Anomalous transport phenomena on the lattice

Eduardo Garnacho Velasco

egarnacho@physik.uni-bielefeld.de

B. B. Brandt, F. Cuteri, G. Endrődi, G. Markó

ICPAQGP 2023, Puri, India, 07-02-2023

- Quantum anomalies $+$ $\begin{array}{l}\text{EM fields}\\ \text{Vorticity}\end{array}$ $\rightarrow$ non-dissipative transport effects:

Anomalous transport phenomena

Introduction

- Quantum anomalies + $\begin{matrix} \text{EM fields} \\ \text{Vorticity} \end{matrix}$ $\rightarrow$ non-dissipative transport effects:

Anomalous transport phenomena

- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - ...

For a review see Kharzeev, Liao, Voloshin, Wang '16

- Quantum anomalies + $\begin{array}{l}\text{EM fields}\\ \text{Vorticity}\end{array}$ $\rightarrow$ non-dissipative transport effects:

  Anomalous transport phenomena

- Examples:
  - Chiral Magnetic Effect (CME)
  - Chiral Separation Effect (CSE)
  - Chiral Electric Separation Effect (CESE)
  - Chiral Vortical Effect (CVE)
  - ...

  For a review see Kharzeev, Liao, Voloshin, Wang '16

- Event-by-event CP-violation $\rightarrow$ non-trivial topology of QCD vacuum

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \to$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:
  - Condensed matter systems Li, Kharzeev, Zhan et al '14

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

- Experimental detection:
  - Condensed matter systems Li, Kharzeev, Zhan et al '14
  - Heavy-ion collisions STAR collaboration '21

Anomalous transport

- Macroscopic manifestations of quantum anomalies
- $U_A(1)$ anomaly origin $\sim G^a_{\mu\nu}\tilde{G}^{\mu\nu}_a \rightarrow$ CP-odd phenomena
- Event-by-event CP-violation:

$$\langle j_{\text{anom.}} \rangle = 0$$
$$\langle j_{\text{anom.}} \rangle_{\text{event}} \neq 0$$

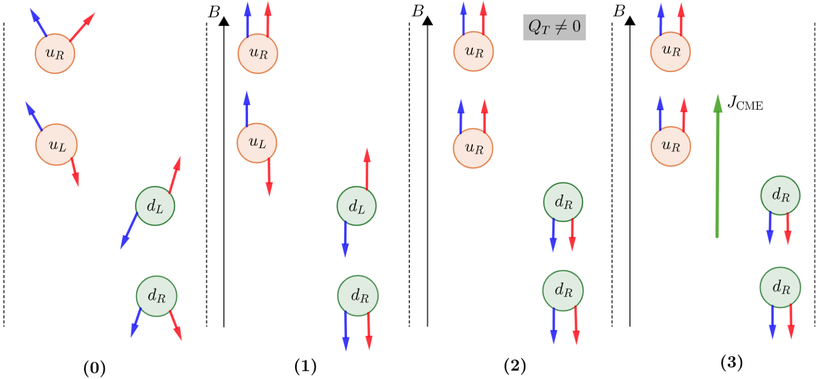
- Experimental detection:
  - Condensed matter systems Li, Kharzeev, Zhan et al '14
  - Heavy-ion collisions STAR collaboration '21

    Latest signals suggest suppression of CME:
    Can we understand this from theory?

Example 1: CME

1. Magnetic field induces polarization
2. Finite chiral density: $N_L - N_R \propto Q_{\text{top}}$
3. **Chiral Magnetic Effect** (CME):

Magnetic field + Finite chiral density → Vector current

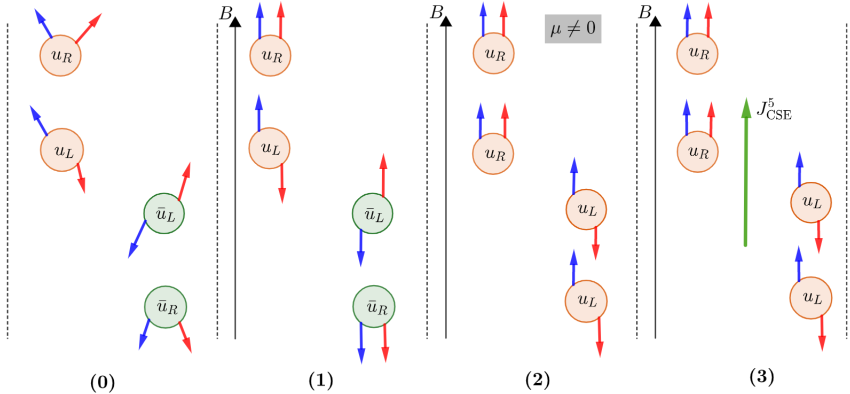


spin, momentum

Example 2: CSE

1. Magnetic field induces polarization
2. Finite density: $N_q - N_{\bar{q}} \propto \mu$
3. **Chiral Separation Effect** (CSE):

Magnetic field + Finite density → Axial current



spin, momentum

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

- Chiral chemical potential μ_5 induces imbalance between n_L and n_R:

$$\mu_5\bar{\psi}\gamma_4\gamma_5\psi$$

Conductivities

- Quark chemical potential μ induces imbalance between n_q and $n_{\bar{q}}$:

$$\mu\bar{\psi}\gamma_4\psi$$

- Chiral chemical potential μ_5 induces imbalance between n_L and n_R:

$$\mu_5\bar{\psi}\gamma_4\gamma_5\psi$$

- Currents linear in B and μ/μ_5 to first order:

$$J^V_{\mathsf{CME}} = C_{\mathsf{CME}}\, eB\mu_5 + \mathcal{O}(\mu_5^3)$$
$$J^A_{\mathsf{CSE}} = C_{\mathsf{CSE}}\, eB\mu + \mathcal{O}(\mu^3)$$

Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\text{CME}} = \frac{1}{2\pi^2}$$

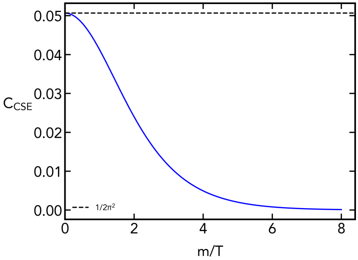
Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \frac{1}{2\pi^2}$$

- CSE Son, Zhitnitsky '04 Metlitski, Zhitnitsky '05:

$$C_{\mathsf{CSE}} = C_{\mathsf{CSE}}(m/T) \xrightarrow{m\to 0} \frac{1}{2\pi^2}$$



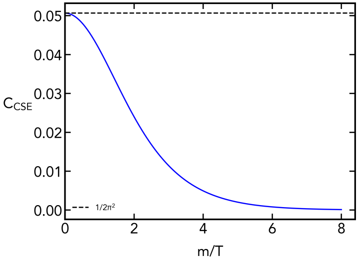
Conductivities

- Analytical predictions for **free fermions**.
- CME Fukushima, Kharzeev, Warringa '08:

$$C_{\mathsf{CME}} = \frac{1}{2\pi^2}$$

- CSE Son, Zhitnitsky '04 Metlitski, Zhitnitsky '05:

$$C_{\mathsf{CSE}} = C_{\mathsf{CSE}}(m/T) \xrightarrow{m\to 0} \frac{1}{2\pi^2}$$



- Problem to solve: Corrections in QCD?

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

- Our setup:
  - Improved staggered fermions, $2+1$ flavors, physical quark masses
  - Background homogenous B field (z direction)

CSE on the lattice

- Some previous results:
  - Overlap: Quenched QCD Puhr, Buividovich '17
    No significant corrections found
  - Wilson/Domain Wall: SU(2) Buividovich, Smith, von Smekal '21
    CSE suppressed at low T

  No results for full QCD with physical masses!

- Our setup:
  - Improved staggered fermions, $2+1$ flavors, physical quark masses
  - Background homogenous B field (z direction)

- Simulations at finite real μ suffer from sign problem

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\bigg|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big] = C_{\mathsf{cse}} eB_z$$

CSE on the lattice

- Measure derivatives of the current:

$$\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\Big|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big] = C_{\mathsf{cse}}\, eB_z$$

- Simulations at $\mu = 0 \rightarrow$ no sign problem

CSE on the lattice

- Measure derivatives of the current:

$$\left.\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\right|_{\mu=0} = \frac{T}{V}\big[\langle \mathsf{Tr}(\gamma_4 M^{-1})\mathsf{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathsf{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\big] = C_{\mathsf{cse}}\, eB_z$$

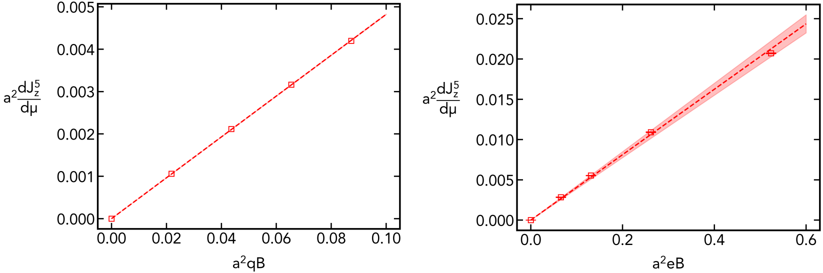
- Simulations at $\mu = 0 \rightarrow$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain C_{cse}:

CSE on the lattice

- Measure derivatives of the current:

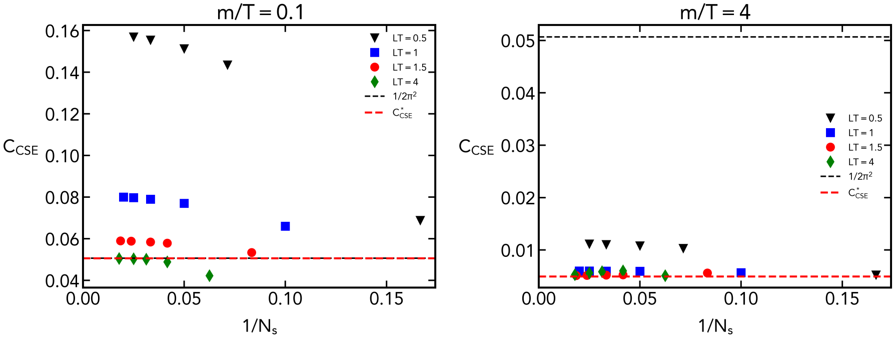
$$\left.\frac{\mathrm{d}\langle J_z^A\rangle}{\mathrm{d}\mu}\right|_{\mu=0} = \frac{T}{V}\left[\langle \mathrm{Tr}(\gamma_4 M^{-1})\mathrm{Tr}(\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0} - \langle \mathrm{Tr}(\gamma_4 M^{-1}\gamma_3\gamma_5 M^{-1})\rangle_{\mu=0}\right] = C_{\mathsf{cse}} eB_z$$

- Simulations at $\mu = 0 \to$ no sign problem
- Numerical derivative (linear fit) w.r.t. B to obtain C_{cse}:



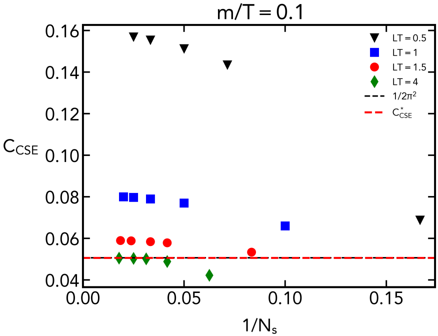
Results for CSE: free case

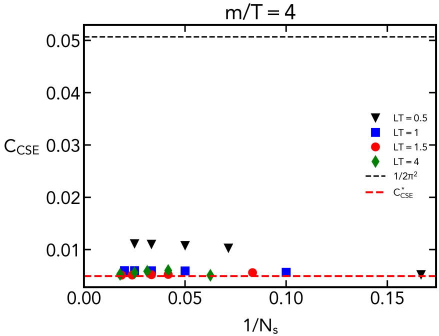
- Consistency check in the free case $(A_\mu = 0 \iff U_\mu = 1)$



Results for CSE: free case

- Consistency check in the free case ($A_\mu = 0 \iff U_\mu = 1$)

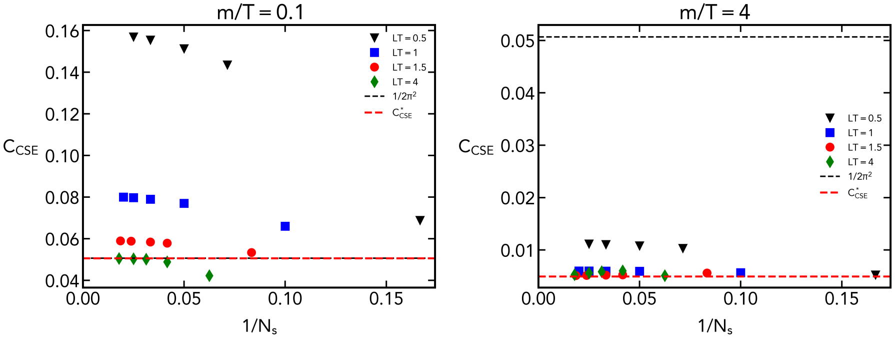




- Finite size effects at $LT \to 0$ sizeable if m/T not large enough

Results for CSE: free case

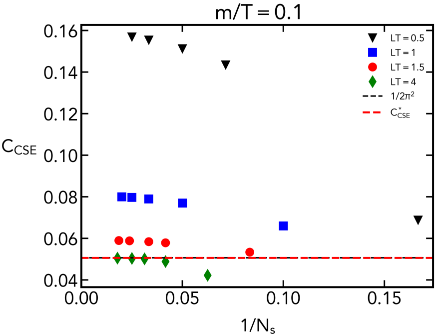
- Consistency check in the free case ($A_\mu = 0 \iff U_\mu = 1$)

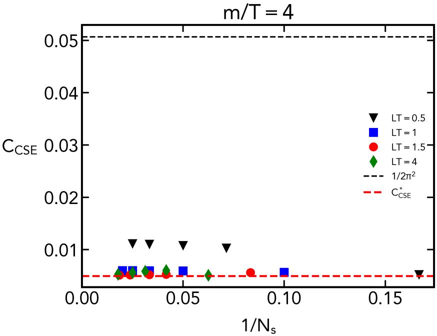


- Finite size effects at $LT \to 0$ sizeable if m/T not large enough
- Importance of continuum limit

Results for CSE: free case

- Consistency check in the free case $(A_\mu = 0 \iff U_\mu = 1)$

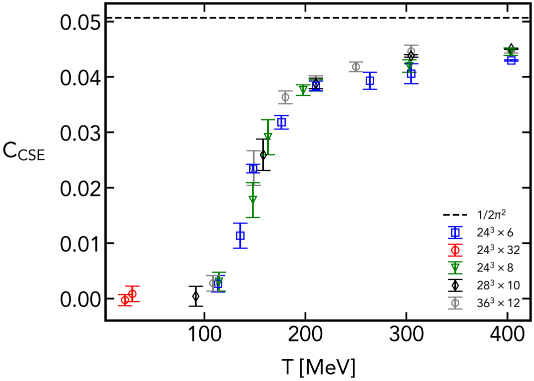




- Finite size effects at $LT \to 0$ sizeable if m/T not large enough
- Importance of continuum limit
- C_{cse} approaches analytical prediction when $LT \to \infty$

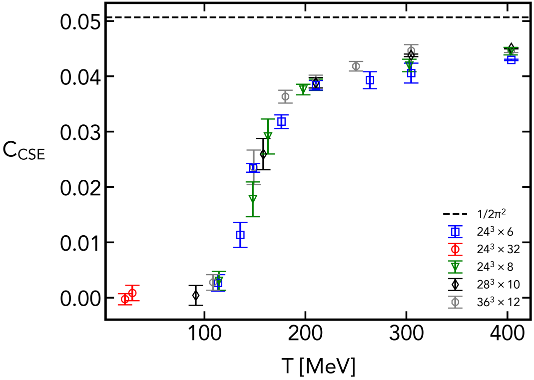
Results for CSE: interacting case

- 2+1 flavors, physical masses



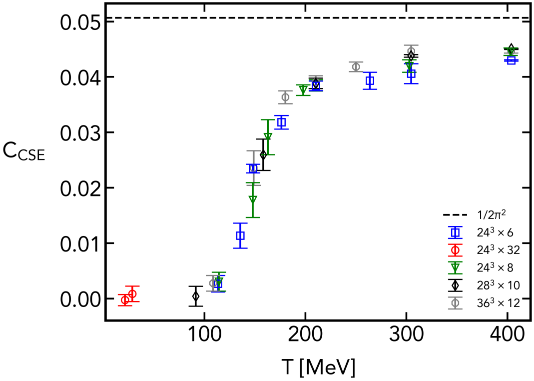
Results for CSE: interacting case

- 2+1 flavors, physical masses



- High T $(T > T_c)$: approaches free case value

- 2+1 flavors, physical masses



- High T $(T > T_c)$: approaches free case value
- Low T $(T < T_c)$: CSE suppressed Buividovich, Smith, von Smekal '21

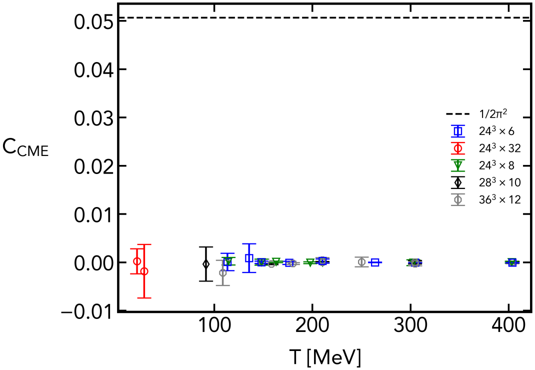
  Chiral effective theories Avdoshkin, Sadofyev, Zakharov '18

And CME?

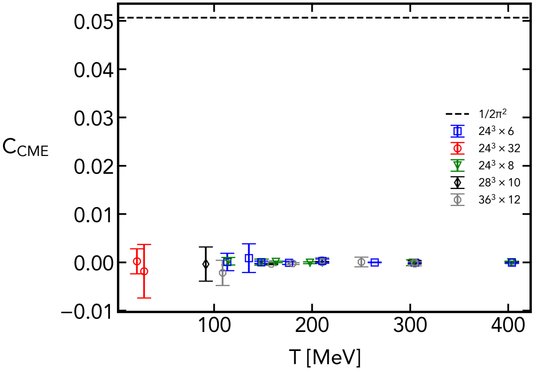
- What about CME?
- Wilson: Quenched and phys. masses Yamamoto '11
  $C_{\text{cme}} = 0.02 - 0.03$ at high T

And CME?

- What about CME?
- Wilson: Quenched and phys. masses Yamamoto '11
  $C_{\text{cme}} = 0.02 - 0.03$ at high T
- We observe $\mathrm{d}J_z^V/\mathrm{d}\mu_5 = 0$



- What about CME?
- Wilson: Quenched and phys. masses Yamamoto '11
  $C_{\text{cme}} = 0.02 - 0.03$ at high T
- We observe $\mathrm{d}J_z^V/\mathrm{d}\mu_5 = 0$



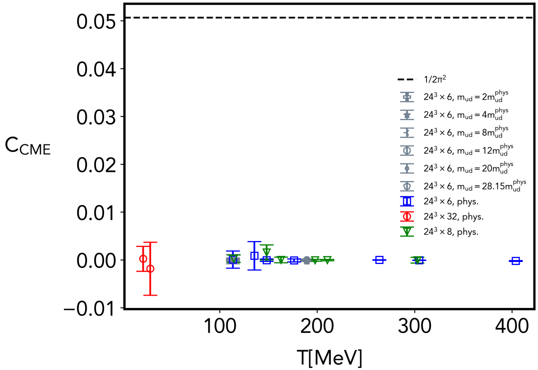
- Response vanishes for a static, homogeneous μ_5 and homogeneous B

CME mass dependence?

- What is the effect of changing the quark mass?

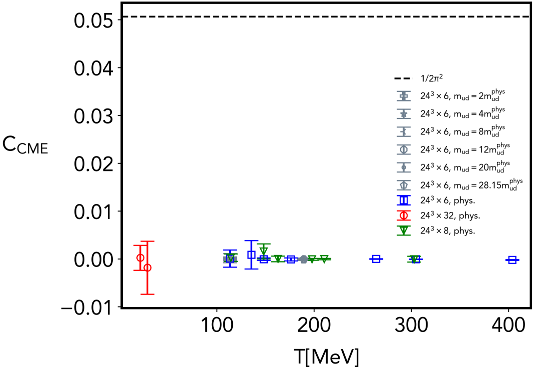
CME mass dependence?

- What is the effect of changing the quark mass?



CME mass dependence?

- What is the effect of changing the quark mass?



- Insensitive to higher than physical pion masses

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16
  - Problems with some lattice discretizations for CME?

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16
  - Problems with some lattice discretizations for CME?
  - Impact of inhomogenous/non-static μ_5 or B?

CME

- Our result for CME is 0 for free fermions and in QCD for physical and higher than physical pion masses
- What is going on?
  - CME is zero in equilibrium? Buividovich '14 Landsteiner '16
  - Problems with some lattice discretizations for CME?
  - Impact of inhomogenous/non-static μ_5 or B?
  - Something else?

Summary & Outlook

Summary

- Study of CSE with staggered fermions, 2+1 flavors
  First result for physical masses and towards the continuum limit
- Free case consistent with analytical prediction
- Full QCD: suppression at low T, approach free case value at high T
- For more details Brandt, Cuteri, Endrődi, Garnacho Velasco, Markó '22
- Zero CME

Summary & Outlook

Summary

- Study of CSE with staggered fermions, 2+1 flavors
  First result for physical masses and towards the continuum limit
- Free case consistent with analytical prediction
- Full QCD: suppression at low T, approach free case value at high T
- For more details 🔗 Brandt, Cuteri, Endrődi, Garnacho Velasco, Markó '22
- Zero CME

Outlook

- Perform continuum limit for C_{CSE}
- Inhomogenous μ_5?
- Inhomogeneous magnetic fields? $\rightarrow$ 🔗 Dean Valois' talk 10/02 12:30
- Clarify CME

Backup slides

- Transport effects:

$$\begin{pmatrix} \vec{J} \\ \vec{J}_5 \end{pmatrix} = \begin{pmatrix} \sigma_{\text{Ohm}} & \sigma_{\text{CME}} \\ \sigma_{\text{CESE}} & \sigma_{\text{CSE}} \end{pmatrix} \begin{pmatrix} \vec{E} \\ \vec{B} \end{pmatrix}$$

- Chiral Vortical Effect: vector/axial current generated by rotation $+ \mu + \mu_5$:

$$\vec{J} = \frac{1}{\pi^2}\mu_5\mu\vec{\omega}$$
$$\vec{J}_5 = \left[\frac{1}{6}T^2 + \frac{1}{2\pi^2}(\mu_5^2 + \mu^2)\right]\vec{\omega}$$

Free case continuum limit

- Continuum limit: $a \to 0$, $V = L^3 = a^3 N_s^3 =$ fixed (L fixed)
- Also keep $T = 1/(N_t a)$, $m = \tilde{m}/a$ and B fixed
- Then $LT =$ fixed so
$$\frac{N_s}{N_t} = \text{fixed}$$
- Also $mL =$ fixed, $m/T =$ fixed so
$$\tilde{m} N_s = \text{fixed}, \quad \tilde{m} N_t = \text{fixed}$$
- And
$$B = \frac{2\pi N_b}{L_y L_x} \Rightarrow N_b = \text{fixed}$$

Staggered gammas

- Staggered gammas $\sim$ Taste singlets $(\gamma_\mu \otimes \mathbb{1})$, $(\gamma_5 \otimes \mathbb{1})$:

$$\Gamma_\mu(x,y) = \frac{1}{2}\eta_\mu(x)[U(x)u(x)e^{a\mu_q}\delta_{x+\hat{\mu},y} + U^\dagger(y)u^\dagger(y)e^{-a\mu_q}\delta_{x-\hat{\mu},y}]$$

$$\Gamma_5(x,y) = \frac{1}{4!}\sum_{\text{perm}} \epsilon_{\text{perm}}\Gamma_1\Gamma_2\Gamma_3\Gamma_4$$

$$(\Gamma_5\Gamma_4)(x,y) = \frac{1}{3!}\sum_{\text{perm}} \epsilon_{\text{perm}}\Gamma_1\Gamma_2\Gamma_3$$

with

$$\eta_\mu(x) = (-1)^{\sum_{\nu<\mu} x_\nu}, \quad \eta_1(x) = 1$$

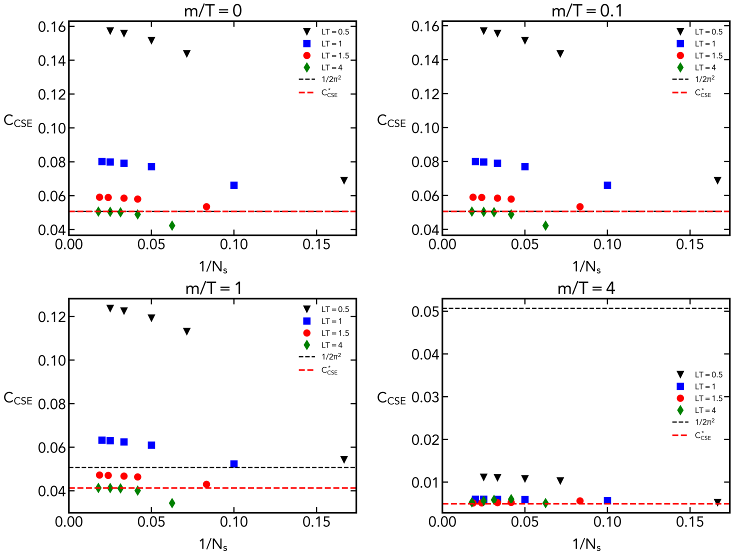
and $U \in \mathrm{SU}(3), u \in \mathrm{U}(1)$

Staggered observable

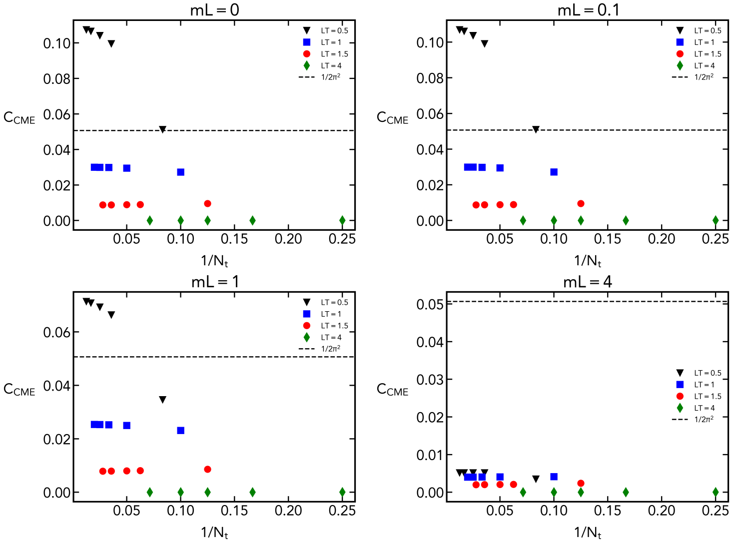
- Staggered observable has an extra term

$$\left.\frac{\mathrm{d}\left\langle J_z^5\right\rangle}{\mathrm{d}\mu}\right|_{\mu=0} = \frac{T}{V}\left[\frac{1}{4}\left\langle \mathsf{Tr}\left(\Gamma_4 M^{-1}\right)\mathsf{Tr}\left(\Gamma_3\Gamma_5 M^{-1}\right)\right\rangle_{\mu=0} - \frac{1}{16}\left\langle \mathsf{Tr}\left(\Gamma_4 M^{-1}\Gamma_3\Gamma_5 M^{-1}\right)\right\rangle_{\mu=0} + \frac{1}{4}\left\langle \mathsf{Tr}\left(\frac{\partial(\Gamma_3\Gamma_5)}{\partial\mu} M^{-1}\right)\right\rangle_{\mu=0}\right]$$

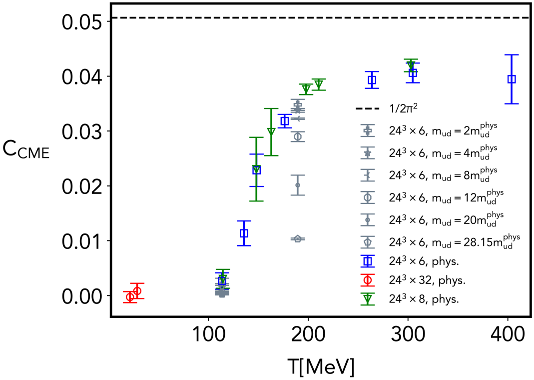
CSE free case



CME free case



CSE masses



0.05
0.04
0.03
0.02
0.01
0.00
C_{CME}
100
200
300
400
T[MeV]
$1/2\pi^2$
$24^3 \times 6$, $m_{ud} = 2m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 4m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 8m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 12m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 20m_{ud}^{phys}$
$24^3 \times 6$, $m_{ud} = 28.15m_{ud}^{phys}$
$24^3 \times 6$, phys.
$24^3 \times 32$, phys.
$24^3 \times 8$, phys.