Confinement-deconfinement transition and Z_N symmetry in SU(N)-Higgs theory

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Plan:

In the "lattice" Partition function of $SU(N)-$Higgs in the path-integral formulation, the partition averages are Z_N invariant, even though the action is not invariant under Z_N.

Discuss possible implications for QCD

Z_N in Pure SU(N) gauge theory: At temperature T

$$
\begin{aligned}
A_\mu &= \lambda_a A^a_\mu, a = 1, 2, ..., N^2 - 1 \\
F_{\mu\nu} &= \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right] \\
S_g &= \int_0^\beta d\tau \int d^3x \left\{\frac{1}{2} Tr\left[F_{\mu\nu} F_{\mu\nu}\right]\right\} \\
\mathcal{Z}(T) &= \int [DA] Exp[-S_g]
\end{aligned}
$$

Functional integral is over A_μ satisfying $A(x, 0) = A(x, \beta)$.
Allowed gauge transformations, $V(x, \tau) \in SU(N)$, satisfy

$$
V(x, 0) = zV(x, \beta), z = \mathbb{I} e^{i2\pi n/N} \in Z_N \subset SU(N),
$$

hence called Z_N gauge transformations.

Z_N symmetry and Polyakov loop: Thermodynamics

$$\begin{aligned} L(x,T) &= \frac{1}{N} Tr\left\{U(x,0;x,\beta)\right\}, \\ U(x,0;x,\beta) &= P\left[Exp\left(ig\int_0^\beta d\tau A_0(x,\tau)\right)\right] \\ \langle |L| \rangle &\propto exp(-F/T) \end{aligned}$$

where F is free energy of a static charge.

Svetitsky and Mclerran, Phys. Rev. D 24 (1981)

$\langle L \rangle = 0 \implies$ confinement, $\langle L \rangle \neq 0 \implies$ deconfinement. So $\langle |L| \rangle$ is an order parameter for the confinement deconfinement phase transition.

$V(x,0) = zV(x,\beta) \implies L(x,T) \longrightarrow zL(x,T)$

Z_N symmetry and Polyakov loop: Thermodynamics

Polyakov loop effective model:- It is possible to describe the CD transition using,

$$\begin{aligned} V(L,T) &= T^4(-aL^2 + cL^4), N = 2 \\ &= T^4(-a|L|^2 + b(L^3 + L^{*3}) + c|L|^4), N = 3 \end{aligned}$$

A. Dumitru, R. D. Pisarski, PLB(2001)

$\langle L \rangle$ is obtained by minimising $V(L,T)$. The free energy of the system,

$$\mathcal{F} = -V(\langle L \rangle, T)$$

By suitable choices a, b, and c, it is possible to reproduce lattice results.

Z_N symmetry in the presence of bosonic(Φ) and fermion(Ψ) matter fields.

$$
\begin{aligned}
\mathcal{Z}(T) &= \int D[A\Phi\Psi]\ Exp[-(S_g + S_B + S_F)] \\
S_B &= \int d^3x \int d\tau \left\{ \frac{1}{2}|D_\mu \Phi|^2 + \frac{m^2}{2}\Phi^\dagger \Phi \right\} \\
S_F &= \int d^3x \int d\tau \left(\not{D} - m \right) \Psi
\end{aligned}
$$

Path-integral is over Φ and Ψ with boundary conditions,

$$
\Phi(x,0) = \Phi(x,\beta), \Psi(x,0) = -\Psi(x,\beta)
$$

But a Z_N gauge transformation leads to $\Phi^g(x,0) = z\Phi^g(x,\beta), \Psi^g(x,0) = -z\Psi^g(x,\beta)$.

Explicit breaking of Z_N

A Z_N transformation can always be written as,

$$V(x,\tau) = g(\tau)U(x,\tau)$$

$U(x,\tau)$ is periodic in $\tau-$direction, $g(0) = zg(\beta)$. $g(\tau)$ can be taken to be,

$$\begin{aligned} g(\tau) = \mathbb{I} exp\{i\alpha(\tau)\} \ \text{with} \\ \alpha(\tau) &= 0, \tau < \beta \\ &= \frac{2\pi n}{N}, \tau = \beta \end{aligned}$$

$n = 0, 1, ..., N-1$.

Supposing $g(\tau)$ is allowed to act only on the gauge fields, i.e $A_0 \rightarrow A_0^g$.The only affected terms in S are,

$$\begin{aligned} |D_0^g \Phi| &\neq |D_0 \Phi| \\ \bar{\Psi}(\gamma^0 D_0^g \Psi) &\neq \bar{\Psi}(\gamma^0 D_0 \Psi) \end{aligned}$$

The effects of matter fields lead to,

$$\begin{aligned} V(L,T) &= V_0(L,T) - h_2 L, \quad \text{for N} = 2 \\ &= V_0(L,T) - h_3 L_r, \ \text{for N} = 3 \end{aligned}$$

h_2 and h_3 are the strength of the explicit breaking. $h_2 \neq 0$ leads to crossover, $h_3 \neq 0$ leads to weak 1st order, second order or even a crossover for large h_3.

What are $h_2(T, m, \lambda, n_f, \mu_B)$ and $h_3(T, m, \lambda, n_f, \mu_B)$ near CD transition?

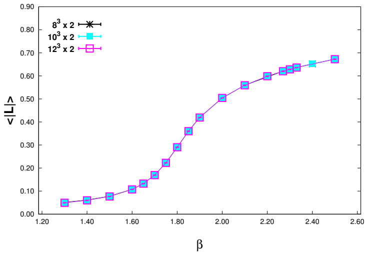
SU(N)+Higgs action on the lattice($\lambda_\phi = 0, m_\phi$)

$$
\begin{aligned}
S_{g,\Phi} &= S_g - \kappa \sum_{\hat{x},n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{x}} U_{n,\hat{x}} \Phi_n\right)\right] + \sum_n \frac{1}{2} \mathrm{Tr}\left(\Phi^\dagger_n \Phi_n\right) \\
&- \kappa \sum_{,n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{\tau}} U_{n,\hat{\tau}} \Phi_n\right)\right], (NS)^3 \times N_\tau
\end{aligned}
$$

$\kappa = \frac{1}{(ma)^2+8}$. Observables computed are averages and distributions of the Polyakov loop.

$$
\begin{aligned}
S_g, L &= \frac{1}{N} \sum_x \mathrm{Tr}\left[\prod_{\mathrm{n}_\tau=1}^{\mathrm{N}_\tau} \mathrm{U}_{\mathrm{x},\hat{\tau}}\right] \\
SK_s &= \textstyle\sum_{\hat{x},n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{x}} U_{n,\hat{x}} \Phi_n\right)\right], \\
SK_\tau &= \textstyle\sum_{\hat{x},n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{\tau}} U_{n,\hat{\tau}} \Phi_n\right)\right]
\end{aligned}
$$

SU(2)-Higgs



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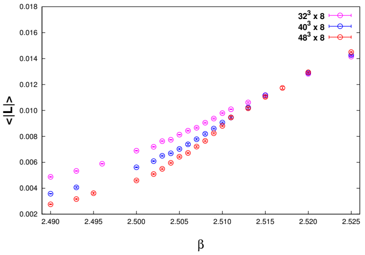
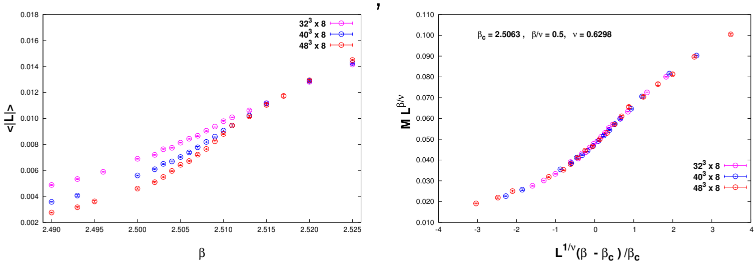


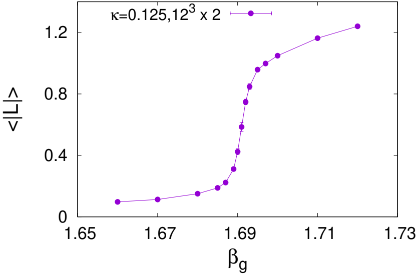
For $N_\tau = 2$ there is no volume dependence in the Polyakov loop average signalling a crossover. In contrast, there is clear volume dependence for $N_\tau = 8$



The Polyakov loop average scales with volume near the critical point suggesting a second-order phase transition.

These results imply $h_2 = 0$. **In $SU(2)-$Higgs theory at $\lambda_\phi = 0$ and $m_\phi = 0$, critical behaviour is only possible if $h_2 = 0$.**

SU(3)-Higgs at $\lambda_\phi = 0$ and $m_\phi = 0$ $N_\tau = 2$



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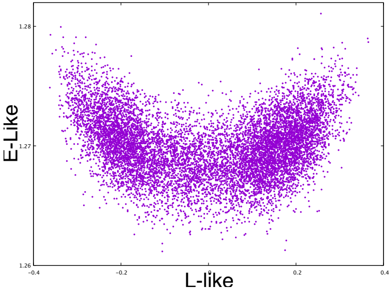
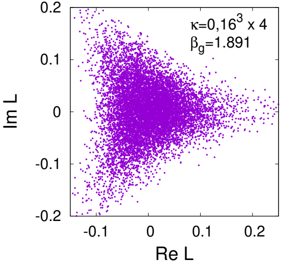
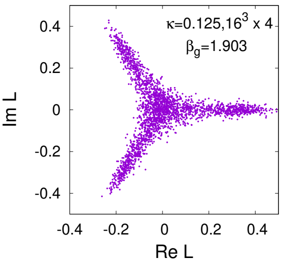


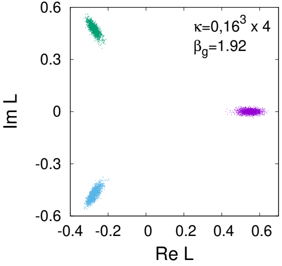
For $N_\tau = 2$ the transition is 2nd order but behaves as an endpoint of first order transition. The universality class is that of 3D Ising model.

$N_\tau = 4$

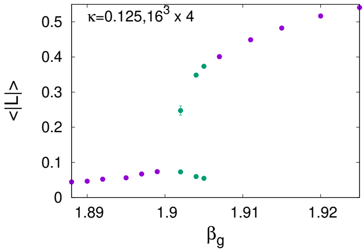

κ=0,16^3 x 4
β_g=1.891
Im L
Re L
0.2
0.1
0
-0.1
-0.2
-0.1
0
0.1
0.2


κ=0.125,16^3 x 4
β_g=1.903
Im L
Re L
0.4
0.2
0
-0.2
-0.4
-0.4
-0.2
0
0.2
0.4

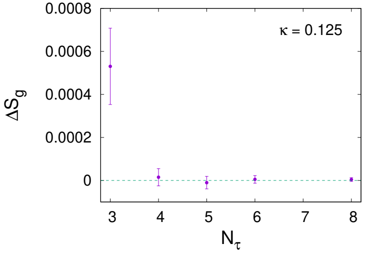
,


κ=0,16^3 x 4
β_g=1.92
Im L
Re L
0.6
0.3
0
-0.3
-0.6
-0.4
-0.2
0
0.2
0.4
0.6

,


κ=0.125,16^3 x 4
<|L|>
β_g
0.5
0.4
0.3
0.2
0.1
0
1.89
1.9
1.91
1.92

Difference of gauge action S_g and gauge Higgs interaction $SK = SK_\tau + SK_4$ between $Arg(L) = 0$ and $Arg(L) = exp(2\pi/3)$ states, as a function of N_τ



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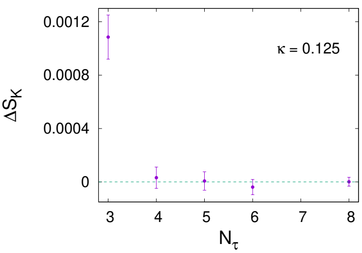


The results suggest that $h_3 \to 0$ in the continuum limit.
Why there is symmetry, even though the action is not invariant under $L \to zL$?

$$\mathcal{Z} = \int dL g(L) e^{-F(L)}$$

Even if the Boltzman factor does not have the $L \to zL$ symmetry if it is dominated by the density of states, $g(L)$, which is symmetric under $L \to zL$, then thermodynamic averages will have Z_N symmetry.

$$\begin{aligned} S_{g,\Phi} &= S_g - \kappa \sum_{\hat{x},n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{x}} U_{n,\hat{x}} \Phi_n\right)\right] + \sum_n \frac{1}{2} \mathrm{Tr}\left(\Phi^\dagger_n \Phi_n\right) \\ &- \kappa \sum_{,n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{\tau}} U_{n,\hat{\tau}} \Phi_n\right)\right] \\ S_{g,\Phi} &= S_g - \kappa \sum_{n} \mathrm{ReTr}\left[\left(\Phi^\dagger_{n+\hat{\tau}} U_{n,\hat{\tau}} \Phi_n\right)\right] + \sum_n \frac{1}{2} \mathrm{Tr}\left(\Phi^\dagger_n \Phi_n\right) \end{aligned}$$

$$S_{g,\Phi} = S_g - \kappa \sum_n \mathrm{ReTr}\left[(\Phi^\dagger_{n+\hat{\tau}} U_{n,\hat{\tau}} \Phi_n)\right] + \sum_n \frac{1}{2} \mathrm{Tr}\left(\Phi^\dagger_n \Phi_n\right)$$

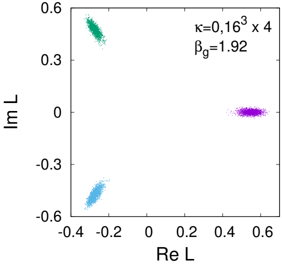
The Φ field can be easily integrated out as the second term is like a collection of non-interacting gauged Higgs chains. The integration gives rise to a determinant,

$$\begin{aligned}
\mathcal{Z} &= \int DAExp[-S_g] Q^V \\
Q(\kappa, L, N_\tau) &= Det\,(M)\,, \\
M &= \begin{pmatrix} A_{N_\tau} & B_{N_\tau} + \kappa W_x \\ B_{N_\tau} + \kappa W_x^\dagger & C_{N_\tau} \end{pmatrix}, W_x = U(x,0;x,\beta)
\end{aligned}$$

$B_{N_\tau} \to 0$ with $N_\tau \to \infty$, so that $Q(L) = Q(zL)$. So in the continuum limit, Z_N symmetry is realised in $SU(N)-$Higgs

Similar calculations, ignoring spatial gauge-fermion interaction terms, in $SU(N)-$KS Fermions show that the strength of explicit breaking decreases with N_τ, but does not vanish in the $N_\tau \to \infty$ limit.

These results suggest that the explicit breaking of Z_3 in QCD will be small, which will lead to Z_N meta-stable states.



Conclusions: Z_3 meta-stable states are possible in heavy-ion collisions.

Collaborators: Minati Biswal, Mridupawan Deka, Vinod Mamale, Sabiar Shaikh, P. S. Saumia