Momentum transport coefficients with chiral dependent quark masses in thermal QCD medium

Pushpa Panday

8^{th} International Conference on Physics and Astrophysics of Quark-Gluon Plasma

Table of contents

1. Quasiparticle Model for Hot and Dense QCD Matter
2. Momentum Transport Coefficients
   - Shear Viscosity
   - Bulk Viscosity
3. Results
4. Conclusions

QCD at Finite Temperature and Weak Magnetic Field

- Medium generated mass of quarks due to interactions among quarks and gluons at asymptotically high temperature for finite chemical potential

$$m_{th}^2 = \frac{1}{8} g^2 C_F \left(T^2 + \frac{\mu^2}{\pi^2} \right) \tag{1}$$

- Effective quark mass for fth flavor in medium :-
  - In pure thermal medium: [J. High Energy Phys.(2007).]

$$m_f^2 = m_{f0}^2 + \sqrt{2} m_{f0} m_{f,th} + m_{f,th}^2$$

  - In magnetic field:

$$m_f^2 = m_{f0}^2 + \sqrt{2} m_{f0} m_{fth,B} + m_{fth,B}^2$$

- Now, we will evaluate $m_{fth,B}$ in weak magnetic field.

Continued

- Quark propagator in weak field limit ($m_{f0}^2 < q_f B < T^2$) in powers of $q_f B$

$$iS(K) = \frac{i\not{K}}{K^2 - m_{f0}^2} - \frac{i\gamma_5[(K.b)\not{u} - (K.u)\not{b}]}{(K^2 - m_{f0}^2)^2}(q_f B) \tag{2}$$

$\Rightarrow u^\mu = (1,0,0,0), \quad b^\mu = (0,0,0,1)$

- One-loop quark self energy upto $\mathcal{O}(q_f B)$

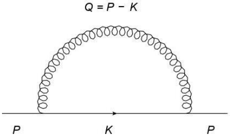


Figure: One-loop quark self energy in hot and magnetized medium

$$\Sigma(P) = g^2 C_F T \sum_n \int \frac{d^3k}{(2\pi)^3} \gamma_\mu \left(\frac{\not{K}}{(K^2 - m_{f0}^2)} - \frac{\gamma_5[(K.b)\not{u} - (K.u)\not{b}]}{(K^2 - m_{f0}^2)^2}(q_f B) \right) \times \gamma^\mu \frac{1}{(P-K)^2} \tag{3}$$

Continued

- Using covariant form of quark self energy, effective quark propagator in terms of chiral projection operator

$$S^*(P) = \frac{1}{2}\left[P_L \frac{\not{L}}{L^2/2} P_R + P_R \frac{\not{R}}{R^2/2} P_L\right]$$

$P_L = \frac{1}{2}(\mathrm{I} - \gamma_5) \Rightarrow$ Left-handed chiral projection operator
$P_R = \frac{1}{2}(\mathrm{I} + \gamma_5) \Rightarrow$ Right-handed chiral projection operator

- Static limit ($p_0 = 0, |\mathbf{p}| \to 0$) of L^2 and R^2 gives

$$m^2_{L(R)} = m^2_{th} \pm 4g^2 C_F M^2 \tag{4}$$

$\Rightarrow m^2_{th} \to$ pure thermal contribution
$\Rightarrow M^2 = \frac{q_f B}{16\pi^2}\left(\frac{\pi T}{2m_{f0}} - \ln 2 + \frac{7\mu^2\zeta(3)}{8\pi^2 T^2}\right) \to$ magnetic contribution to thermal mass

The degenerate left- and right-handed chiral modes of quarks get separated out in presence of weak magnetic field in contrast to the case of strong magnetic field.

Momentum Transport Coefficients

- Momentum transport occurs due to the irreversible transfer of momentum from points where velocity is large to those where it is small, resulting in the dissipation effects in fluids.
- Particle flow vector (N^μ) and energy-momentum tensor ($T^{\mu\nu}$) for dissipative fluid is defined as

$$N^\mu = N^\mu_{(0)} + n^\mu \qquad ; T^{\mu\nu} = T^{\mu\nu}_{(0)} + \tau^{\mu\nu} \tag{5}$$

$N^\mu_{(0)}, T^{\mu\nu}_{(0)} \to$ ideal currents ; $n^\mu, \tau^{\mu\nu} \to$ dissipative currents

- Decomposing the $\tau^{\mu\nu}$ into its irreducible form

$$\tau^{\mu\nu} = -\Pi\Delta^{\mu\nu} + 2u^{(\mu}h^{\nu)} + \pi^{\mu\nu} \tag{6}$$

$\Pi \to$ bulk viscous pressure ; $h^\mu \to$ energy-diffusion four-current

$\pi^{\mu\nu} \to$ shear-stress tensor ; $u^\mu \to$ fluid four-velocity

- We have used the Landau definition of fluid four-velocity, i.e. $h^\mu = 0$. Therefore,

$$N^\mu = nu^\mu + \underbrace{n^\mu}_{\text{dissipative}} \qquad ; T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu} \underbrace{- \Pi\Delta^{\mu\nu} + \pi^{\mu\nu}}_{\text{dissipative}} \tag{7}$$

Shear viscosity

- Dissipative part of $T^{\mu\nu}$ can be expressed in terms of away from equilibrium part of distribution function, $\delta f << f_{eq}$ as

$$\Pi = -\frac{\Delta_{\alpha\beta}}{3}\int dp p^{\alpha} p^{\beta}(\delta f + \delta\bar{f})$$
$$\pi^{\mu\nu} = \Delta^{\mu\nu}_{\alpha\beta}\int dp p^{\alpha} p^{\beta}(\delta f + \delta\bar{f})$$

- Relativistic-Boltzmann transport equation in presence of an external electromagnetic field

$$p^{\mu}\partial_{\mu} f(x,p) + qF^{\mu\nu}p_{\nu}\frac{\partial f(x,p)}{\partial p^{\mu}} = C[f] \tag{8}$$

  - Relaxation time approximation: $C[f] \simeq -\frac{p^{\mu}u_{\mu}}{\tau}(f - f_{eq})$

- Expressing the RBTE in terms of gradients of flow velocity and temperature and considering the general form of δf [Phys. Rev. D 102, 016016 (2020)]

$$\delta f = \sum_{r=0}^{4} a_r A^{(r)}_{\mu\nu\alpha\beta} p^{\mu} p^{\nu} V^{\alpha\beta}$$

$V^{\alpha\beta} = \frac{1}{2}\left(\frac{\partial u^{\alpha}}{\partial x^{\beta}} + \frac{\partial u^{\beta}}{\partial x^{\alpha}}\right)$; $A^{(r)}_{\mu\nu\alpha\beta} \rightarrow$ fourth-rank projection tensor

7

Continued

- We obtain the five a's coefficients and then finally compare the integral form of shear viscous tensor with $\pi^{\mu\nu} = \eta^{\mu\nu\alpha\beta} V_{\alpha\beta}$, which gives the five shear viscosity coefficients

$$\eta_0 = \sum_f \frac{g_f}{15T} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\mathbf{p}^4}{\varepsilon_f^2}\right) \left(f_{eq,f}\left(1 - f_{eq,f}\right)\tau_f + \bar{f}_{eq,f}\left(1 - \bar{f}_{eq,f}\right)\tau_{\bar{f}}\right),$$

$$\eta_1 = \sum_f \frac{g_f}{15T} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\mathbf{p}^4}{\varepsilon_f}\right) (u.p) \left[\frac{f_{eq,f}\left(1 - f_{eq,f}\right)\tau_f}{(u.p)^2 + (q_f B\tau_f)^2} + \frac{\bar{f}_{eq,f}\left(1 - \bar{f}_{eq,f}\right)\tau_{\bar{f}}}{(u.p)^2 + (q_{\bar{f}} B\tau_{\bar{f}})^2}\right]$$

$$\eta_2 = \sum_f \frac{g_f}{15T} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\mathbf{p}^4}{\varepsilon_f}\right) \left[\frac{(q_f B) f_{eq,f}\left(1 - f_{eq,f}\right)\tau_f^2}{\left((u.p)^2 + (q_f B\tau_f)^2\right)} + \frac{(q_{\bar{f}} B)\bar{f}_{eq,f}\left(1 - \bar{f}_{eq,f}\right)\tau_{\bar{f}}^2}{\left((u.p)^2 + (q_{\bar{f}} B\tau_{\bar{f}})^2\right)}\right]$$

$$\eta_3 = \sum_f \frac{g_f}{15T} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\mathbf{p}^4}{\varepsilon_f}\right) \left[\frac{(u.p) f_{eq,f}\left(1 - f_{eq,f}\right)\tau_f}{\left((u.p)^2 + (2q_f B\tau_f)^2\right)} + \frac{(u.p)\bar{f}_{eq,f}\left(1 - \bar{f}_{eq,f}\right)\tau_{\bar{f}}}{\left((u.p)^2 + (2q_{\bar{f}} B\tau_{\bar{f}})^2\right)}\right]$$

$$\eta_4 = \sum_f \frac{2g_f}{15T} \int \frac{d^3p}{(2\pi)^3} \left(\frac{\mathbf{p}^4}{\varepsilon_f}\right) \left[\frac{(q_f B) f_{eq,f}\left(1 - f_{eq,f}\right)\tau_f^2}{\left[(u.p)^2 + (2q_f B\tau_f)^2\right]} + \frac{(q_{\bar{f}} B)\bar{f}_{eq,f}\left(1 - \bar{f}_{eq,f}\right)\tau_{\bar{f}}^2}{\left[(u.p)^2 + (2q_{\bar{f}} B\tau_{\bar{f}})^2\right]}\right]$$

$\eta_0 \rightarrow$ longitudinal component; $\eta_1, \eta_3 \rightarrow$ transverse components; $\eta_2, \eta_4 \rightarrow$ Hall components

8

Bulk viscosity

- The general form of δf associated with bulk viscosity is [Phys. Rev. D 102, 016016 (2020)]

$$\delta f = \sum_{r=1}^{3} a_r A_{(r)\mu\nu} \partial^{\mu} u^{\nu}$$

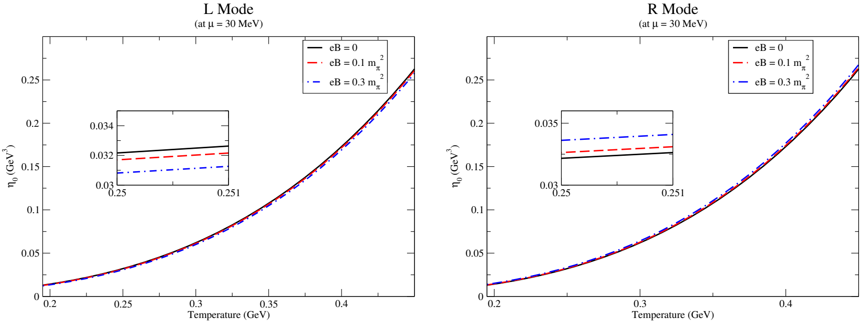
- Employing the RBTE and comparing the integral form of bulk viscous pressure with $\Pi = \zeta^{\mu\nu} \partial_{\mu} u_{\nu}$, we obtain the two bulk viscosity coefficients

$$\zeta_1 = \zeta_2 = \sum_f \frac{g_f}{3T} \int \frac{d^3p}{(2\pi)^3} \frac{X^2}{\varepsilon_f^2} \left[\tau_f f_{eq,f} \left(1 - f_{eq,f}\right) + \tau_{\bar{f}} \bar{f}_{eq,f} \left(1 - \bar{f}_{eq,f}\right)\right] \tag{9}$$

$\zeta_1 \rightarrow$ longitudinal component; $\zeta_2 \rightarrow$ transverse component

- We have embodied the quasiparticle masses of left- and right-handed chiral modes of quarks for the estimation of shear and bulk viscosity.
- Similarly, we have obtained the gluon contribution to the shear and bulk viscous coefficients.

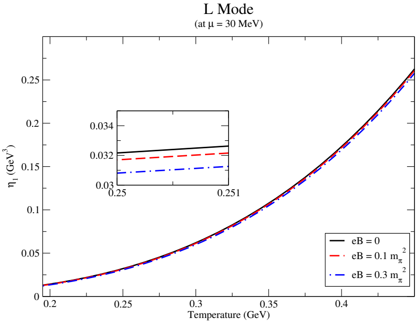
Results



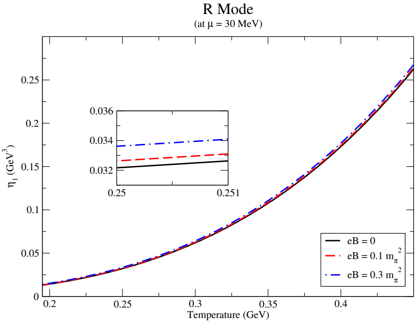
(a) (b)

η_0 (longitudinal) decreases for L mode and increases for R mode with magnetic field.

Continued



(a)

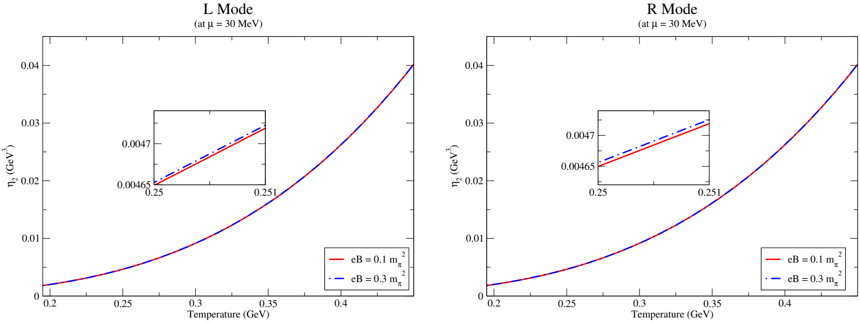


(b)

L mode η_1 (transverse) decreases with magnetic field and whereas increases for R mode.

η_3 (transverse) is also found to follow the same trend as η_1 with magnetic field for both modes.

Continued

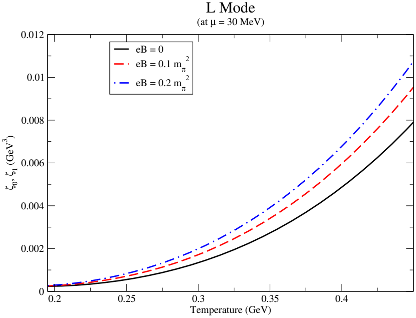


(a) (b)

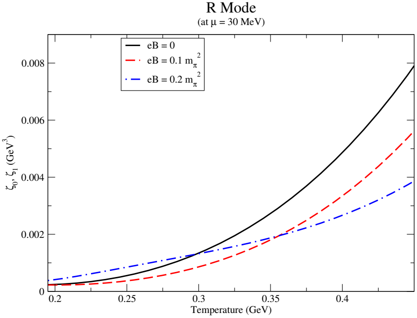
η_2 (Hall) for both L and R modes increases with magnetic field.

Another Hall component η_4 also follows the same trend as η_2 with magnetic field.

Continued



(a)



(b)

Both ζ_0 (longitudinal) and ζ_1 (transverse) increases with magnetic field for L mode.

For R mode, upto $T \sim 0.35$GeV, both ζ_0 and ζ_1 at $eB = 0.2m_\pi^2$ has high magnitude than at $eB = 0.1m_\pi^2$, whereas beyond T ~0.35 GeV, it has high magnitude at $eB = 0.1m_\pi^2$ than at $eB = 0.2m_\pi^2$.

Conclusions

- Left- and right-handed chiral modes of quarks gets separated out in presence of a weak magnetic field due to their different masses contrary to the case of strong magnetic field.
- Magnetic field breaks the isotropy of the medium and hence we obtain the five shear and two bulk viscous components.
- The decrement/increment of η_0, η_1 and η_3 with magnetic field for L/R mode is attributed to the different values of effective quark mass for both modes.
- The increment of η_2 and η_4 with magnetic field is due to the direct dependence of magnetic field on Hall components.
- ζ_0 and ζ_1 increases with B for L mode, while for the R mode, it has higher magnitude at $eB = 0.2m_\pi^2$ than at $eB = 0.1m_\pi^2$ until $T \sim 0.35$ GeV, after which it becomes smaller than at $eB = 0.1m_\pi^2$.

THANKING YOU FOR YOUR KIND ATTENTION.