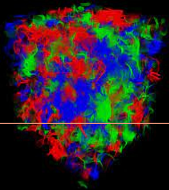
# Lattice approach to inhomogeneous magnetic fields as probes of QCD thermodynamics

ICPAQGP 2023 - Puri, India



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Department of Physics
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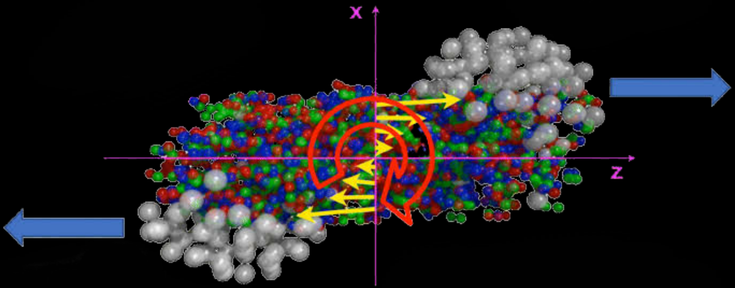
# OUTLINE

1. Motivation
2. Lattice QCD and magnetic fields
3. Lattice simulations
4. Summary & Conclusions

# Motivation

# PERIPHERAL HICS




$$\sqrt{eB} \sim 0.1 \text{ - } 0.5 \text{ GeV}$$

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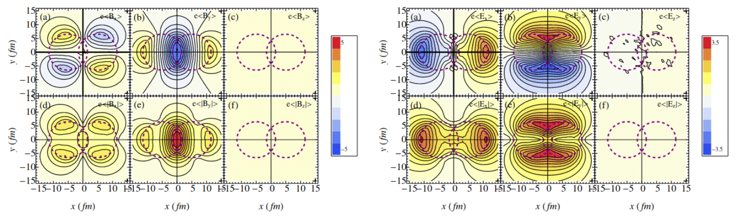

**Figure 1:** Spatial distributions **B** (left) and **E** (right) fields for an impact parameter $b = 10$ fm. Deng and Huang 2012.

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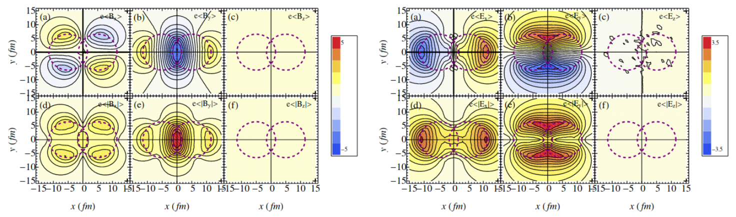


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Caveats:

- highly non-homogeneous background.
- **E** leads to sign problem.
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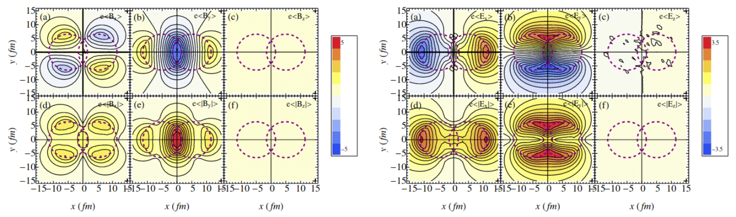


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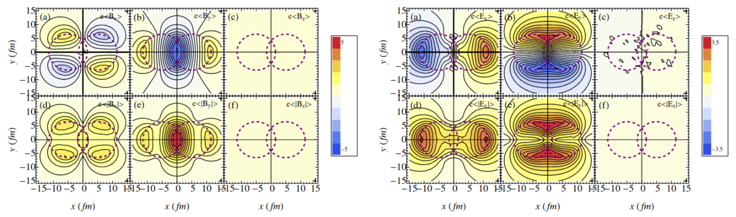

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$B$ changes $T_c$: could the system be in different phases at different $x$?

# Lattice QCD and magnetic fields

# Lattice QCD in a nutshell

# LATTICE QCD IN A NUTSHELL

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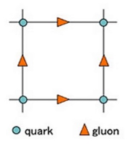

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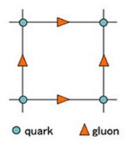

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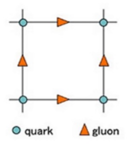

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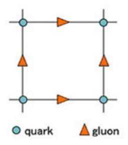


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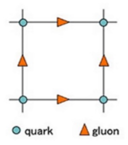


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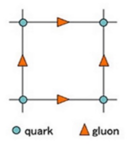


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- magnetic field $B \longrightarrow u_\mu = e^{iaqA_\mu} \in$ U(1) (BACKGROUND!)

# Uniform magnetic field on the lattice

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Flux quantization in a box

$$qB = \frac{2\pi N_b}{L_x L_y}$$

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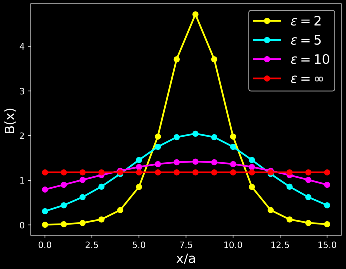
# Inhomogeneous magnetic field on the lattice

$$\mathbf{B} = \frac{B}{\cosh\left(\frac{x-L_x/2}{\epsilon}\right)^2}\hat{z}$$

Motivated by HIC scenarios Deng and Huang 2012, Cao 2018.

$$qB = \frac{\pi N_b}{L_y \epsilon \tanh\left(\frac{L_x}{2\epsilon}\right)} \qquad N_b \in \mathbb{Z}$$

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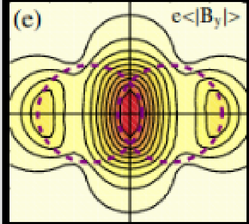
$$u_y = e^{iaqB\epsilon\left[\tanh\left(\frac{x-L_x/2}{\epsilon}\right)+\tanh\left(\frac{L_x}{2\epsilon}\right)\right]}$$

$$u_z = u_t = 1$$



# Lattice simulations

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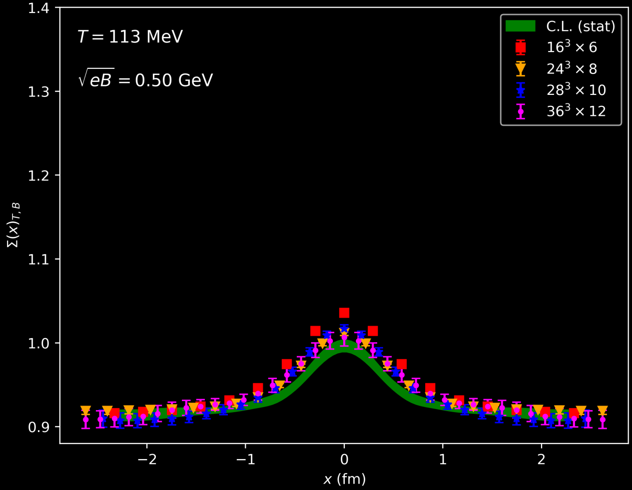
- Quantities: $\Sigma(x)_{T,B} \quad P(x)_{T,B} \quad \mathbf{J}(x)_{T,B}$.

# CHIRAL CONDENSATE

# Chiral condensate




Enchancement of the condensate: magnetic catalysis.

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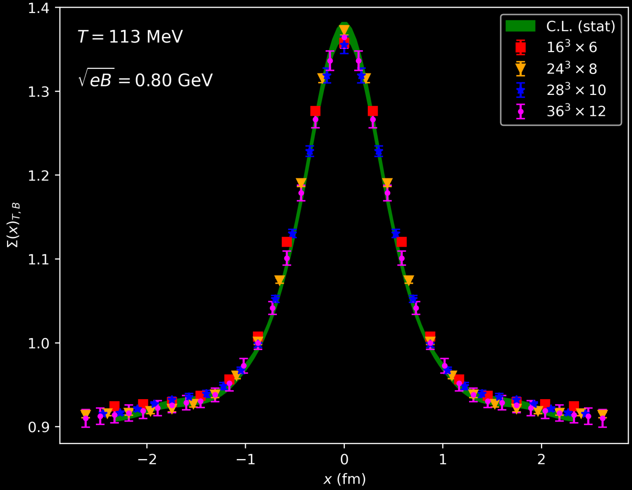


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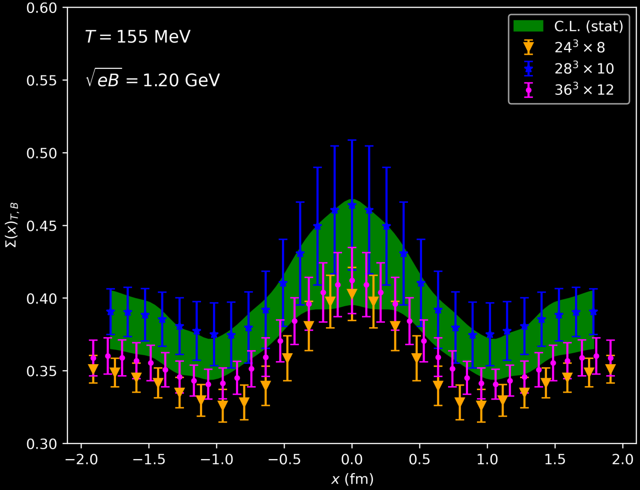


$\Sigma(x)_{T,B}$ is deformed (dips appear!).

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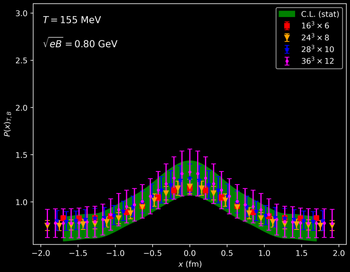

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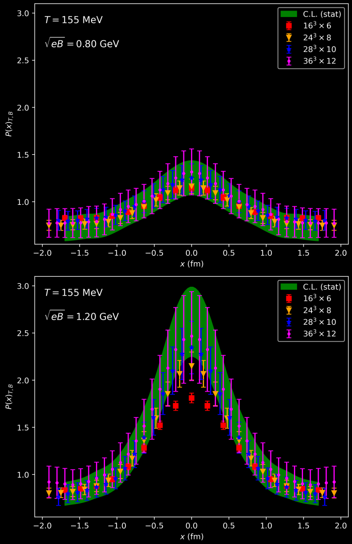

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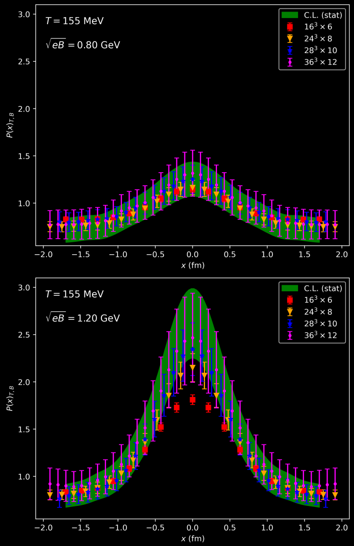


$P(x)_{T,B}$ is broader than the chiral condensate.

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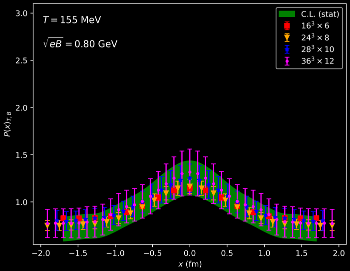


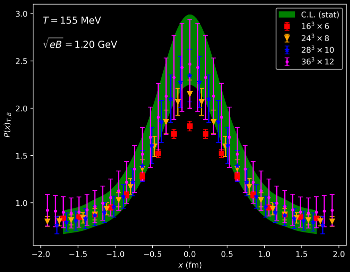


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$$C(x, x') = \frac{1}{m_\pi^3} \langle\, \bar{\psi}\psi(x) P(x')\, \rangle_c$$

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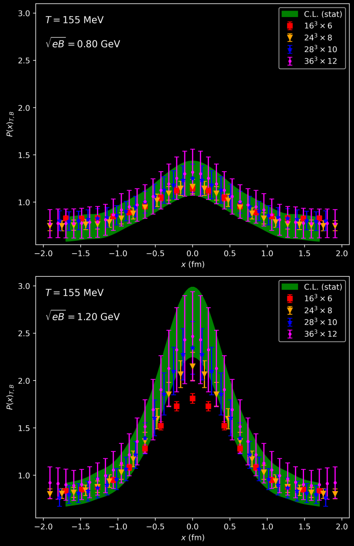


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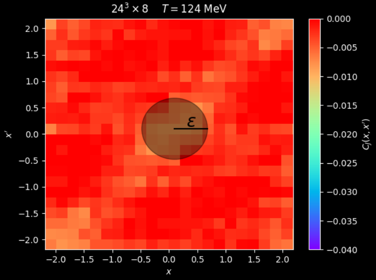

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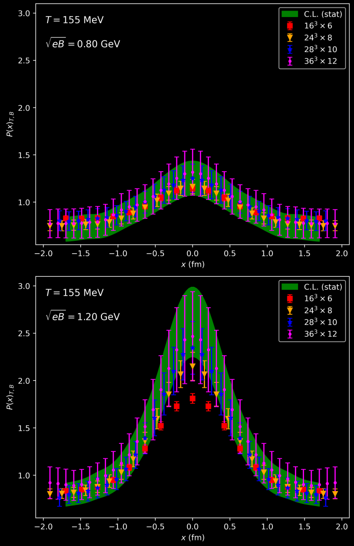


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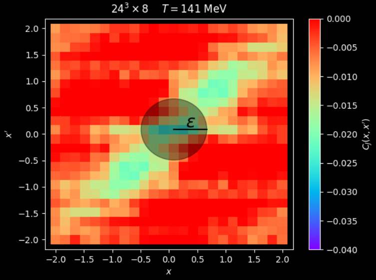

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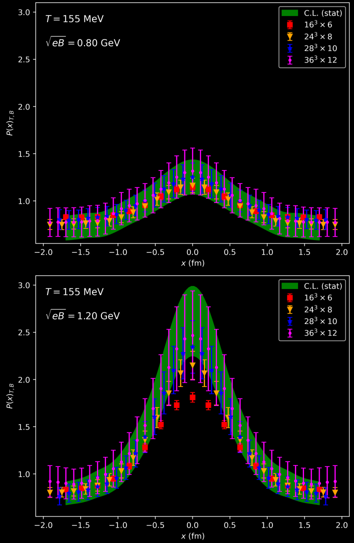


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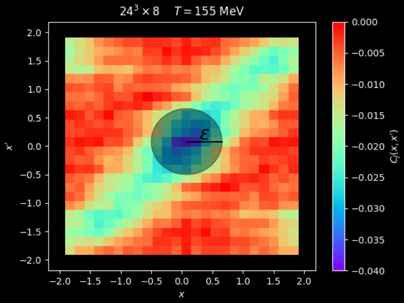

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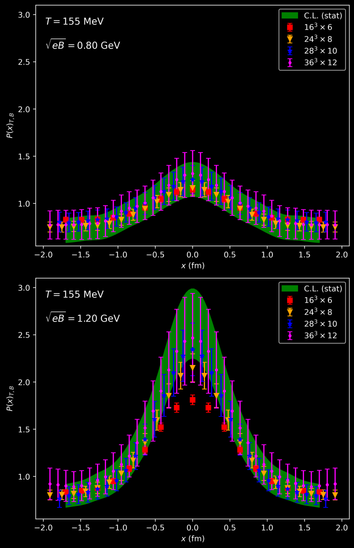


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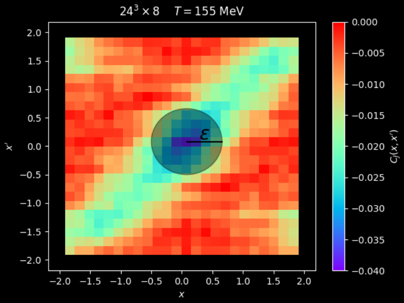


Interaction with $P$ causes the dips! (Local inverse magnetic catalysis)

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We compare $\bar{\psi}\psi(x,B)$ and $\bar{\psi}\psi(B)$.

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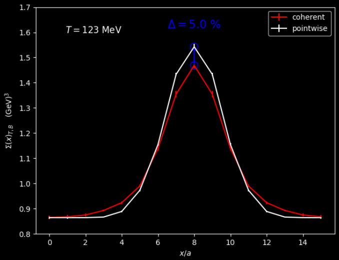

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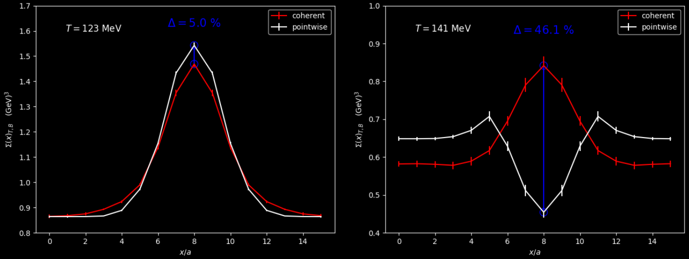

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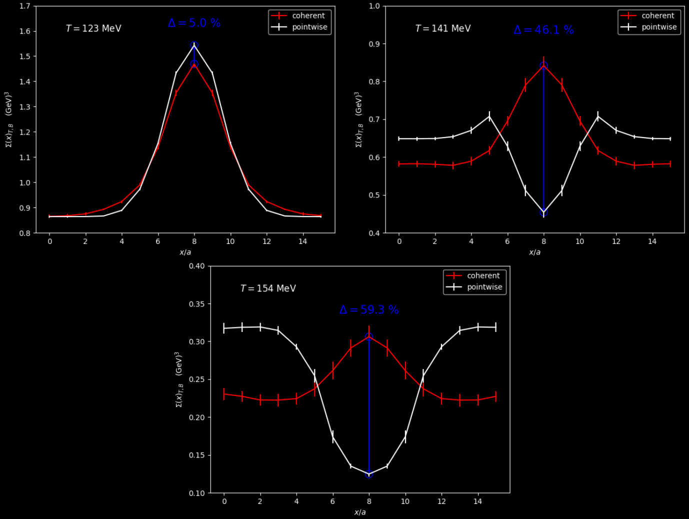

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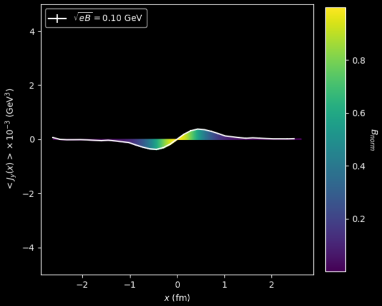

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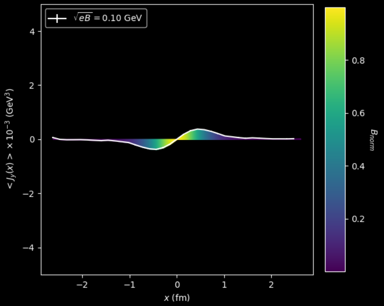


**Figure 4:** Lattice electric currents for RHIC-like ($\sqrt{eB} = 0.1$ GeV) and LHC-like ($\sqrt{eB} = 0.5$ GeV) magnetic fields, respectively.

# (BARE) MAGNETIC SUSCEPTIBILITY

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Linear response term:

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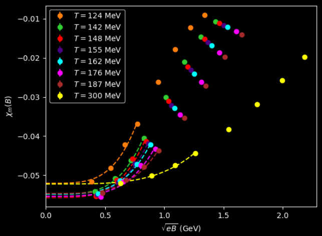

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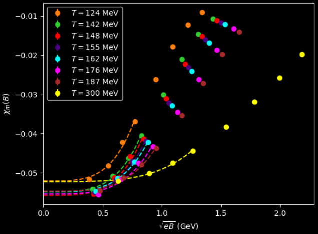


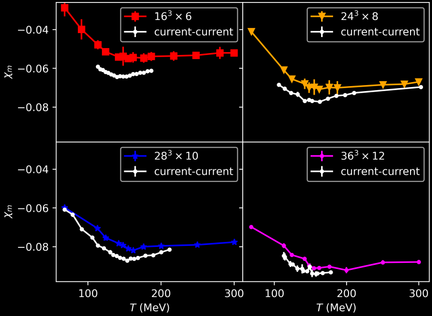

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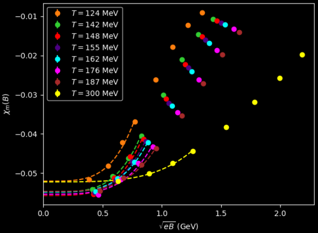


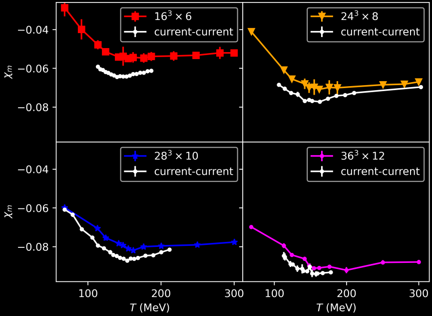


The susceptibility contains an additive divergence $\chi_m \sim \log(a)$

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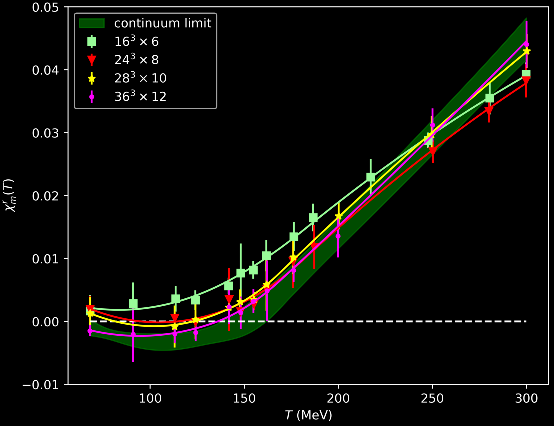

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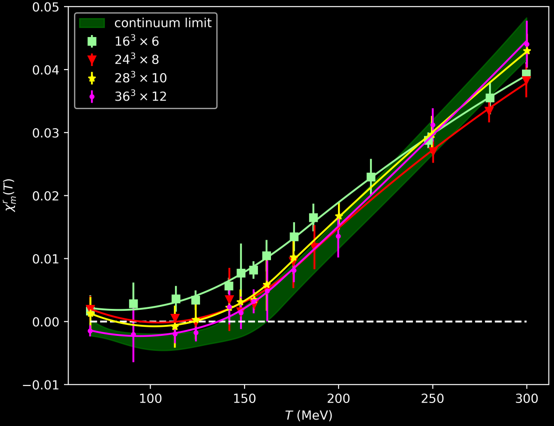


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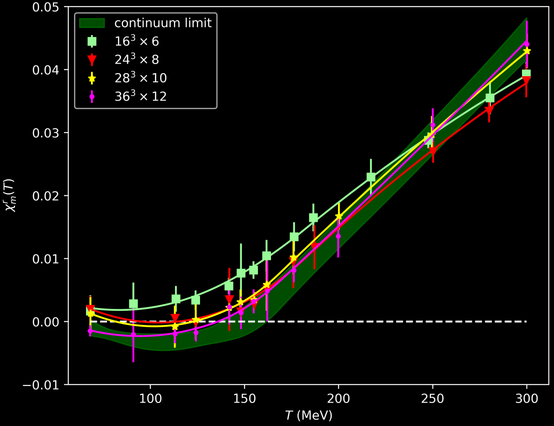


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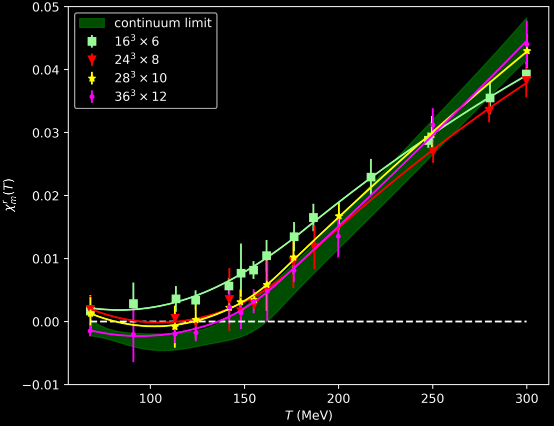


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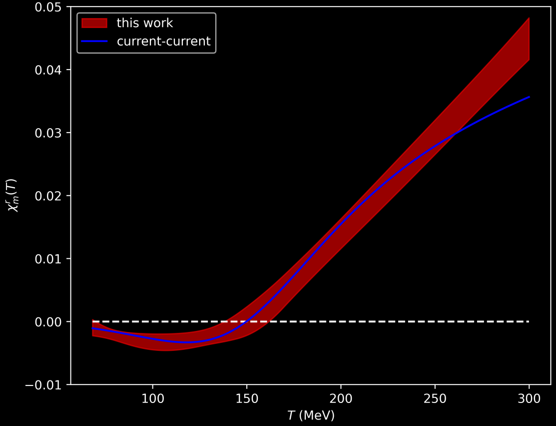


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Great agreement other predictions Bali, Endrődi, and Piemonte 2020

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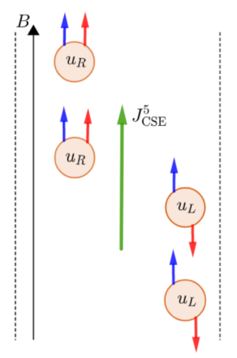
$$\left.\frac{\partial J^5}{\partial \mu}\right|_{\mu=0} = C_{\mathrm{CSE}}\, eB$$

See the talk by E. Garnacho (Tuesday at 17:40).



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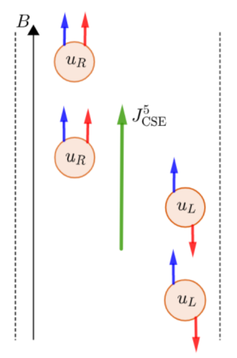


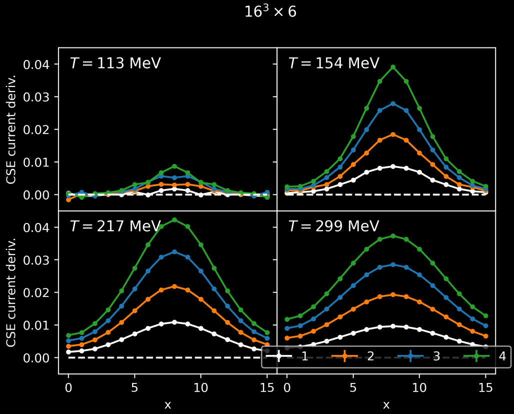

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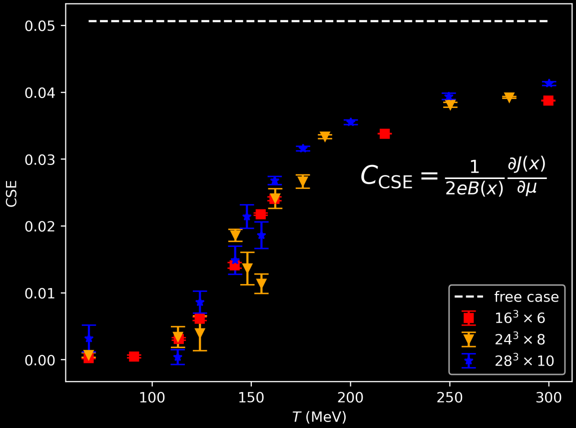

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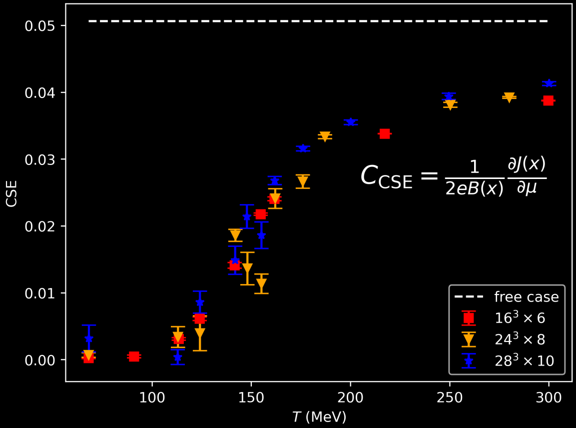


What about CME? Work in progress!

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धन्यवाद!

# Bibliography I

## References


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