eta = 10^{-3}$, the oscillation amplitude θ_m will be of order $10^6\ \epsilon$. This results in approximately 1° amplitude for $\epsilon = 10^{-8}$.

Numerical Simulation for Studying the Pulse Modulations :



- We assume the standard conical shape geometry for the pulse emission region.
- The angles of magnetic axis and the line of sight pointing towards earth with the rotation axis are denoted by θ_r and θ_e, respectively.

Numerical simulation...



- The angle θ_p changes with time as the pulse emission cone sweeps across the line of sight with rotation frequency $\dot{\phi}_r(t) = \omega$.
- The evolution of θ_p will be reflected in the observed intensity of the pulses (Gaussian of width w),

$$I(\theta_p) = I_0 \; e^{-(\theta_p^2/w^2)}.$$

- In the presence of density fluctuations, the profile will be modulated due to the precession of pulsars.

Results :

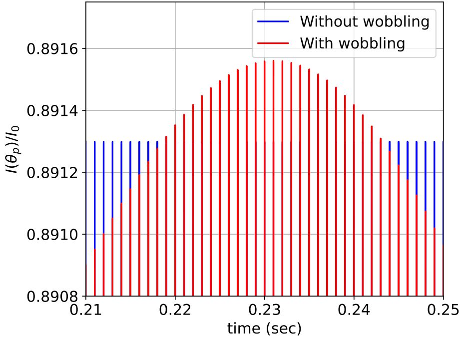
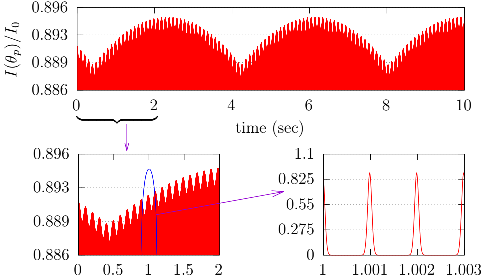


Figure: Evolution of pulse intensity for a millisecond pulsar for the set of parameters : $\eta = 10^{-2}, \epsilon = 10^{-5}, w = 15^\circ, \theta_r = 45^\circ, \theta_e = 40^\circ$

Results...



Time evolution of $I(\theta_p)/I_0$ in the presence of density fluctuation induced modulation.

Top plot : Pulse profile for a long time duration, showing two different modulation time scales [$T_\Omega \simeq \frac{T_\omega}{\eta} \simeq 0.1$ s ; $T_m \simeq (\frac{\eta}{\epsilon}) T_\omega \simeq 1$ s].

Bottom left : For a smaller time duration for a better resolution

Bottom right : Further resolved to observe full profiles of a few individual millisecond pulses.

Results...

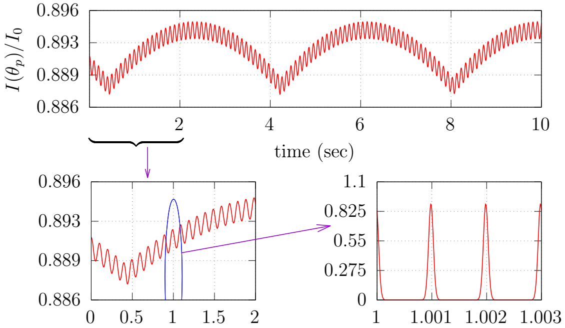


Figure: Top part of the pulse profiles for clear visibility of the modulated pulse shape details.

Results...

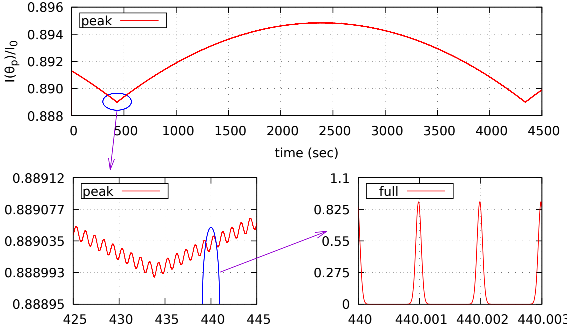


Figure: Parameter set : $\eta = 10^{-3}, \epsilon = 10^{-8}, w = 15^\circ, \theta_r = 45^\circ, \theta_e = 40^\circ$

Observational Aspects of the Results and Conclusion :

- Transient Changes in Pulse Profile :

  One should look for transient changes in pulse profile for any signal of phase transitions.

- Identifying the Nature of the Phase Transition :

  The time scale of the *first modulation* ($T_\Omega \simeq \frac{T_\omega}{\eta}$), with a shorter time scale should be possible to see in the pulsar data easily.

  Possibility of observation of the longer time *second modulation* [$T_m \simeq (\frac{\eta}{\epsilon}) T_\omega$] depends on the time scale of the completion of the phase transition. This important feature may help in identifying the nature of the phase transition (first order/continuous, or even more interesting possibilities may arise if there are topological defects produced in phase transition).

Observational aspects...

- Memory Effect:

  After the density fluctuations fade away, the original rotation state will be restored. However, one expects a shift in the angular position of the emitting region. Thus, there should be a *residual time shift* in the pulsar signal for any time after the end of the phase transition. The presence of such residuals can be a signal for an earlier phase transition stage, which could have been missed in direct detection.

THANKS

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Physics dept., BITS - Pilani for financial support
and conference organizers for the hospitality

Analytical Estimates...

- The Effects of Precession on the Observed Fluxes :
  By taking the angular flux distribution from the pulsar azimuthal symmetric (about the centre of the emission region) and Gaussian (width w),

$$F(\alpha) = F_0 \; e^{-\frac{\alpha^2}{w^2}} .$$

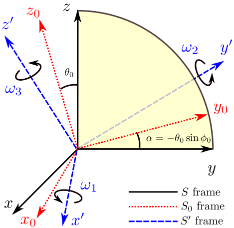
  α is the angle of the radial vector of the emission point from the central axis of the cone.

- If the perturbation results in the change of α by δ, the corresponding change in flux becomes

$$F'(\phi) = F_0 \; e^{-\frac{(\alpha+\delta)^2}{w^2}} \simeq F(\phi) \left(1 - 2\delta \frac{\alpha}{w^2}\right) .$$

- Fractional change of flux $\left(\frac{\Delta F}{F}\right) \sim O(\delta) \simeq \theta_m \simeq \epsilon/\eta^2$.

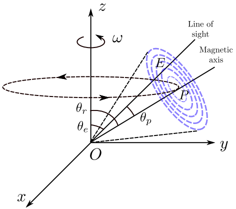
Algorithm in Brief (See ref. *MNRAS 513, 2794 (2022)* for details)



- Step-1: Diagonalization of the perturbed MI matrix to get the new set of principal axes (S_0-frame). The orientation of S_0 relative to S is obtained through R_0.

$$R_0 = \begin{pmatrix} 1 & 0 & -\theta_0 \cos\phi_0 \\ 0 & 1 & -\theta_0 \sin\phi_0 \\ \theta_0 \cos\phi_0 & \theta_0 \sin\phi_0 & 1 \end{pmatrix}$$

Algorithm...



- Step - 2 : $\theta_i(\Delta t)$, $(i = 1, 2, 3)$ is obtained for a time step Δt. The matrix R_1 that describes the orientations of new body-fixed frame relative to S_0-frame is determined.
- The radiation point $P(\theta^*, \phi^*)$ at time $t = \Delta t$ relative to space-fixed fixed observer is obtained through coordinate transformation.
- θ_p is calculated at time $t = \Delta t$ and hence, the intensity of the pulse $I(\theta_p)$.
- ”Step-2” is repeated successively for a long time duration to get the modulated profile.