Axions, topology and electromagnetic fields on Lattice QCD

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Outline

- Topology in QCD with EM fields
- Axion mechanism and observables
- Computation of $g_{a\gamma\gamma}$ and χ_{top} on the lattice
- Preliminary results: $g_{a\gamma\gamma}$ and χ_{top}
- Conclusions and further work

Topology in QCD with EM fields

Effect of EM fields on QCD topology

- Definition of Q_{top}:

$$Q_{\text{top}} = \int d^4x \, q_{\text{top}}(x), \quad q_{\text{top}} = \frac{1}{32\pi^2} \epsilon^{\mu\nu\rho\sigma} \mathrm{Tr} \, G_{\mu\nu} \, G_{\rho\sigma} \, .$$

- Adding electric or magnetic fields *separately*: no changes in topology.

$$\langle Q_{\text{top}} \rangle = 0.$$

- If $F_{\mu\nu} \neq 0$ such that $\vec{E} \cdot \vec{B} \neq 0$ it can be interpreted as an effective θ-therm D'Elia et al., 2012.
- Hence non-orthogonal EM fields $\Longleftrightarrow$ non-trivial topology.

$$\langle Q_{\text{top}} \rangle \neq 0.$$

- In particular, for $\vec{E} \cdot \vec{B} > 0 \Longrightarrow Q_{\text{top}} < 0$.

Axion mechanism and observables

Strong CP problem

- In principle, the QCD Lagrangian could include an extra term:

$$\mathcal{L}_{\mathrm{QCD}+\theta} = \mathcal{L}_{\mathrm{QCD}} + \theta\, q_{\mathsf{top}}.$$

  - This term is CP odd.
  - Induces an electric dipole moment in n: $|\theta| < 10^{-10}$ Abel et al 2020.
- Why don't we see it 😞? $\longrightarrow$ Axions 🤔? Peccei, Quinn '77

$$\mathcal{L}_a = \frac{1}{2}\partial_\mu\, a\partial^\mu a + \frac{a}{f_a} q_{\mathsf{top}} + \mathcal{L}_{\mathrm{int}}.$$

- Essence of the idea: new pseudoscalar a whose minimum is $\langle a\rangle = -\theta f_a$.

Topological susceptibility χ_{top}

- Is the second moment of Q_{top}: $\chi_{\text{top}} = \frac{\langle Q_{\text{top}}^2 \rangle}{V_4}$
- It is also the mass of the axion:

$$f_a^2 \frac{\delta^2}{\delta a^2} \log \mathcal{Z}(a) \Big|_{a=0} = \frac{\partial^2}{\partial \theta^2} \log \mathcal{Z}(\theta) \Big|_{\theta=0} \longleftrightarrow m_a^2 f_a^2 = \chi_{\text{top}}.$$

- Hence, an analysis of χ_{top} gives information on m_a.
- Current estimate from ChPT at zero T: $\chi_{\text{top}}(LO) = \left[75.5(5)\text{MeV}\right]^4$ Cortona et al 2016.
  - Lattice calculations give almost the same central value but with a bigger error, $\chi_{\text{top}} = \left[75.6(1.8)(0.9)\text{MeV}\right]^4$ Borsanyi et al 2016.
  - ChPT also predicts a mild enhancement with B at low T Adhikari 2022.
- It also gives us cosmological information about a.

The photon-axion coupling $g_{a\gamma\gamma}^{QCD}$

- The axion couples directly and indirectly to photons.
- ChPT calculations show that the coupling decomposes into two terms, one model *dependent* and one model *independent*.
- Current estimate from ChPT.: $g_{a\gamma\gamma} = g_{a\gamma\gamma}^{0} + g_{a\gamma\gamma}^{QCD} = \frac{\alpha_{\text{em}}}{2\pi f_a}\left(\frac{E}{N} - 1.92(4)\right)$ Cortona et al 2016.
- We want to compute the QCD dependent part of the coupling $\longrightarrow$ no need to include axions on the lattice!

Q_{top} and $g_{a\gamma\gamma}^{QCD}$

- If we include both electric and magnetic background fields, the only CP odd operators in our theory are:

$$\mathrm{Tr}\, G_{\mu\nu}\widetilde{G}^{\mu\nu} \;\;\&\;\; \mathbf{E}\cdot\mathbf{B}.$$

So by symmetry arguments, Q_{top} can only be (for weak fields):

$$Q_{\text{top}} \propto \mathbf{E}\cdot\mathbf{B} + \mathcal{O}\left([\mathbf{E}\cdot\mathbf{B}]^3\right).$$

- By looking at $\mathcal{Z}$:

$$\left.\frac{\delta \log \mathcal{Z}(a)}{\delta a}\right|_{a=0} = \frac{\langle Q_{\text{top}}\rangle_{E,B}}{f_a} \longrightarrow g_{a\gamma\gamma}^{QCD} f_a = \frac{T}{V}\frac{\partial}{\partial(\mathbf{E}\cdot\mathbf{B})}\langle Q_{\text{top}}\rangle_{E,B}\Big|_{\mathbf{E},\mathbf{B}=0}$$

- So for homogeneous, static and weak EM fields

$$\frac{T}{V}\langle Q_{\text{top}}\rangle_{E,B} \approx \frac{g_{a\gamma\gamma}^{QCD}\cdot f_a}{e^2} e^2\mathbf{E}\cdot\mathbf{B} \;\; \textit{and} \;\; g_{a\gamma\gamma}^{QCD} < 0.$$

Computation of χ_{top} and $g_{a\gamma\gamma}^{QCD}$ on the lattice

Lattice artifacts of χ_{top}

- Index theorem says $\not{D}$ has zero modes when $Q_{\text{top}} \neq 0$.
- Staggered operator lacks these zero modes $\longrightarrow$ huge lattice artifacts, specially for high temperatures (we are talking of several orders of magnitude!).
- One possible solution: substitute the smallest eigenvalues of D_{stagg} with their continuum values Borsanyi et al 2016.
- Reweight each configuration by:

$$\prod_f \prod_{i=1}^{2|Q_{\text{top}}|} \prod_{\sigma=\pm} \left(\frac{2m_f}{i\sigma\lambda_i + 2m_f} \right)^{n_f/4} .$$

- We have explored two ways for computing $g_{a\gamma\gamma}^{QCD}$:
  - Correlator method: computing $\frac{T}{V}\frac{\partial^2 \log Z}{\partial E \partial a}\Big|_{a,E=0}$ and fitting to $\mathbf{B}$.

$$\mathrm{Im}\left[\int d^4x \langle q_{\mathrm{top}}(0) j_4(x) x_3 \rangle\right] = \frac{g_{a\gamma\gamma} f_a}{e} eB.$$

  - Electric method: measuring $\langle Q_{\mathrm{top}} \rangle_{E,B}$ and fitting to $\mathbf{E} \cdot \mathbf{B}$

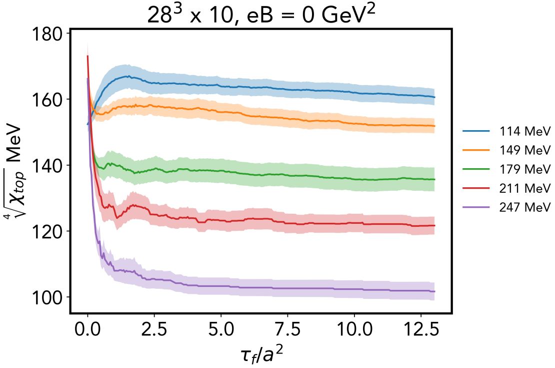
$$\frac{T}{V} \langle Q_{\mathrm{top}} \rangle_{E,B} = \frac{g_{a\gamma\gamma}^{QCD} f_a}{e^2} e^2 \mathbf{E} \cdot \mathbf{B}.$$

To take into account...

- Simulations: improved staggered quarks with 2+1 flavours and physical masses.
- We also have to deal with two issues:
  1. The sign problem. We can't simulate real electric fields, so $\mathbf{E} \longrightarrow i\mathbf{E}$,
  2. UV fluctuations of the gluon fields $\longrightarrow$ *gradient flow* Lüscher 2010.

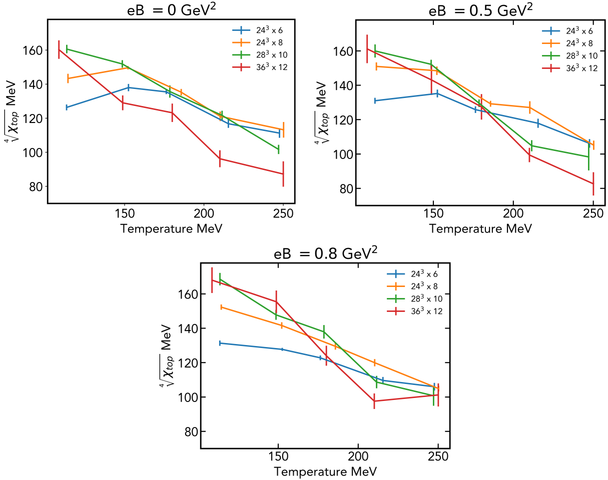
Preliminary results

Wilson evolution of χ_{top}. Note the plateaus.



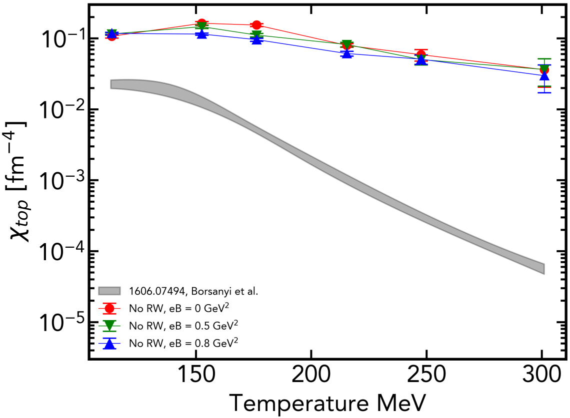
$\chi_{\text{top}}(B)$ vs T: preliminary results

$\chi_{\text{top}}(B)$ as a function of T. Note that $\sqrt{\chi_{\text{top}}} = m_a f_a$.



$\chi_{\text{top}}(B)$ vs T: preliminary results

Comparison with continuum result. Effect of reweighting.



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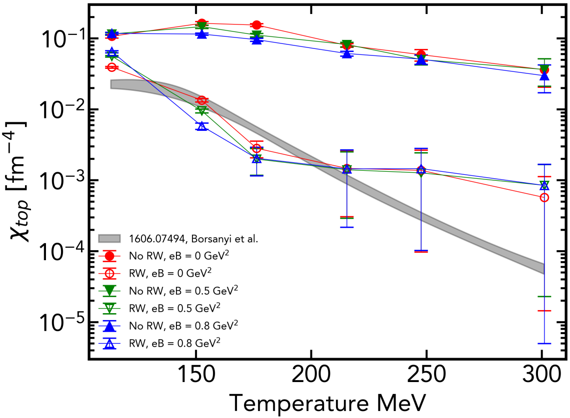
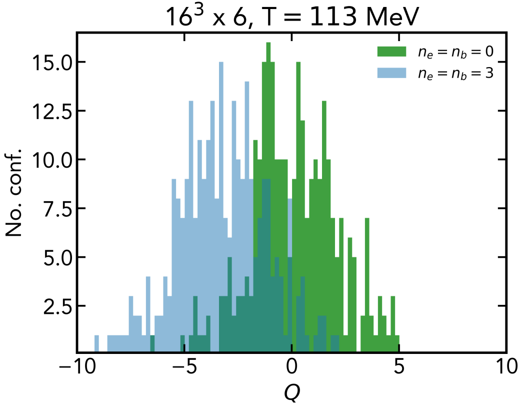
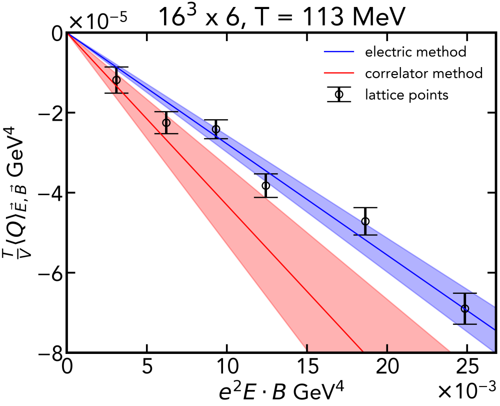


Shift of Q_{top} at non-zero $\mathbf{E}\cdot\mathbf{B}$. Effect also shown in D'Elia et al 2016.



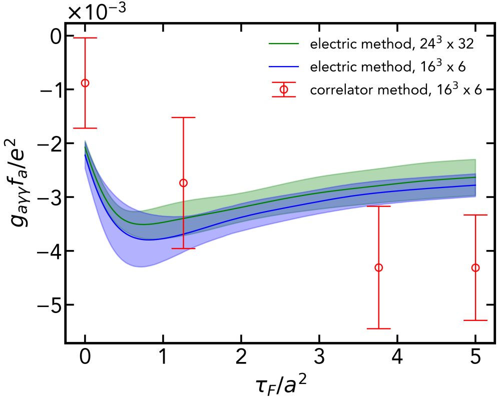
$\langle Q_{\text{top}} \rangle_{E,B}$ vs $\mathbf{E} \cdot \mathbf{B}$: preliminary results

$\langle Q_{\text{top}} \rangle_{E,B}$ as a function of $\mathbf{E} \cdot \mathbf{B}$.



$g_{a\gamma\gamma}^{QCD}$ flow: preliminary results

$g_{a\gamma\gamma}^{QCD}$ as a function of flow time for the two different methods.
$g_{a\gamma\gamma}^{QCD} f_a/e^2 = -0.0243(5)$ from ChPT.



Conclusions and further work

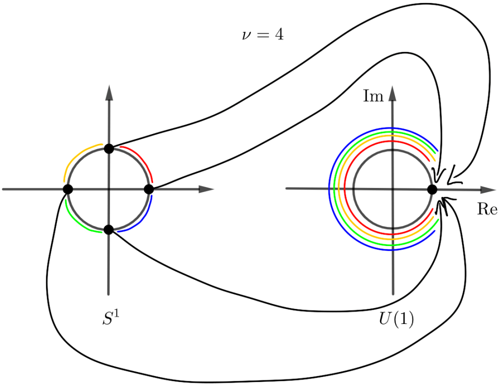
Conclusions and further work

- We have shown:
  - why there is an interplay between EM fields and $SU(3)$ topology.
  - that there is a linear response of $\langle Q_{\text{top}} \rangle$ with $\mathbf{E} \cdot \mathbf{B}$ for weak fields.
  - the effect of the would-be zero modes of D_{stagg} on the lattice artifacts.
  - preliminary results for χ_{top} as a function of $\mathbf{B}$, T as well as for $g_{a\gamma\gamma}^{QCD}$.
- Further work:
  - further understand the would-be zero modes at low temperatures.
  - generate more statistics and perform the continuum limit for both observables.
  - use experimental bounds and lattice results to constrain axion models.

Thank you for your attention!

Backup slides

Winding numbers in $U(1)$



Adding EM fields (II)

- EM fields can induce topologies in the gluon sector. But how? $\longrightarrow$ Index theorem.
- The index theorem says (for QCD) Atiyah, Singer '71:

$$\mathrm{Index}(\not{D}) \equiv n_- - n_+ = Q_{top}$$

  Since in QCD $\langle Q_{top} \rangle = 0$, we don't see imbalances in chirality.
- But after including electromagnetic fields the situation is different:

$$\mathrm{Index}(\not{D}) \equiv n_- - n_+ = Q_{top} + Q_{U(1)}.$$

  We have two different topological contributions to the zero modes.
- Path integral favours as little zero modes as possible: $\det M \uparrow\uparrow$.
- Hence, it selects gluon field configurations such that:

$$Q_{U(1)} \uparrow \Longleftrightarrow \; Q_{top} \downarrow .$$