Bound to unbound states transitions of heavy quarkonia in the cooling phase of QGP

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Outline of Talk

- Introduction:
  - Review of Conventional Mechanism of quarkonia suppression
  - Non Adiabaticity: An Important Aspect of Evolution of Quarkonia
- Bound to unbound states transitions:
  - For Real Potential
  - For Complex Potential
    - Results
- Summary

Conventional Mechanism for Quarkonia Suppression

- Quarkonia is bound states of heavy quark and its anti-quark.
- The quarks are sufficiently heavy so that we can use nonrelativistic schrödinger equation to solve the bound states.

- The most common phenomenological potential between quarks and antiquarks (in vacuum) is . $V(r) = -\frac{\alpha}{r} + \sigma r$
- J/ψ is bound states of c and $\bar{c}$.

Conventional Mechanism for Quarkonia Suppression

Continued...

- Matsui and Satz[1] proposed J/ψ suppression as a signal for QGP due to Debye screening of the potential between $q\bar{q}$.
- If at a temperature T_D, the Debye screening length of the medium becomes less than the radius of quarkonia, then $q\bar{q}$ may not form bound states.
- In the above picture, suppression of quarkonia occurs when the temperature of QGP achieves a value higher than T_D.
- If the QGP temperature remains below T_D, no quarkonia suppression is expected due to color screening(?) in the conventional mechanism.

[1] T. Matsui and H. Satz, Phys.Lett. B178,416 (1986)

Real-time dynamics of quarkonia

There are mainly two ways to look into that problem

1. Modelling of heavy quarkonia as open quantum systems.
   - By solving master equations
   - By solving stochastic schrödinger equation
     - Medium is static
     - Dissociation probability has not been calculated so far
2. Solving schrödinger equation in the potential model

Adiabatic Approximation

Description of quarkonia through effective potential

- $q\bar{q}$ potential changes slowly from initial temperature ($V(T = T_i)$) to the final temperature ($V(T_f)$).
- Initial quarkonium state evolves to the state corresponding to $V(T_f)$ which is also a bound state for $T_f < T_D$ with same quantum number as initial state, hence no quarkonium suppression for $T < T_D$. $\Longrightarrow$ Adiabatic

Evolution of Fireball Created in Heavy Ion Collisions

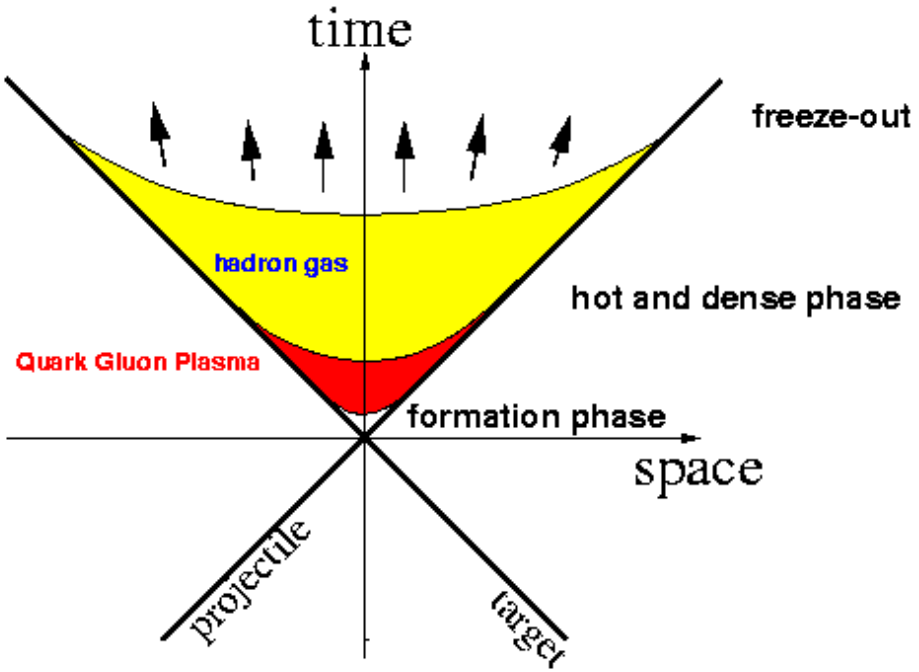

Figure: Nuclear collision evolution epoch.

Some Important Observations

- The fireball created in Heavy Ion Collision is rapidly evolving with time.
- If quarkonia is described in potential model then $q\bar{q}$ potential is no-doubt time dependent.

Note:

Matsui and Satz picture considers the static QGP only.

- **One need to solve Schrödinger equation for time-dependent Hamiltonian.**

Violation of Adiabatic Approximation

Why?

- We question this assumption of adiabatic evolution for ultra-relativistic heavy-ion collisions, such as at RHIC, and especially at LHC.

Quarkonia evolution during thermalisation

- At LHC or RHIC energy collision, thermalization is achieved in time scale less than 1 fm (from elliptic flow measurements) and may be as short as 0.25 fm for RHIC and about 0.1 fm for LHC[2]
- For J/ψ and even for Υ, typical time scale of $q\bar{q}$ dynamics will be at least 1-2 fm from the size of the bound state (as can be seen from the extent of the wave function and the fact that $q, \bar{q}$ have non-relativistic velocities).

[2] D.M. Elliott and D.H. Rischke, Nucl. Phys. A 671 , 583 (2000)

Violation of Adiabatic Approximation

Continued

- Also, ΔE between J/ψ and its next excited state (χ) is about 300 MeV (400 MeV for Υ states), leading to transition time scale ~ 0.7 fm (0.5 fm for Υ).
- Thus, the time scale of change in potential from $V(T = 0)$ to $V(T)$ is at most the same as the transition time scale between relevant states.

Time dependent potential for studying quarkonia wave function evolution

- The evolution of the wave function, thus, cannot be taken to be adiabatic and it should be treated in terms of a time dependent perturbation theory
- Survival probability of quarkonia should be calculated under this perturbation
- Adiabatic assumption has been questioned earlier by Dutta and Borghini [2] for the cooling part.

[2] Dutta and Borghini, Mod.Phys.Lett. A30 (2015) no.37, 1550205

Time dependent potential.....continued

- It is clear that even if the final temperature remains less than T_D (above which Debye screening can melt quarkonia), if the change in potential is fast enough invalidating the adiabatic assumption then transition of initial quarkonium state to other excited states, or unbound state, will occur
- These excited states will have larger profile and will easily melt in medium.
- Thus: Quarkonia melting can occur even when QGP temperature remains below T_D.

Quarkonia Suppression Due to Rapid Thermalisation of QGP

- As Thermalisation may happen in very short time, about 0.25 fm for RHIC and 0.1 fm for LHC, it seems reasonable to use the *sudden* perturbation approximation instead of adiabatic approximation.
- The initial wave function of the quarkonium cannot change under this quick change of potential.
- Thus, as soon as thermalization is achieved the initial quarkonium wave function is no longer an energy eigen state of the new Hamiltonian with the $q\bar{q}$ potential corresponding to temperature T of QGP.

Results

- we calculate wave functions for J/ψ and Υ at different temperatures
- Debye screened $q\bar{q}$ potential[4]

$$V(r) = -\frac{\alpha}{r} exp(-\omega_D r) + \frac{\sigma}{\omega_D} r(1 - exp(-\omega_D r))$$

$$\omega_D = T\sqrt{6\pi\alpha_s}, \quad \alpha = 0.471 = \frac{4}{3}\alpha_s \quad \sigma = 0.192 GeV^2$$

[4] F. Karsch, M.T. Mehr, and H. Satz, Z. Phys. C **37**, 617 (1988).

Results: Continued....

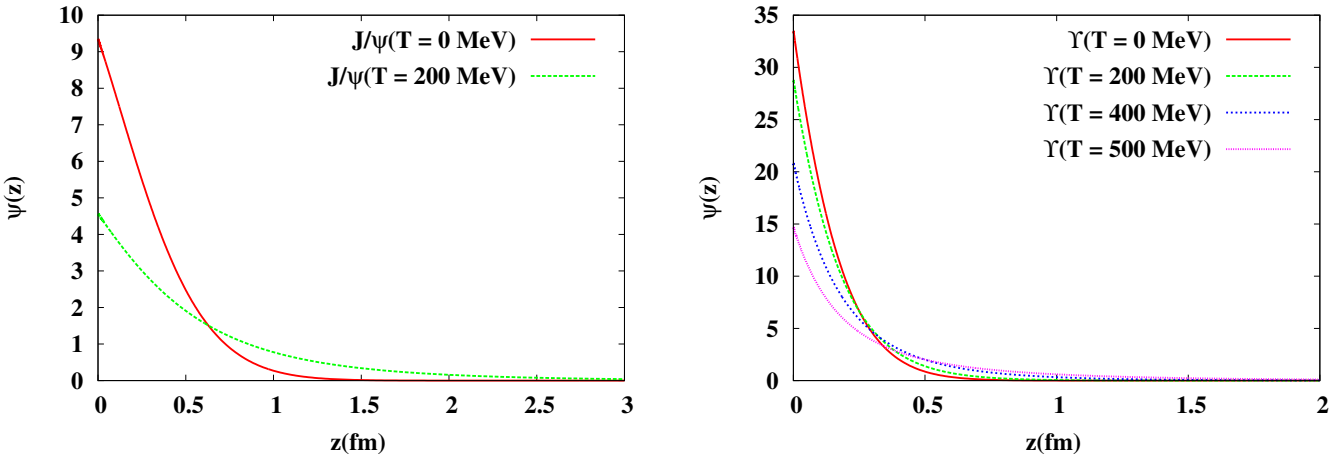

Figure: Wave functions for J/ψ and Υ at different temperatures.

Results: Continued....

- As a result, one can find overlap with the new (instantaneous) eigen states, giving us the survival probability of the quarkonium as well as the probability of its transition to other excited states.
- We note dramatic decrease in survival probabilities down to about 10 % as temperature increases to about 260 MeV and 590 MeV respectively for J/ψ and Υ.

Results: Continued....

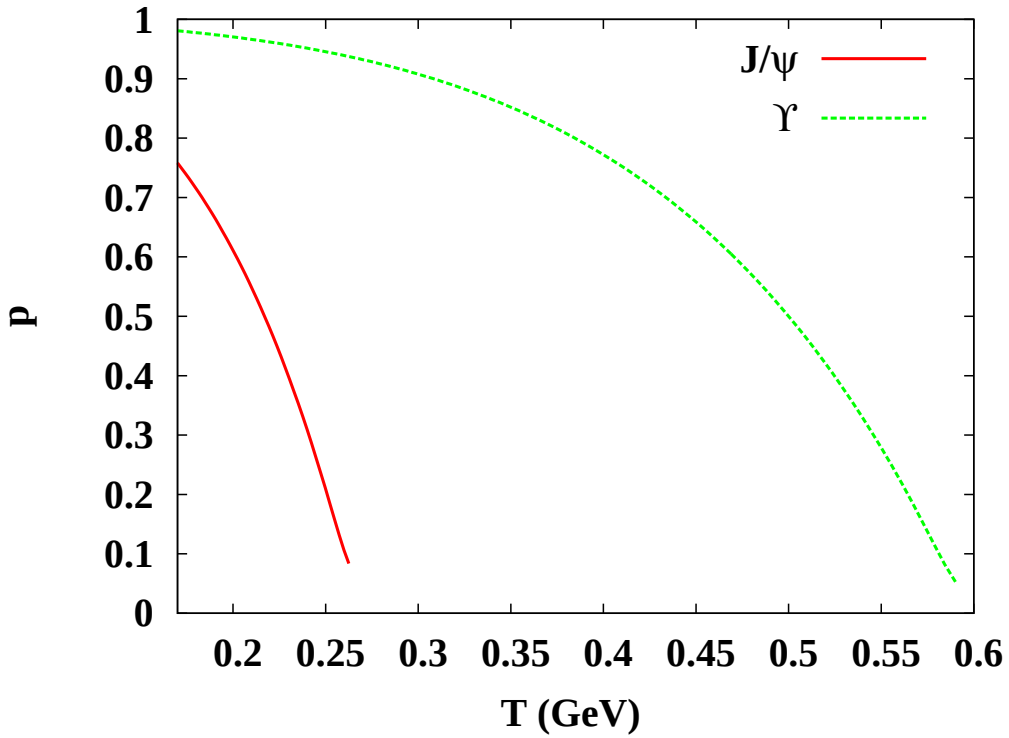

Figure: Survival Probability p of J/ψ and Υ vs. temperature of medium. Plots are given upto the temperature T_D for J/ψ and Υ .

Error Calculation

- We have also estimated the error[5] in using this sudden approximations by calculating the probability ζ of transition of the original quarkonium state to some other state during the time scale τ of the change of the potential.
- $$\zeta = \tau^2 \Delta \bar{H}^2$$
$$\Delta \bar{H} = < \bar{H}^2 > - < \bar{H} >^2$$
$$\bar{H} = \frac{1}{\tau} \int_0^\tau H(t) dt$$
- Here, $<>$ denotes the expectation value in the initial quarkonium state.

[5] A. Messiah, *Quantum Mechanics*, Dover Publications (1999)

Error Calculation

Continued

- This error remains below 8 % for the thermalization time used as 0.5 *fm*.

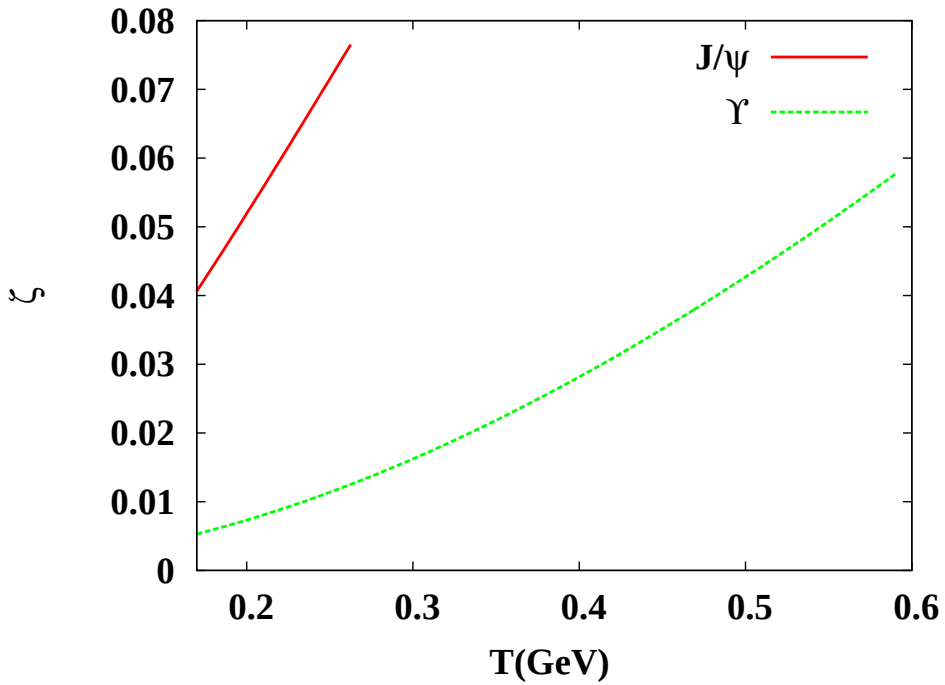

Figure: Plot of the probability ζ encoding the error in making the sudden approximation.

Quarkonia Evolution During Cooling Phase of QGP

- Initially the QGP temperature can be 400 to 500 MeV.
- The temperature decays very fast.
- Potential between quark and anti-quark depends on medium temperature
- The evolution of the wave function again cannot be taken to be adiabatic and also it can not be taken as sudden.
- It should be treated in terms of a time dependent perturbation theory (is one of the tool).

Bound to unbound states transition

Framework

- The time dependent perturbation at any instant t can be considered as $H^1(t) = V(r,t) - V(r,0)$.
- The unbound states are the plane wave states lying in the continuum regime of energy
- Unbound state of momentum k can be written like $\Psi_k = \frac{1}{\sqrt{\Omega}} e^{i\vec{k}.\vec{r}}$
- Transition amplitude after a certain time t from initial state $|\Psi_i\rangle$ of the potential $V(r,0)$ to $|\Psi_k\rangle$ is $a_{ik} = \int \frac{d}{dt}\langle\Psi_k|H^1(t)|\Psi_i\rangle \frac{e^{i(E_i-E_k)t}}{(E_i-E_k)} dt$.

Bound to unbound states transition

Framework-Continued

- The number of unbound states between the continuum momentum k and $k + dk$ over 4π solid angle is given by, $dn = \left(\frac{L}{2\pi}\right)^3 k^2 dk = \frac{\Omega}{(2\pi)^3} k^2 dk$
- The total transition probability to all continuum states is given by,
  $P = \int_{k=0}^{\infty} |a_{ik}|^2 \frac{\Omega}{(2\pi)^3} k^2 dk$

Framework

Complex potential

- If the quark anti-quark potential is Complex then the transition amplitude is taken as,
  $\bar{a}_{ik} = \int_0^t \frac{d}{dt}\langle\Psi_k|H^1_{com}(t)|\Psi_i\rangle \frac{e^{i(E_i-E_k)t}}{(E_i-E_k)-i\frac{\Gamma}{2}} e^{\frac{\Gamma}{2}t} dt$
- $H^1_{com}(t)$ is the perturbation in complex potential
- The additional quantity Γ appears due to the imaginary part in the potential $V_{im}(r)$ and is proportional to the imaginary contribution to the energy of the initial bound state [3] which can be estimated through the following equation,
  $\Gamma \approx 2\int dr V_{im}(r)|\Psi_i(r)|^2$.

[3] S. Kajimoto, Y. Akamatsu, M. Asakawa, and A. Rothkopf, Phys. Rev. D97, 014003 (2018),

Results

- we calculate wave functions for J/ψ and Υ by using Numerov's method
- Debye screened $q\bar{q}$ potential[4]

$$V(r) = -\frac{\alpha}{r} exp(-\omega_D r) + \frac{\sigma}{\omega_D} r(1 - exp(-\omega_D r))$$

$$\omega_D = T\sqrt{6\pi\alpha_s}, \quad \alpha = 0.471 = \frac{4}{3}\alpha_s \quad \sigma = 0.192 GeV^2$$

[4] F. Karsch, M.T. Mehr, and H. Satz, Z. Phys. C **37**, 617 (1988).

Results: Continued....

- The complex potential is given by, $V_{com}(r) = V(r) + i(-\alpha T \Phi(m_D r))$,
- Where $\Phi(x) = 2\int_0^\infty dz \frac{z}{(z^2+1)^2}\left(1 - \frac{\sin(zx)}{zx}\right)$.

Results: Continued....

- For an optimum performance of our numerical calculation, we have considered the initial temperature of the thermalised deconfined medium at $t = 0$ to be around 400MeV.
- Temperature decreases due to Bjorken expansion like, $T(t) = T_0\left(\frac{\tau_0}{\tau_0+t}\right)^{\frac{1}{3}}$.
- $\Upsilon(1S)$ was there at time $t = 0$
- J/ψ starts to form in medium when medium temperature decays to ~ 1.7 Tc
- $\Upsilon(2S)$ starts to form in medium when medium temperature decays to ~ 1.5 Tc

Results

Dissociation Probability

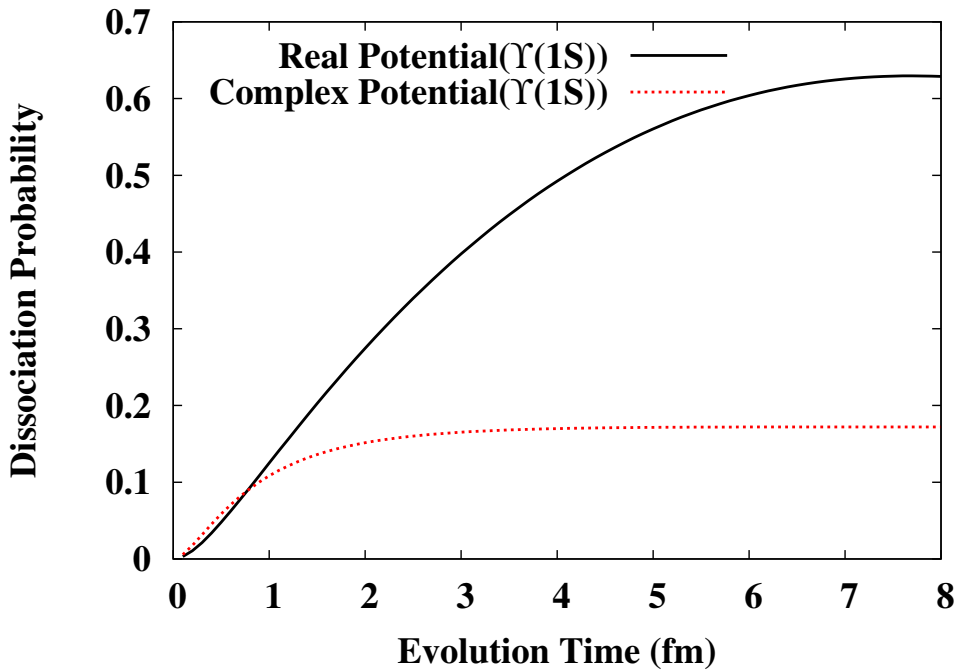

Figure: The solid black line shows that dissociation probability (transition to unbound states) of $\Upsilon(1S)$ increases with time in real valued potential and the dotted red curve shows the same in the complex valued potential.

Results

Dissociation Probability- Continued

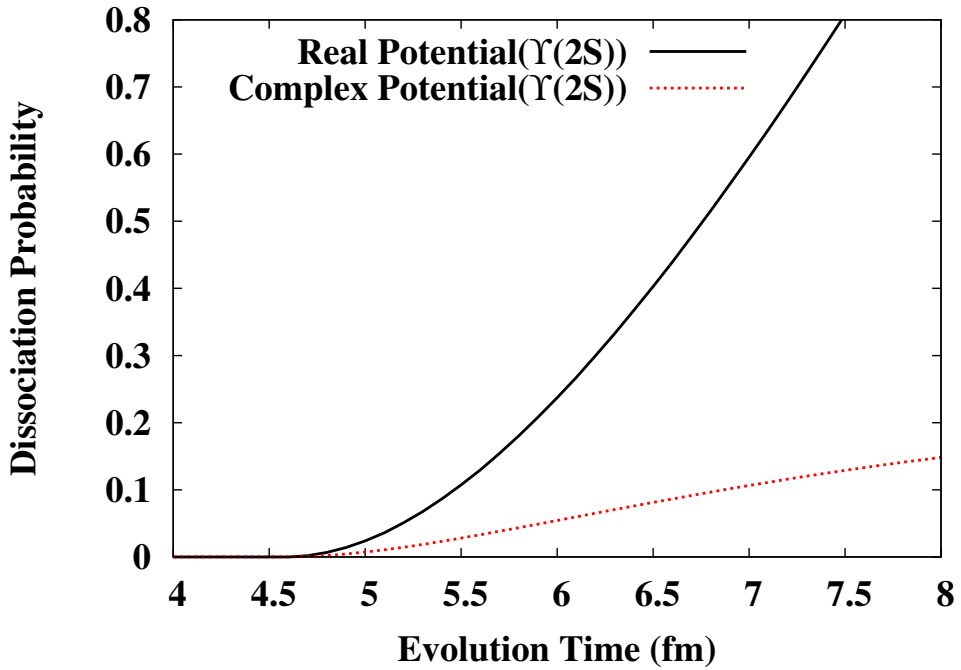

Figure: The solid (black) and dotted (red) lines show the dissociation probabilities of $\Upsilon(2S)$ below 1.5 T_c as a function of time in real and complex valued heavy quark potential respectively.

Results

Dissociation Probability- Continued

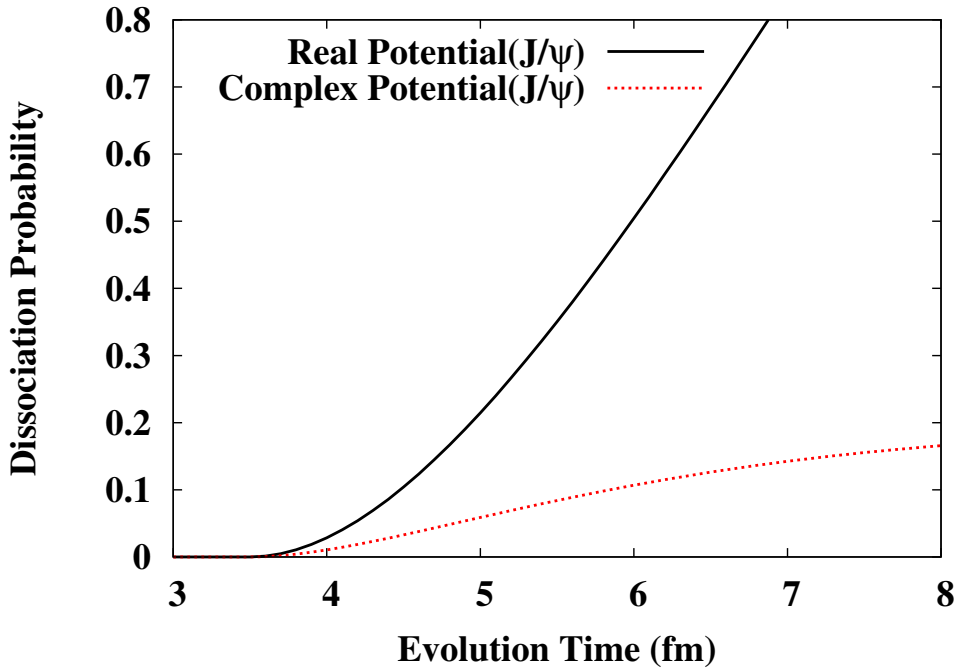

Figure: he solid (black) and dotted (red) lines show the dissociation probabilities of J/Ψ below 1.7 T_c as a function of time in real and complex valued heavy quark potential respectively.

Results

Dissociation Rate

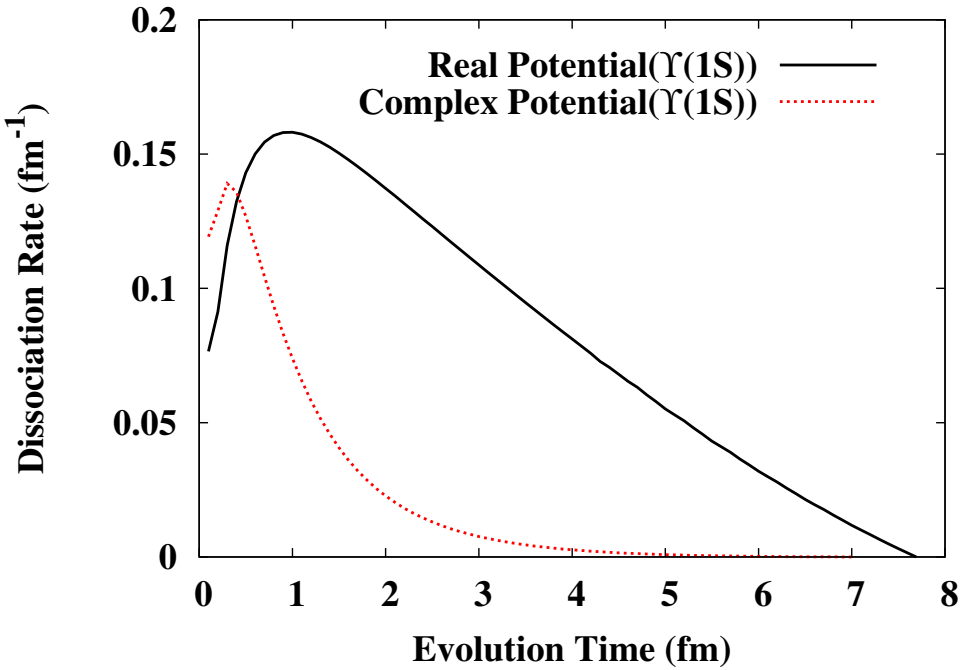

Figure: The solid (black) and dotted (red) lines show that transition rate of $\Upsilon(1S)$ to unbound states with time in real and complex valued potential respectively.

Results

Dissociation Rate-Continued

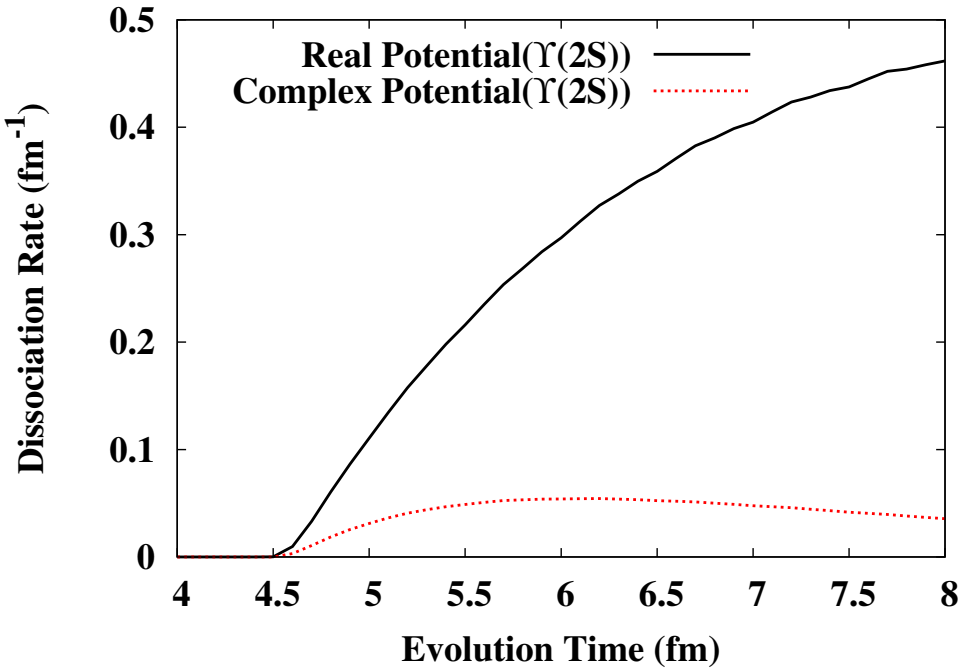

Figure: The solid (black) and dotted (red) lines show that transition rate of $\Upsilon(2S)$ to unbound states with time in real and complex valued potential respectively.

Results

Dissociation Rate-Continued

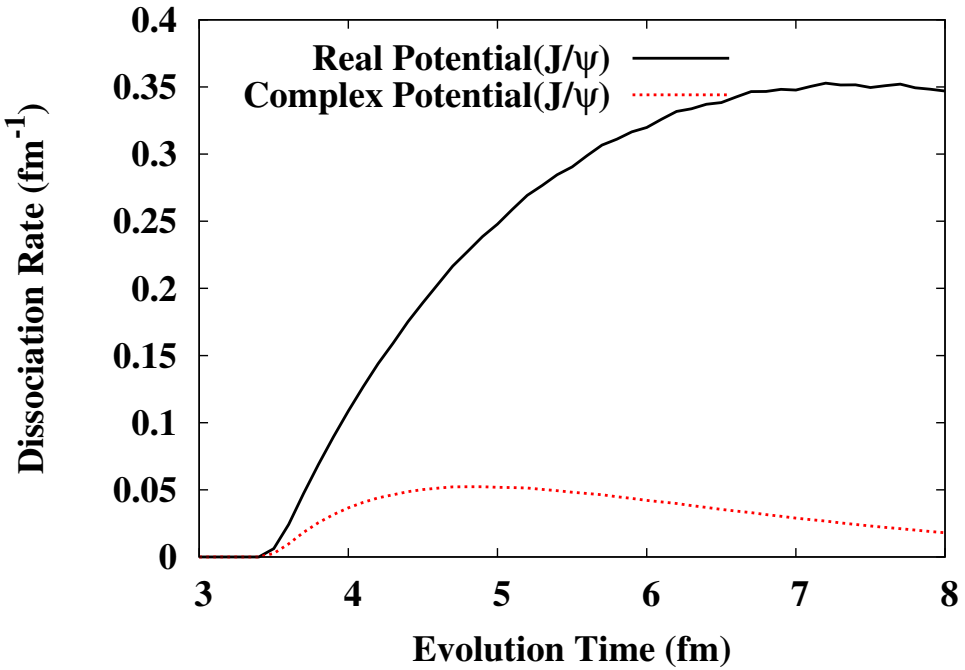

Figure: The solid (black) and dotted (red) lines show that transition rate of J/Ψ to unbound states with time in real and complex valued potential respectively.

Summary

- The initially survived $\Upsilon(1S)$ and later produced $\Upsilon(2S)$ and J/Ψ can further be dissociated in the medium at a temperature even below their respective dissociation thresholds.
- The medium induced transition was previously considered to study the bound state to bound state transition for a static QGP [5] using the solution of master equation but the transition to unbound states was untreated.
- In this work we have addressed the problem of bound state to unbound state transitions and that also for a more realistic scenario of evolving medium.

[5] Borghini and C. Gombeaud, Eur. Phys. J. C72, 2000 (2012)

Summary- Continued

- Our calculation in real valued potential gives an approximate result which is valid to describe short time behaviour.
- For long time behaviour, one must look into a non perturbative computation which at the moment does not appear feasible.
- On the other hand, in the complex potential scenario, the additional decay governed by the imaginary part makes the perturbation technique extremely reliable almost for the whole time span of the cooling phase of QGP.

Thank You !