Eigenvalue spectrum of 2+1 flavor QCD using highly improved staggered quarks

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Reference: O. Kaczmarek, R.S.,S. Sharma, arXiv:2301.11610

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- However $U_A(1)$ part of the chiral symmetry is anomalous hence it is not clear whether it is effectively restored along with its non-singlet part.
- There are some evidence that show $U_A(1)$ remains effectively broken at T_c in $2+1$ flavor QCD with physical quark mass m [A. Bazavov et al., 12, V. Dick et al., 15] even when $m \rightarrow 0$ [O. Kaczmarek, L. Mazur, and S. Sharma, 21] but also there are some contrary results [S. Aoki et. al., JLQCD coll, 15, 17, 21, B. Brandt et. al., 16, T. W. Chiu, 13].

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- The eigenvalue spectrum on the lattice depends on the choice of the fermion discretisation. What will happen with staggered fermions?
- It will be interesting to check the properties of the eigenvalue spectrum by carefully performing a continuum extrapolation, in the large volume limit.

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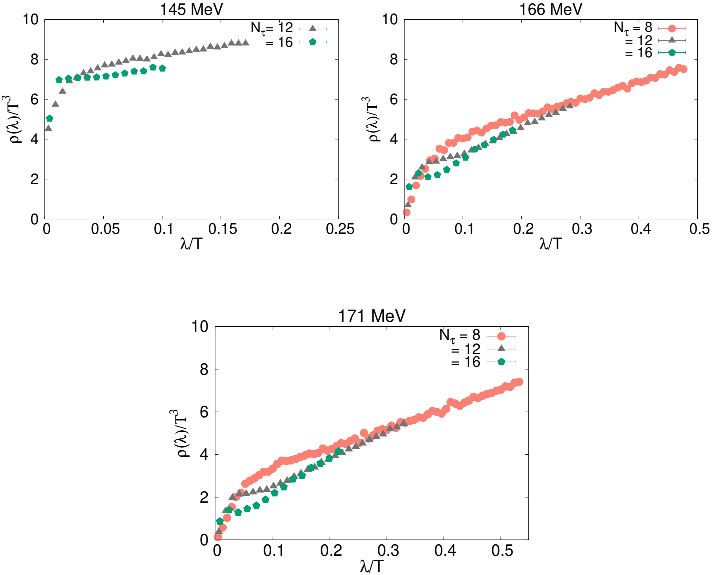
- We next measure $60 - 200$ eigenvalues of the massless HISQ Dirac matrix per configuration.

Our results: Eigenvalue density as a function of T



Characterizing Bulk modes

- The bulk eigenvalue density is characterized as

  [S. Aoki, H. Fukaya, and Y. Taniguchi, 12]

$$\frac{\rho(\lambda)}{T^3} = \frac{\rho_0}{T^3} + \frac{\lambda}{T} . \frac{c_1(T,m)}{T^2} + \frac{\lambda^2}{T^2} . \frac{c_2(T,m)}{T} + \frac{\lambda^3}{T^3} c_3(T,m) .$$

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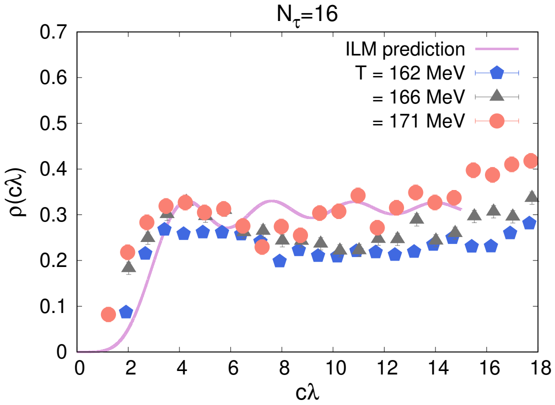
- Taking continuum ($\sim 1/N_\tau^2$) extrapolation of c_1 at different T, the T and m dependence is given by $c_1(m,T)/T^2 = 16.8(4)$.
- We perform a fit to the bulk part i.e. all eigenvalues $\lambda > \lambda_0$ with $\frac{\rho(\lambda)}{T^3} = \frac{\lambda}{T}.\frac{c_1(T,m)}{T^2} + \frac{\rho_0}{T^3}$.

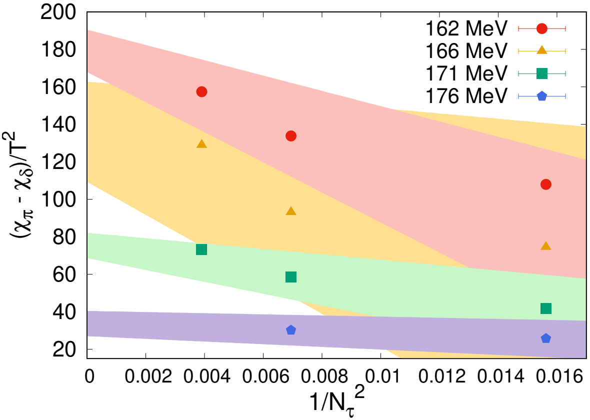
- Near zero modes distribution at zero temperature from Instanton Liquid model [J. J. M. Verbaarschot, 1997]

$$\rho(c\lambda) = \frac{c\lambda}{2}\left[J^2_{N_f}(c\lambda) - J_{N_f+1}(c\lambda)J_{N_f-1}(c\lambda)\right] \quad , \; c = \langle 0|\bar{\psi}\psi|0\rangle V/T \; .$$



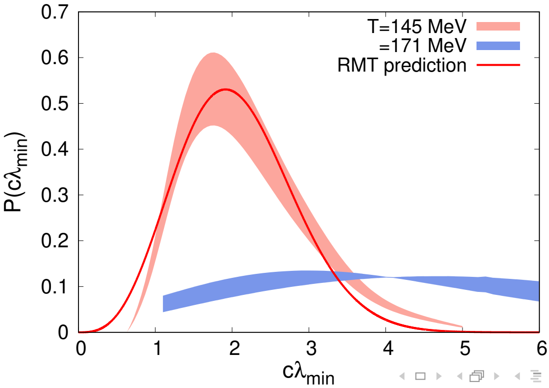
When is $U_A(1)$ effectively restored?

- The observable we look is $\chi_\pi - \chi_\delta$, where χ_π and χ_δ is defined as $\chi_\pi = \int d^4x\, \langle \pi^i(x)\pi^i(0)\rangle$ and $\chi_\delta = \int d^4x\, \langle \delta^i(x)\delta^i(0)\rangle$
  [E. V. Shuryak, 93]
- $U_A(1)$ restoration happens around temperature $\sim 1.14\,T_c$



- For a random matrix ensemble at $T = 0$ the smallest eigenvalue is distributed according to,

$$P(c\lambda_{\text{min}}) = \sqrt{\frac{\pi}{2}}(c\lambda_{\text{min}})^{3/2} I_{3/2}(c\lambda_{\text{min}}) e^{-\frac{1}{2}(c\lambda_{\text{min}})^2} \,, \text{[P. J. Forrester, 93]}$$



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- We find that at $T \sim 1.15\,T_c$ the bulk and near-zero modes completely separates.
- Incidentally this is the same temperature at which $U_A(1)$ is effectively restored.

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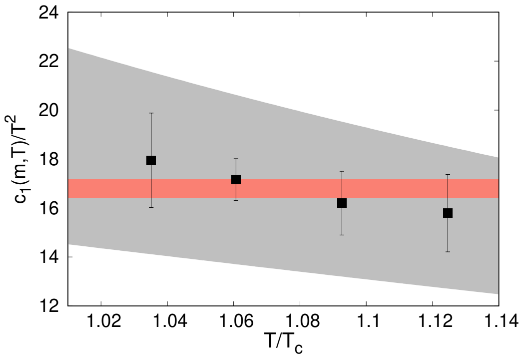
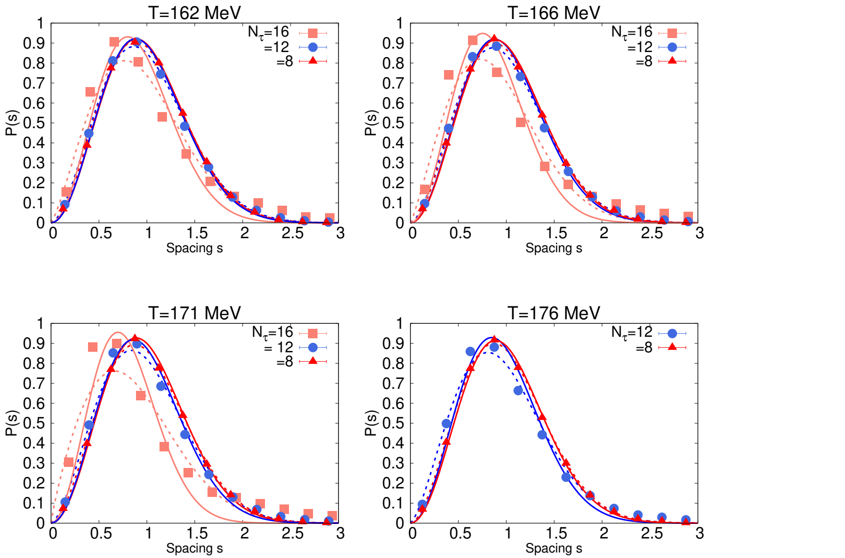
- $T \gtrsim T_c$:Interactions are liquid-like $\rightarrow$ small eigenvalues have oscillatory behavior.
- $T \sim 1.15\, T_c$:Interactions further reduce $\rightarrow$ Near-zero and bulk modes completely separates.
- This is the probably the reason why $U_A(1)$ is effectively restored around the same temperature $\rightarrow$ it is essential to take the continuum limit.

Thanks

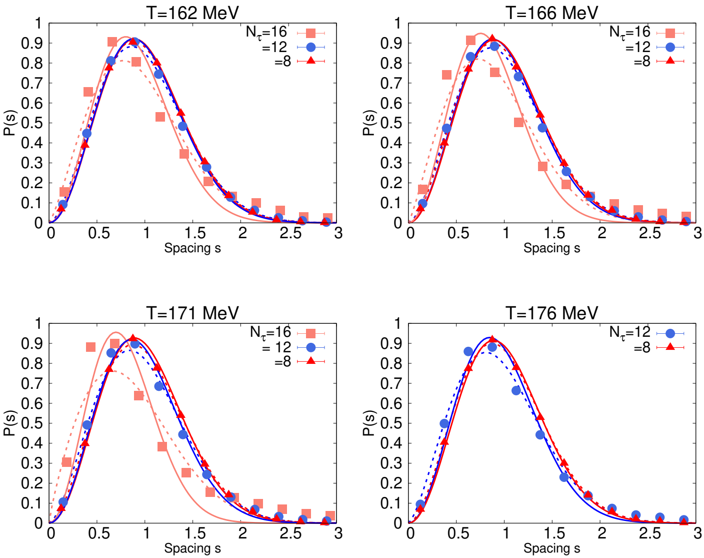


Figure

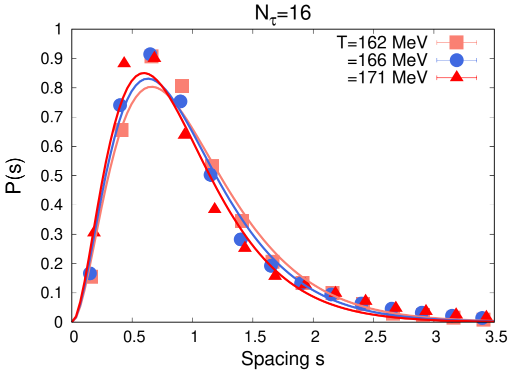
- Temperature dependence of c_1 in continuum.



- Spacing distribution of bulk modes.



- Spacing distribution of bulk modes.
- Dotted $\rightarrow f(s) = as^b \mathrm{e}^{-\mathrm{c}s^2}$, solid $\rightarrow f(s) = ps^2 \mathrm{e}^{-\mathrm{q}s^2}$



Figure

- spacing distribution using a mixed ansatz $\rightarrow$ $P(s) \sim s^2 \exp\{(-\alpha s)\}$.