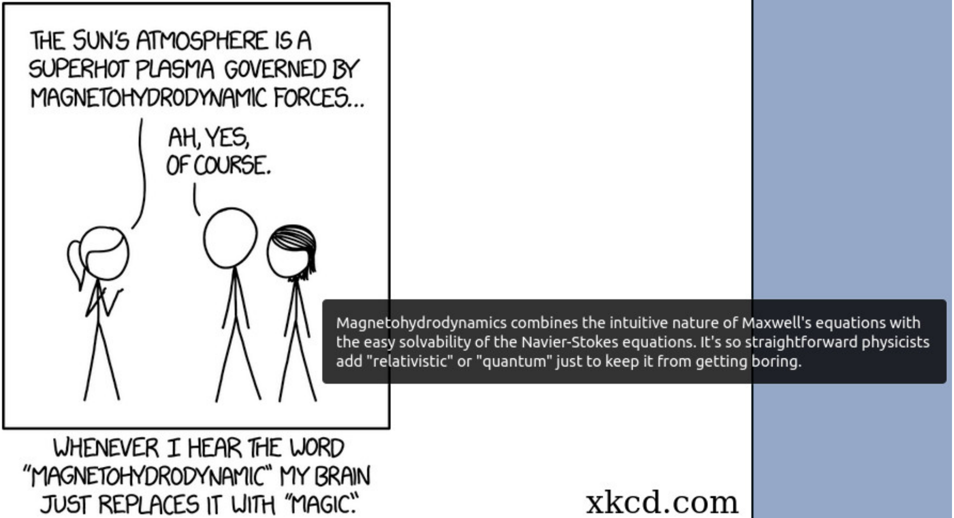
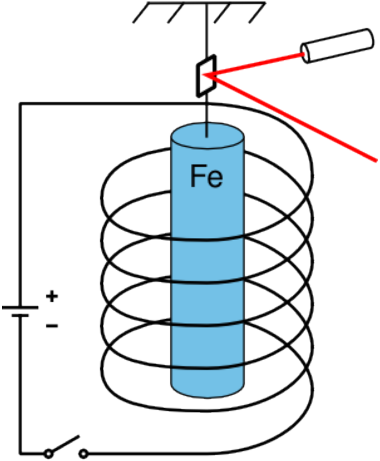
Relativistic spin-magnetohydrodynamics



xkcd.com

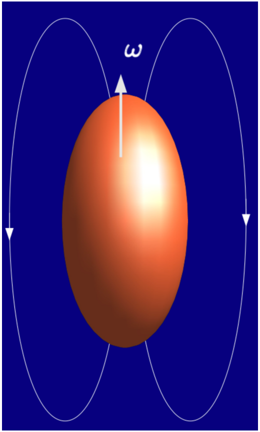
ICPAQGP Puri, February 7, 2023

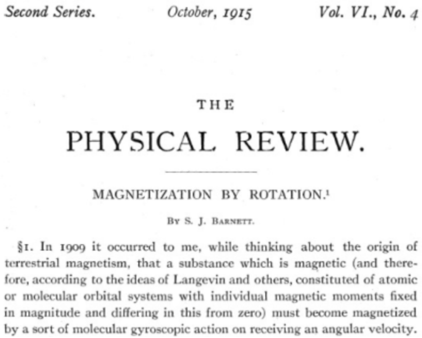
Einstein-de Haas effect



Electron spins get aligned in external magnetic field which is compensated by rotation of the ferromagnetic material.

Converse: Barnett effect





Second Series. *October, 1915* *Vol. VI., No. 4*

THE

PHYSICAL REVIEW.

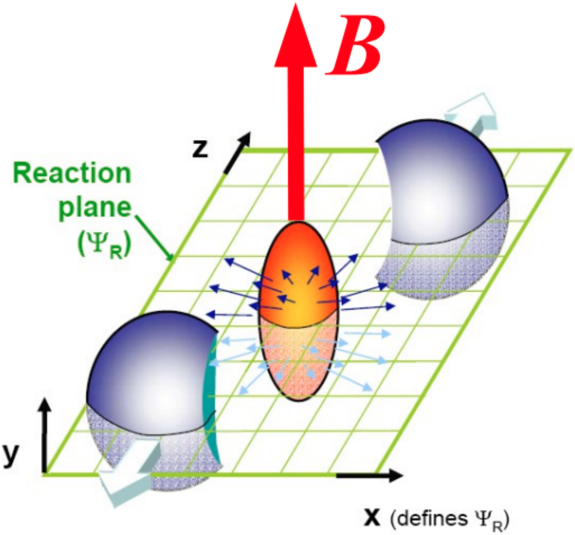
MAGNETIZATION BY ROTATION.[1]

BY S. J. BARNETT.

§1. In 1909 it occurred to me, while thinking about the origin of terrestrial magnetism, that a substance which is magnetic (and therefore, according to the ideas of Langevin and others, constituted of atomic or molecular orbital systems with individual magnetic moments fixed in magnitude and differing in this from zero) must become magnetized by a sort of molecular gyroscopic action on receiving an angular velocity.

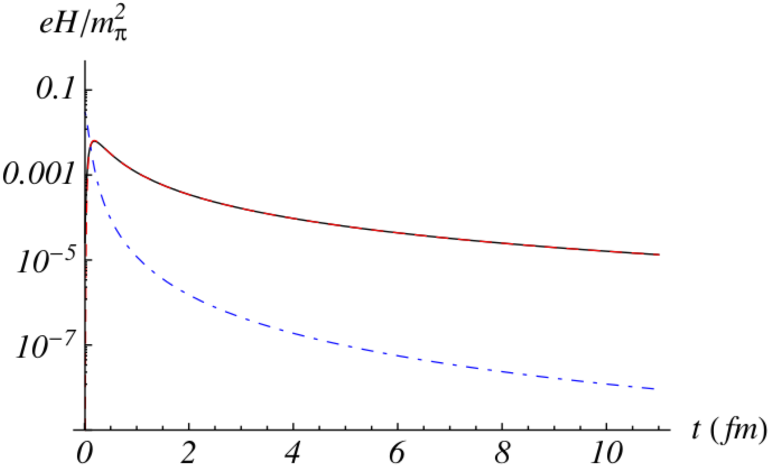
Spontaneous magnetization when spun around. Transformation of orbital angular momentum into spin alignment. Angular velocity decreases with appearance of magnetic field. Explanation appeals to spin-orbit coupling.

Generation of magnetic field in heavy ion collisions



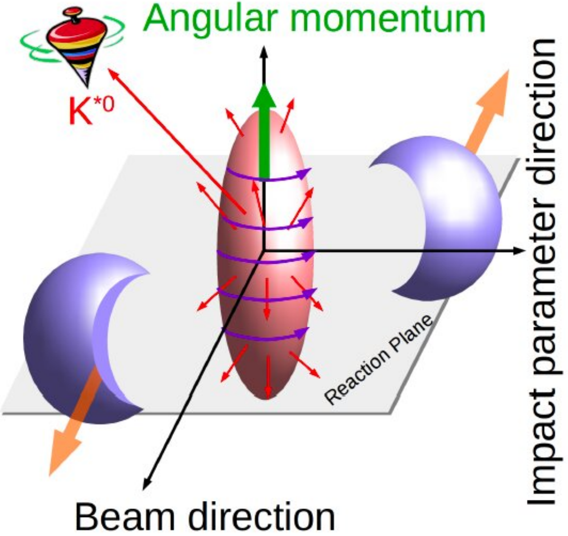
[Adapted from D. Kharzeev @ CPOD 2013.]

Magnetic field time evolution



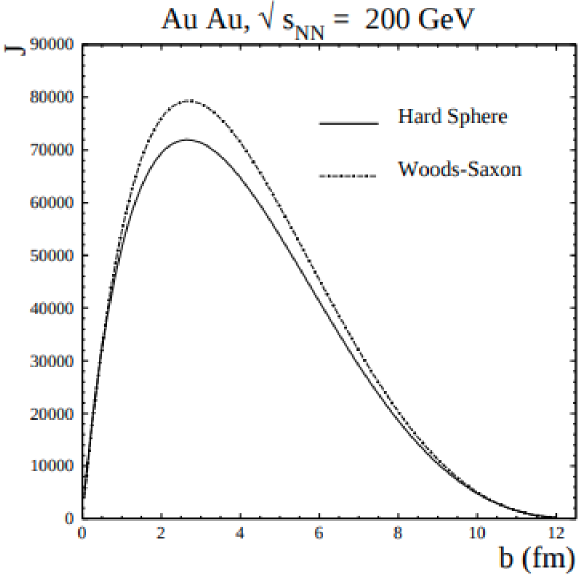
[K. Tuchin, Int. J. Mod. Phys. E23, 1430001 (2014).]

Global angular momentum in heavy ion collisions



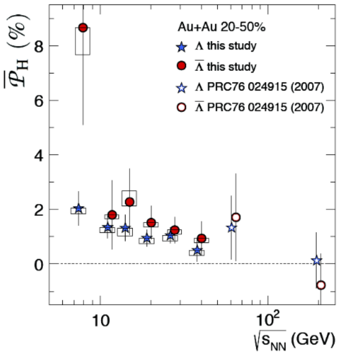
[B. Mohanty, ICTS News 6, 18-20 (2020).]

Angular momentum generation in non-central collisions



[F. Becattini, et al., Phys. Rev. C77, 024906 (2008).]

STAR Collaboration, *Global Lambda hyperon polarization in nuclear collisions*, Nature 548 62-65, 2017



First evidence of a quantum effect in (relativistic) hydrodynamics

Adapted from F. Becattini
'Subatomic Vortices'

Relativistic kinetic theory

- Kinetic theory: calculation of macroscopic quantities by means of statistical description in terms of distribution function.
- Let us consider a system of relativistic particles of rest mass m with momenta $\mathbf{p}$ and energy p^0

$$p^0 = \sqrt{\mathbf{p}^2 + m^2}$$

- For large no. of particles, $f(x,p)$ gives a distribution of the four-momenta $p = p^\mu = (p^0, \mathbf{p})$ at each space-time point.
- $f(x,p)\Delta^3 x\, \Delta^3 p$ gives average no. of particles in the volume element $\Delta^3 x$ at point x with momenta in the range $(\mathbf{p}, \mathbf{p} + \Delta\mathbf{p})$.
- Statistical assumptions:
  - No. of particles contained in $\Delta^3 x$ is large ($N \gg 1$).
  - $\Delta^3 x$ is small compared to macroscopic volume ($\Delta^3 x / V \ll 1$).
- The equilibrium distribution: $f_{eq}(x,p,s) = \left[\exp\left(\beta \cdot p - \xi\right) \pm 1\right]^{-1}$

Extended phase-space for spin degrees of freedom

- The phase-space for single particle distribution function gets extended $f(x, p, s)$.
- The equilibrium distribution for Fermions is given by

$$f_{eq}(x,p,s) = \frac{1}{\exp\left[\beta\cdot p - \xi - \frac{1}{2}\,\omega : s\right] + 1} \qquad \begin{cases}\beta\cdot p \equiv \beta_\mu p^\mu \\ \omega : s \equiv \omega_{\mu\nu} s^{\mu\nu}\end{cases}$$

- Quantities $\beta^\mu = u^\mu/T,\ \xi = \mu/T,\ \omega_{\mu\nu}$ are functions of x.
- $\xi,\ \beta^\mu,\ \omega^{\mu\nu}$: Lagrange multipliers for conserved quantities.
- $s^{\mu\nu}$: Particle spin, similar to particle momenta p^μ.
- Hydrodynamics: average over particle momenta and spin.
- Classical treatment of spin.

Bhadury et. al., PLB 814, 136096 (2021); PRD 103, 01430 (2021).

Conserved currents and spin-hydrodynamics

- Express hydrodynamic quantities in terms of $f(x,p,s)$.

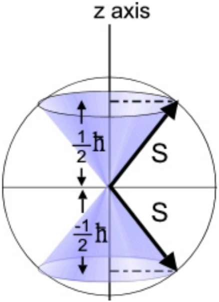
$$T^{\mu\nu}(x) = \int dP dS\; p^\mu p^\nu \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$N^{\mu}(x) = \int dP dS\; p^\mu \left[f(x,p,s) - \bar{f}(x,p,s)\right]$$

$$S^{\lambda,\mu\nu}(x) = \int dP dS\; p^\lambda s^{\mu\nu} \left[f(x,p,s) + \bar{f}(x,p,s)\right]$$

$$dP \equiv \frac{d^3p}{E_p(2\pi)^3}, \quad dS \equiv m\frac{d^4s}{\pi\,\mathfrak{s}}\,\delta(s\cdot s + \mathfrak{s}^2)\,\delta(p\cdot s)$$

$$\int dS = 2; \quad \mathfrak{s}^2 = \frac{1}{2}\left(\frac{1}{2}+1\right) = \frac{3}{4}; \quad s^\mu \equiv \frac{1}{2m}\epsilon^{\mu\nu\alpha\beta}p_\nu s_{\alpha\beta}$$



- Classical treatment of spin: internal angular momentum.
- Equations of motion: $\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0.$
- Non-dissipative spin hydrodynamics: $f(x,p,s) = f_{eq}(x,p,s)$.

Spin Magnetohydrodynamics Bhadury et. al., PRL 129, 192301 (2022)

- The particle four-current and its conservation is given by

$$N^{\mu} = nu^{\mu} + n^{\mu}, \qquad \partial_{\mu} N^{\mu} = 0$$

- Total stress-energy tensor of the system: $T^{\mu\nu} = T^{\mu\nu}_{\rm f} + T^{\mu\nu}_{\rm int} + T^{\mu\nu}_{\rm em}$

$$T^{\mu\nu}_{\rm f} = \epsilon u^{\mu} u^{\nu} - (P + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu},$$

$$T^{\mu\nu}_{\rm int} = -\Pi^{\mu} u^{\nu} - F^{\mu}{}_{\alpha} M^{\nu\alpha}$$

$$T^{\mu\nu}_{\rm em} = -F^{\mu\alpha} F^{\nu}{}_{\alpha} + \frac{1}{4} g^{\mu\nu} F^{\alpha\beta} F_{\alpha\beta}$$

- Maxwell's equation: $\partial_{\mu} H^{\mu\nu} = J^{\nu}$ and $H^{\mu\nu} = F^{\mu\nu} + M^{\mu\nu}$,

$$\partial_{\mu} T^{\mu\nu}_{\rm em} = F^{\nu}{}_{\alpha} J^{\alpha}$$

- Current generating external field, $J^{\mu} = J^{\mu}_{\rm f} + J^{\mu}_{\rm ext}$ where $J^{\mu}_{\rm f} = \mathfrak{q} N^{\mu}$,

$$\partial_{\mu} T^{\mu\nu} = -f^{\nu}_{\rm ext}, \qquad f^{\nu}_{\rm ext} = F^{\nu}{}_{\alpha} J^{\alpha}_{\rm ext}$$

Equations of motion

- Divergence of matter part of energy-momentum tensor,

$$\partial_\nu T_{\mathrm{f}}^{\mu\nu} = F^{\mu}{}_{\alpha} J_{\mathrm{f}}^{\alpha} + \frac{1}{2} \left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}$$

- Next, consider total angular momentum conservation:

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\mu} + S^{\lambda,\mu\nu}$$

- In presence of external torque its divergence leads to,

$$\partial_\lambda J^{\lambda,\mu\nu} = -\tau_{\rm ext}^{\mu\nu}, \qquad \tau_{\rm ext}^{\mu\nu} = x^\mu f_{\rm ext}^{\nu} - x^\nu f_{\rm ext}^{\mu}$$

- Torque due to moment of external force; "pure" torque ignored.
- The orbital part of angular momentum and its divergence is

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}, \qquad \partial_\lambda L^{\lambda,\mu\nu} = -\tau_{\rm ext}^{\mu\nu}$$

- Spin part of the total angular momentum is conserved

$$\partial_\lambda S^{\lambda,\mu\nu} = 0$$

- Along with particle four-current conservation, $\partial_\mu N^\mu = 0$.

Boltzmann equation

- Boltzmann equation (BE) in relaxation-time approximation (RTA)

$$\left(p^\alpha \frac{\partial}{\partial x^\alpha} + m\,\mathcal{F}^\alpha \frac{\partial}{\partial p^\alpha} + m\,\mathcal{S}^{\alpha\beta} \frac{\partial}{\partial s^{\alpha\beta}}\right) f = C[f] = -\,(u\cdot p)\,\frac{f - f_{\rm eq}}{\tau_{\rm eq}}$$

- The force term is:

$$\mathcal{F}^\alpha = \frac{\mathfrak{q}}{m}\,F^{\alpha\beta}p_\beta + \frac{1}{2}\left(\partial^\alpha F^{\beta\gamma}\right) m_{\beta\gamma}, \qquad m^{\alpha\beta} = \chi s^{\alpha\beta}$$

- There is a "pure" torque term:

$$\mathcal{S}^{\alpha\beta} = 2\,F^{\gamma[\alpha}\,m^{\beta]}{}_\gamma - \frac{2}{m^2}\left(\chi - \frac{\mathfrak{q}}{m}\right) F_{\phi\gamma}\,s^{\phi[\alpha}\,p^{\beta]}p^\gamma$$

- We ignore this "pure" torque term for now.
- Employ the Boltzmann equation to obtain $\delta f = \delta f_1$.
- Evolution equations for spin-magnetohydrodynamics.

Hydrodynamic equations from kinetic theory

- Impose Landau frame and extended matching conditions

$$u_\mu T^{\mu\nu} = \epsilon u^\nu, \quad \epsilon = \epsilon_{\rm eq}, \quad n = n_{\rm eq}, \quad u_\lambda \delta s^{\lambda,\mu\nu} = 0$$

- Zeroth, first and "spin" moment of the RTA collision vanishes

$$\int dPdS\, C[f] = \int dPdS\, p^\mu\, C[f] = \int dPdS\, s^{\mu\nu} C[f] = 0$$

- Using definitions of hydro quantities, these moments of BE gives

$$\partial_\mu N^\mu = 0, \quad \partial_\nu T_{\rm f}^{\mu\nu} = F^\mu{}_\alpha J_{\rm f}^\alpha + \frac{1}{2}\left(\partial^\mu F^{\nu\alpha}\right) M_{\nu\alpha}, \quad \partial_\lambda S^{\lambda,\mu\nu} = 0$$

- Same equations as obtained from macroscopic arguments.
- Polarization/magnetization emerge naturally at gradient order.
- Boltzmann equation → dissipative spin-magnetohydodynamics.

Einstein-de Haas and Barnett effects

- One can define the polarization-magnetization tensor as

$$M^{\mu\nu} = m \int dP dS \, m^{\mu\nu} \left(f - \bar{f} \right)$$

- The equilibrium polarization-magnetization tensor is

$$M^{\mu\nu}_{eq} = m \int dP dS \, m^{\mu\nu} \left(f_{eq} - \bar{f}_{eq} \right)$$

- Magnetic dipole moment $m^{\mu\nu} = \chi s^{\mu\nu}$.
- χ: resembles the gyromagnetic ratio.
- Integrating over the momentum and spin degrees of freedom,

$$M^{\mu\nu}_{eq} = a_1 \, \omega^{\mu\nu} + a_2 \, u^{[\mu} u_\gamma \omega^{\nu]\gamma}$$

- In global equilibrium, $\omega^{\mu\nu}$ corresponds to rotation of the fluid.
- Rotation produces magnetization (Barnett effect) and vice versa (Einstein-de Hass effect).

Other relevant works in spin hydrodynamics

- Other parallel approaches from Wigner function [N. Weickgenannt, X.-l. Sheng, E. Speranza, Q. Wang and D. Rischke, PRD 100 (2019) 056018].
- Appraoch based on chiral kinetic theory [S. Shi, C. Gale and S. Jeon, PRC 103 (2021) 044906].
- Appraoch based on Lagrangian method [D. Montenegro and G. Torrieri, PRD 100 (2019) 056011].
- Formulation with torsion in metric [A. D. Gallegos, U. Gürsoy and A. Yarom, SciPost Phys. 11, 041 (2021); M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, H.-U. Yee, JHEP 11 (2021) 150].
- Useful reviews on spin hydro: [W. Florkowski, R. Ryblewski and A. Kumar, Prog.Part.Nucl.Phys. 108 (2019) 103709; S. Bhadury, J. Bhatt, A. Jaiswal and A. Kumar, Eur.Phys.J.ST 230 (2021) 3, 655-672].
- Relativistic spin-magnetohydrodynamics: unexplored area.
- Much work needed in this direction.

Thank you!