Quarkonium dynamics in the non-static limit

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based on: arXiv 2302.00508
and ongoing work

in collaboration with:
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Outline

- Quarkonium as multi-scale system
- Scale hierarchies in a thermal medium
- Formalism
- Results
- Implications in quantum dynamics
- Conclusions

Scale separation

- Quarkonia are bound states of heavy quark (Q) and anti-quark ($\bar{Q}$)

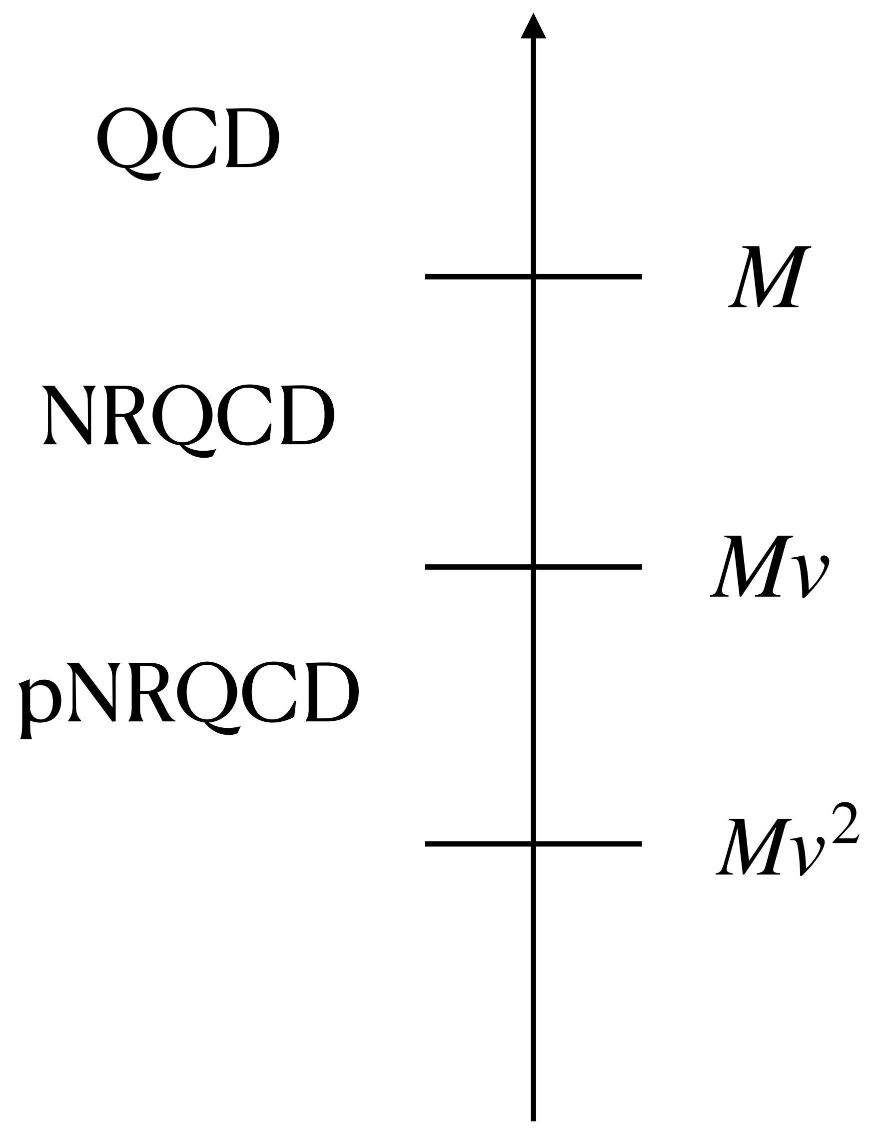
- For Coulombic bound state

$$M \sim 5 \text{ GeV}, \quad 1/r \sim 1.5 \text{ GeV and } E_b \sim 0.5 \text{ GeV}$$

- Characterised by the energy scales $\quad M \gg 1/r \gg E_b$

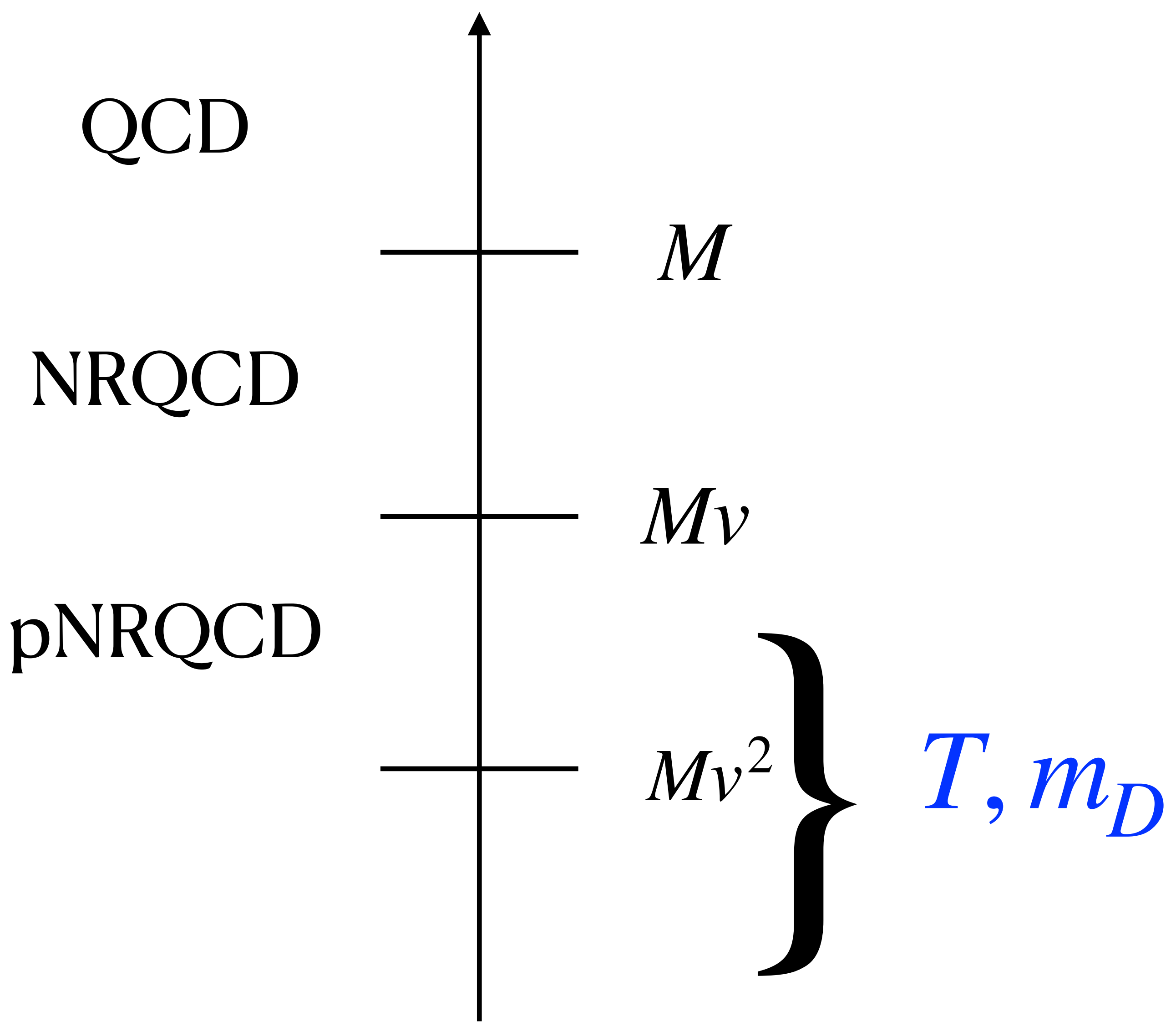
- $M \gg 1/r$ ensures that the bound states are non-relativistic

- $1/r \gg E_b$ means that at leading order in M, the $Q\bar{Q}$ interaction is a potential

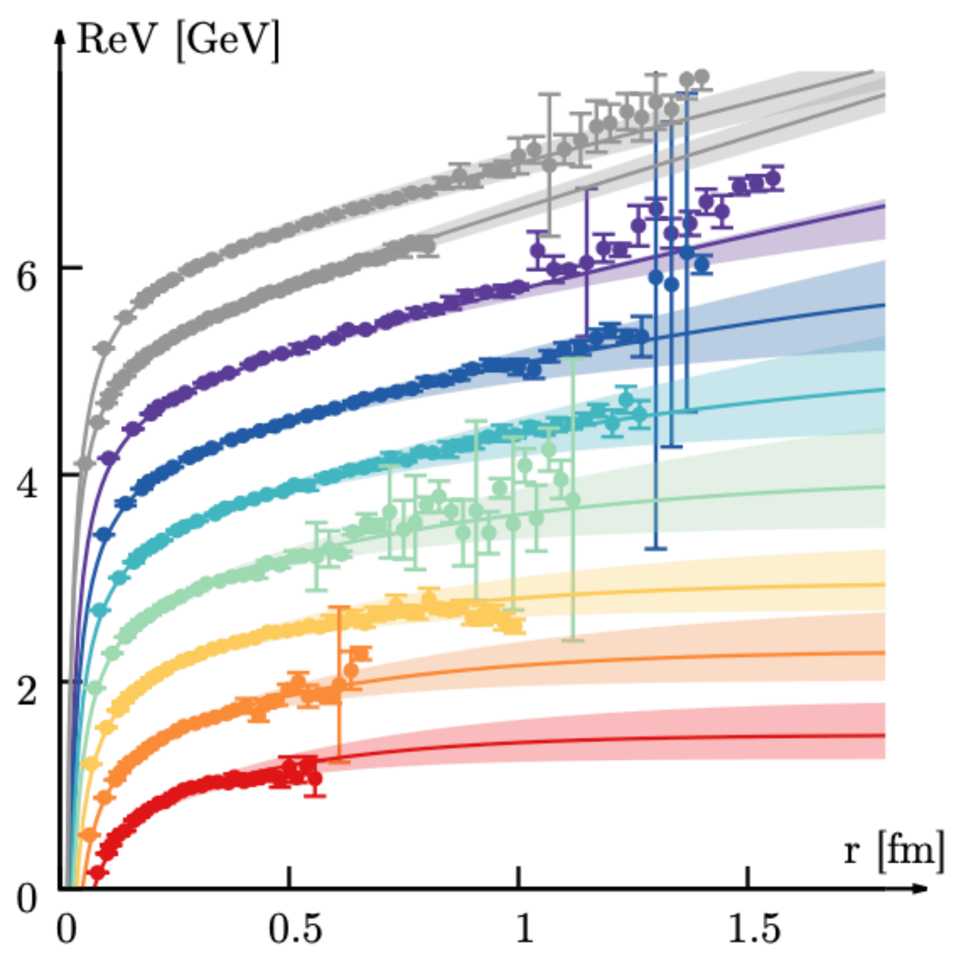


$Q\bar{Q}$ pair in a thermal medium

- Medium introduces extra scales: m_D and T
- Quarkonia dynamics is governed by the hierarchy between thermal scales and E_b
- In QGP these should be compared with $T \sim (0.2 - 0.5)$ GeV
- Depending on whether $E_b \gg T$ or $E_b \ll T$ different processes dominate the dynamics of quarkonia in the QGP



Static screening

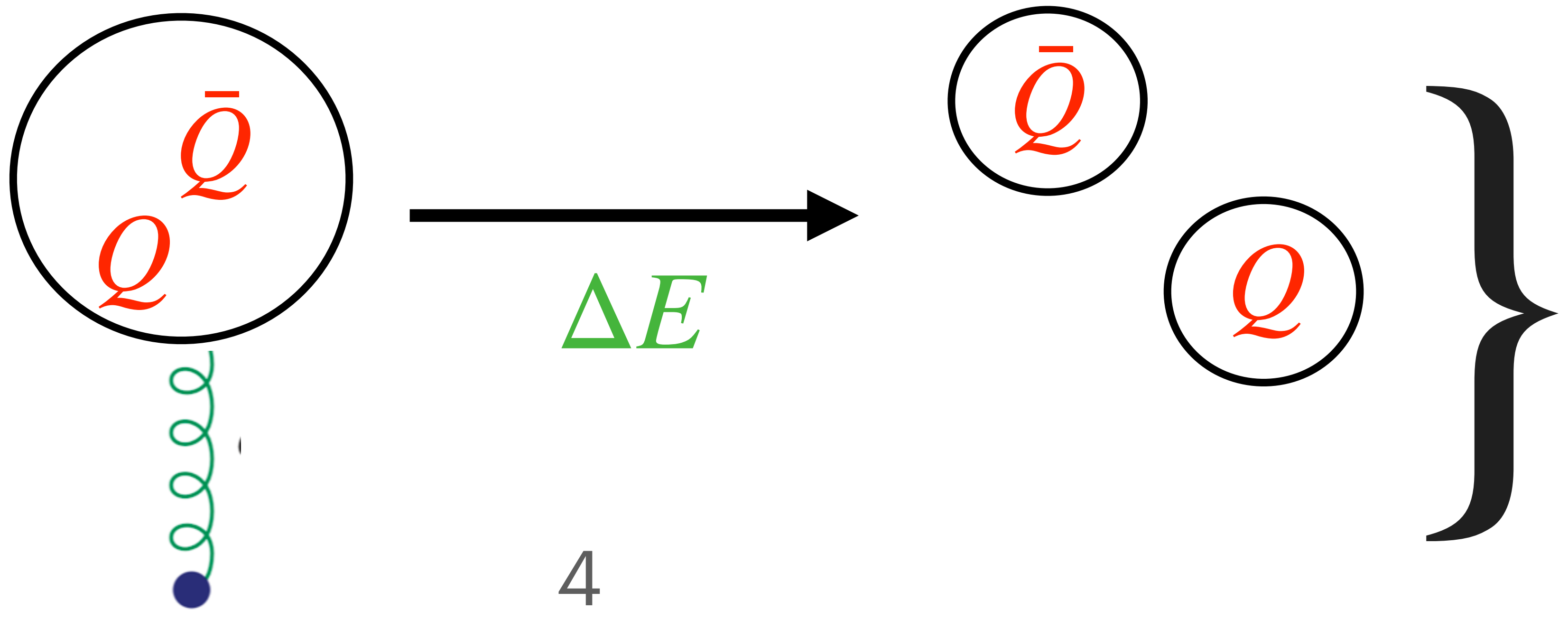


D. Lafferty, A. Rothkopf (2020)

Dynamical processes

Scattering/absorption with medium constituents

$\longrightarrow$ Dissociation of bound states



Landau damping and gluo-dissociation

N. Brambilla et.al (2011)

Potential non-relativistic QCD (pNRQCD)

- pNRQCD is obtained from full QCD via NRQCD by first integrating out hard modes (M) and soft mode (Mv) where $v \ll 1$

- Degrees of freedom are singlet (S) and octet (O) states and gluons (A^μ)

- Small radius (r) and large M allows a double expansion in r and $1/M$ at Lagrangian level

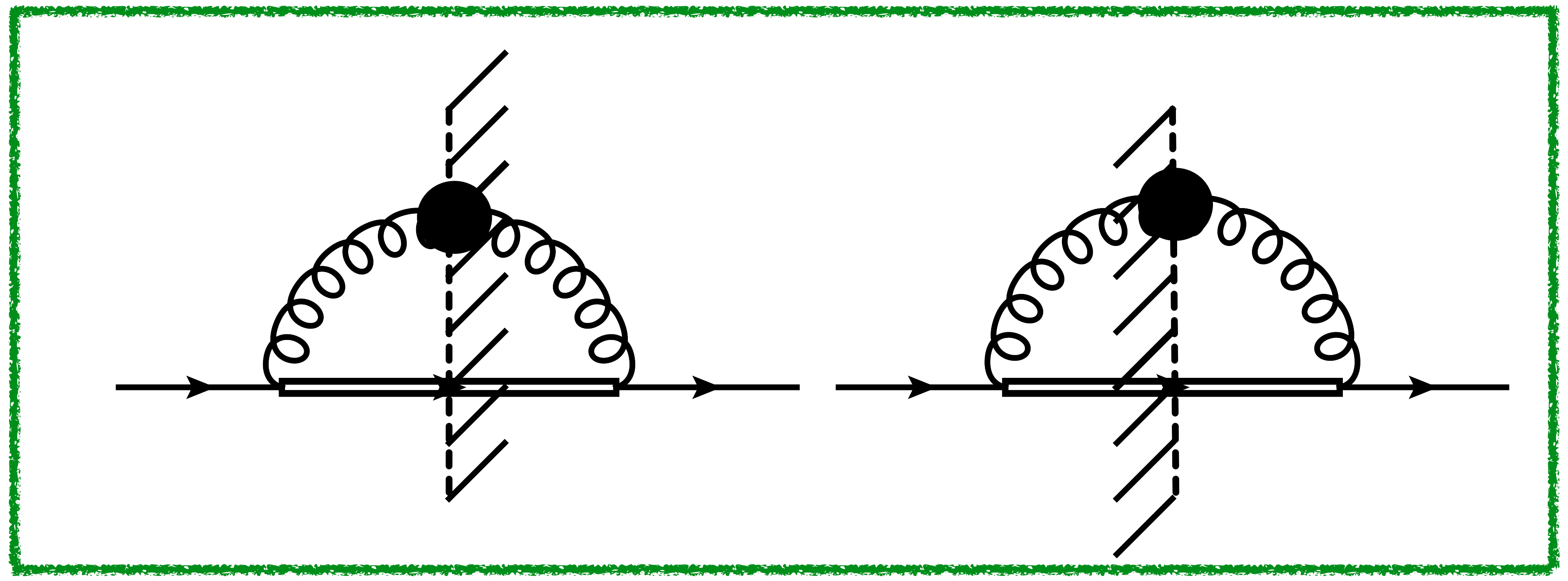
- The Lagrangian in terms of singlet and octet states is (pNRQCD)

$$\mathscr{L} = S^\dagger\left(i\partial_t + \frac{\nabla^2}{M} - V_s(r)\right)S + O^\dagger\left(i\partial_t + \frac{\nabla^2}{M} - V_o(r)\right)O + gV_A(r)[S^\dagger r \cdot EO + O^\dagger r \cdot ES] + gV_B(r)[O^\dagger r \cdot EO + O^\dagger O r \cdot E] + \cdots$$

where r is separation between Q and $\bar{Q}$, E is chromo-electric field

Formalism

- At leading order in r singlet and octet fields interact via chromo-electric field. E



- At any given T, decay width is

$$\Gamma = 2\,\langle \phi \,|\, \Im\Sigma_{11} \,|\, \phi \rangle$$

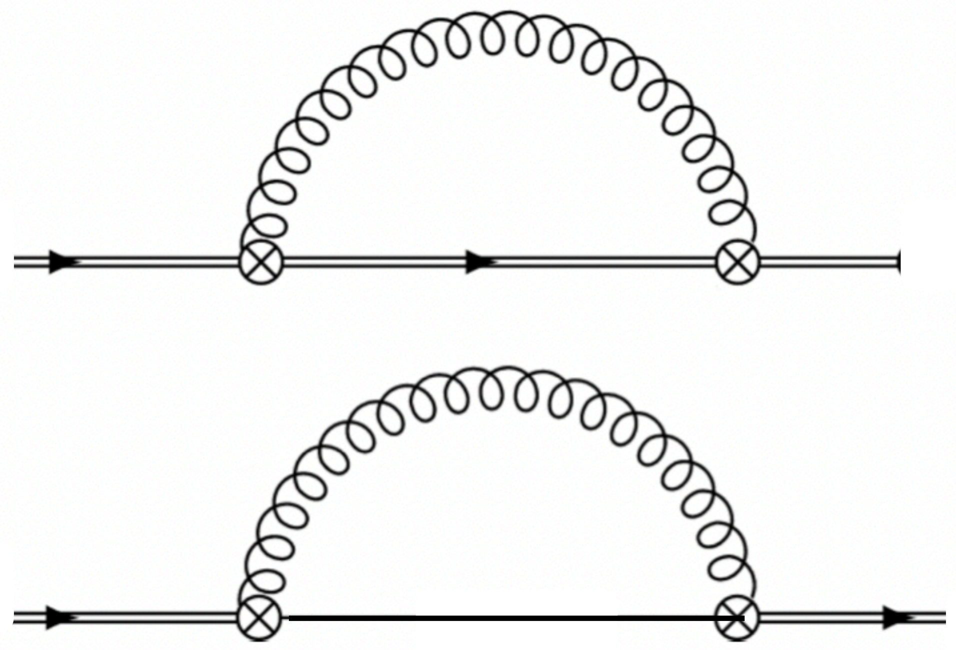
$$\Gamma = 2\,\sum_{o} \langle \phi \,|\, r\hat{\mathcal{O}}(p_0, p, r) \,|\, o \rangle \langle o \,|\, r \,|\, \phi \rangle$$

where ϕ is singlet state wave function

$$\Im\Sigma_{11}(p_0, p, r) = \frac{g^2 C_F}{6} r_i \left(\int \frac{d^4k}{(2\pi)^4} \left\{ \rho_o \theta(q_0)(\theta(-k_0) + f(|k_0|))[k_0^2 \rho_{jj}(k_0, k) + k_j^2 \rho_{00}(k_0, k)] \right\} \right) r_i$$

where $\rho_o = 2\pi\delta(k_0 + p_0 - \hat{q}_0)$ and $\hat{q}_0 = V_o + \dfrac{\nabla^2}{M}$

$$\rho^{\mu\nu}(k_0, k) = \int dt d^3x e^{i(k_0 t - k\cdot x)} \left\langle \frac{1}{2} \left[A^\mu(t, x), A^\nu(0) \right] \right\rangle$$



Spectral functions

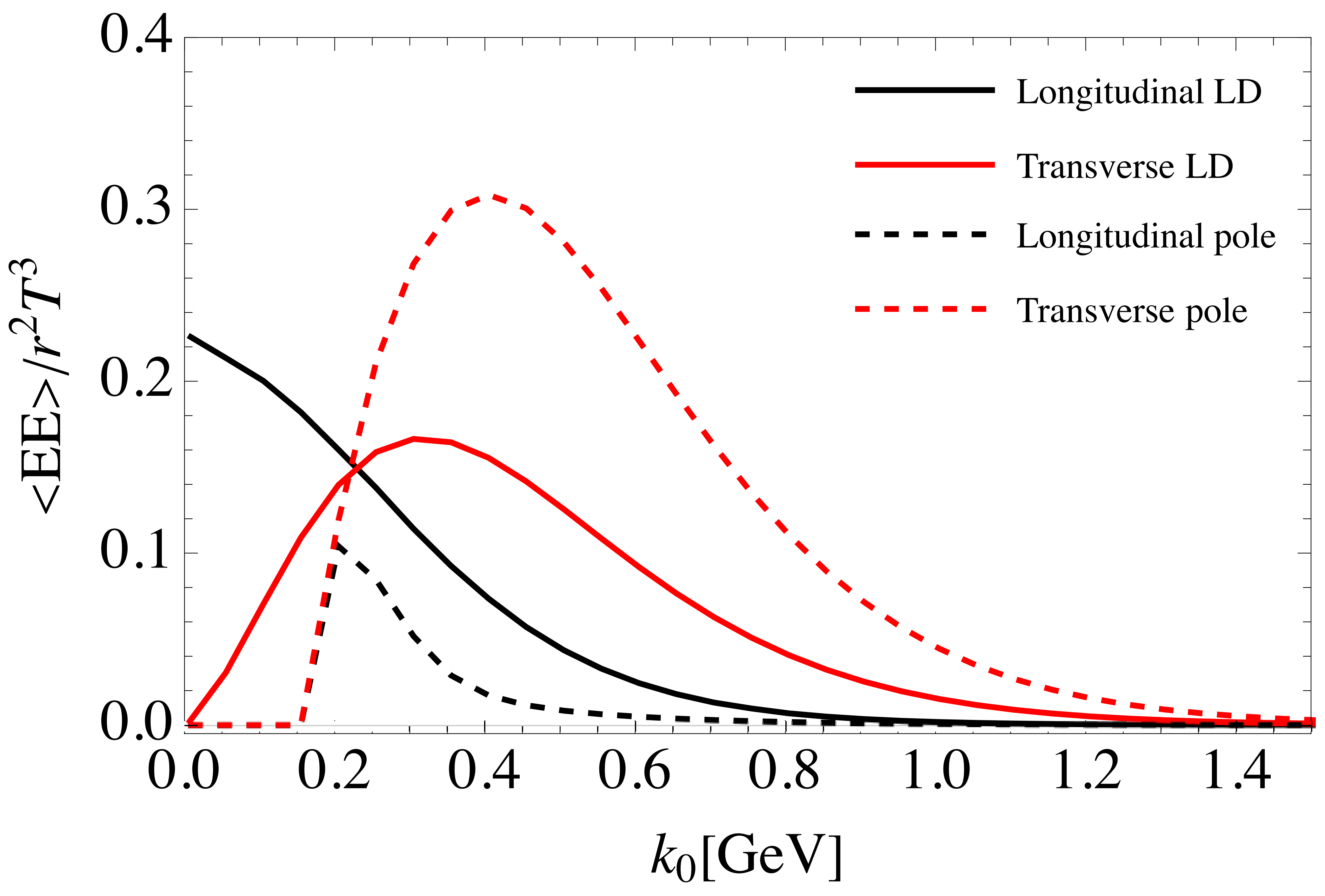
Heavy quark momentum diffusion coefficient

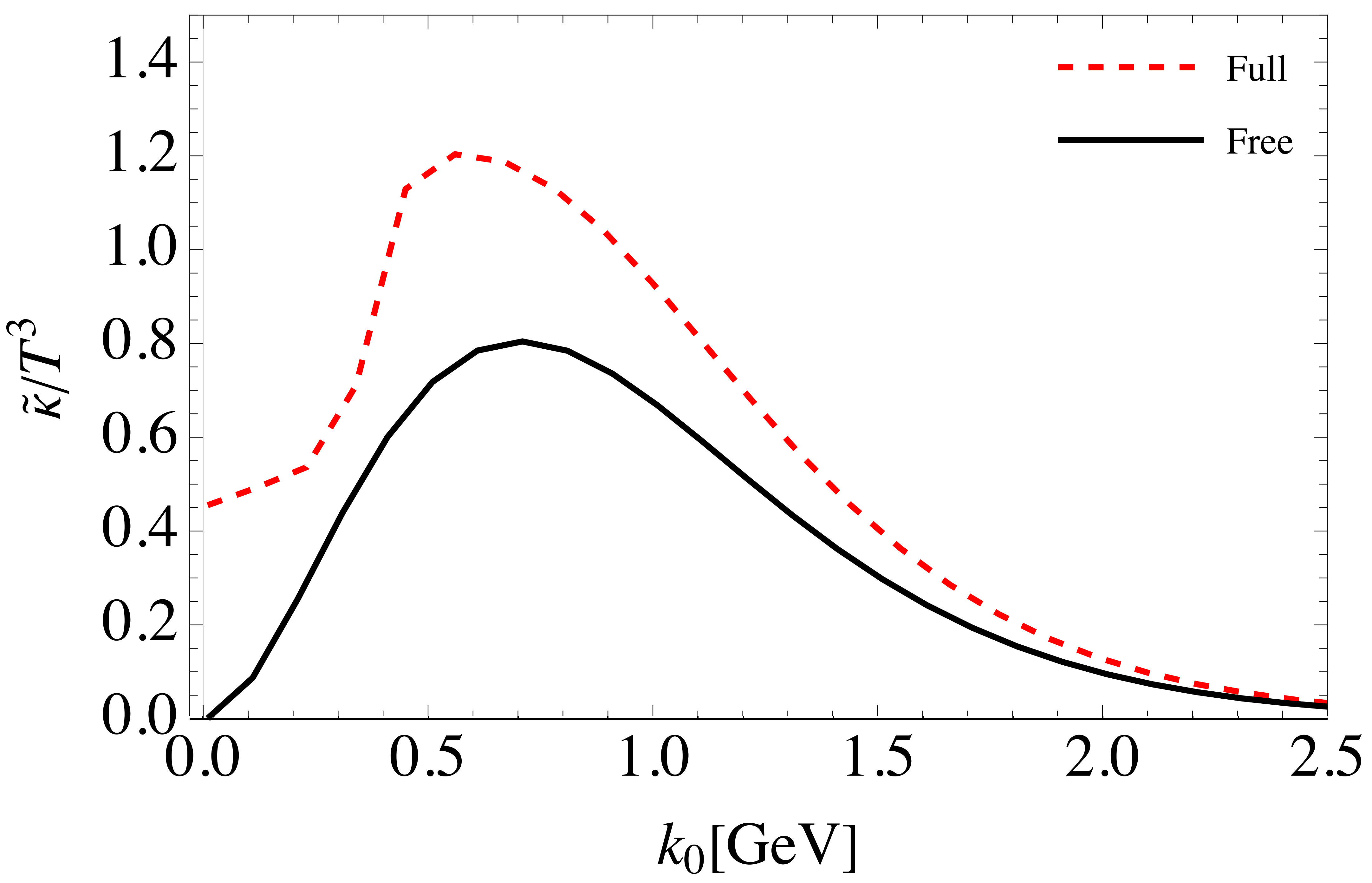
$$\kappa = \frac{g^2}{3N}\int_{-\infty}^{\infty} dt\,\mathrm{Tr}\langle U(-\infty,t)E_i(t)U(t,0)E_i(0)U(0,-\infty)\rangle$$

J. Casalderry-Solana, D. Teaney (2020)

For quarkonium

$$\tilde{\kappa}(k_0) = \frac{2(\Im\Sigma_{11}(k_0)\theta(k_0-k) + \Im\Sigma_{11}(k_0)\theta(k-k_0))}{r^2}$$





D. Banerjee et.al (2022)

Real and Imaginary potentials

- Both singlet and octet potentials are complex

Real part → Debye screening in the medium

Imaginary part → Landau damping

D. Bala, S. Dutta (2020)

- At large r, real part of singlet and octet potential approach each other

Y. Akamatsu (2013)

$$\Re V_s(r,T) = -\frac{C_F g^2}{4\pi}\left(m_D + \frac{e^{-m_D r}}{r}\right) \qquad \Re V_o(r,T) = \frac{g^2}{4\pi}\left(-C_F m_D + \frac{1}{2N}\frac{e^{-m_D r}}{r}\right)$$

- For singlet state, we use lattice inspired potential

A. Islam, M. Strickland (2020)

$$V_s(r,T) = -\frac{a}{r}(1+m_D r)e^{-m_D r} + \frac{2\sigma}{m_D}(1-e^{-m_D r}) - \sigma r e^{-m_D r}$$

A. Rothkopf et. Al (2017)

where $a = 0.409$, $\sigma = 0.21\ \mathrm{GeV}^2$ and $m_D = \sqrt{(1+N_f/6)}gT$

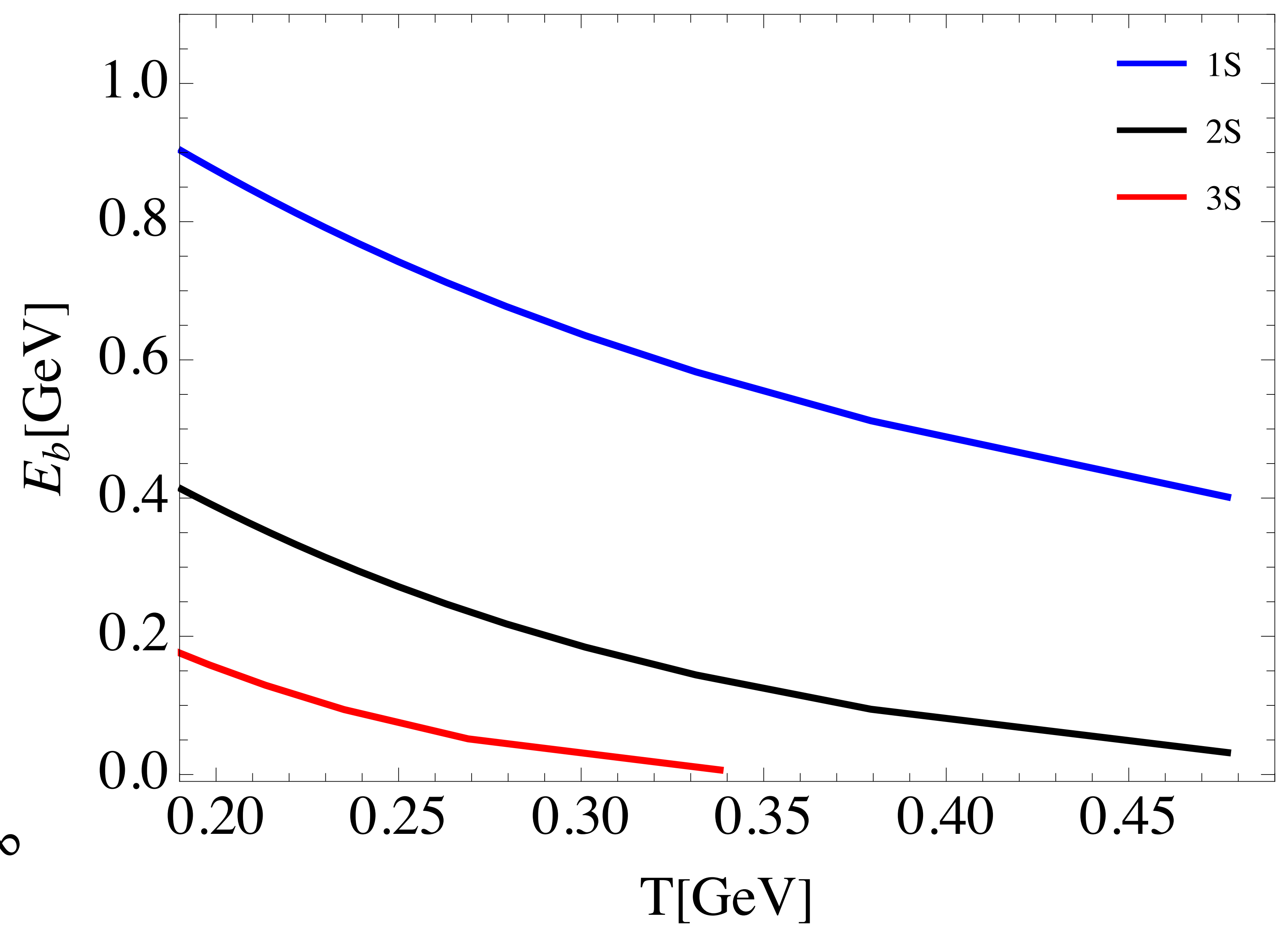
- For octet state, a well motivated choice is

$$V_o(r,T) = \frac{g^2}{2N}\frac{e^{-m_D r}}{r} + V_\infty$$

Real and Imaginary potentials

- We assume the singlet states are in the eigenstates of the real part of the potential

$$\left(\frac{p^2}{M} + \Re V_s(r,T)\right)|\psi\rangle = E|\psi\rangle$$



- Octet state lies in continuum

$$\left(\frac{q^2}{M} + \Re V_o(r,T)\right)|o\rangle = E_o|o\rangle$$

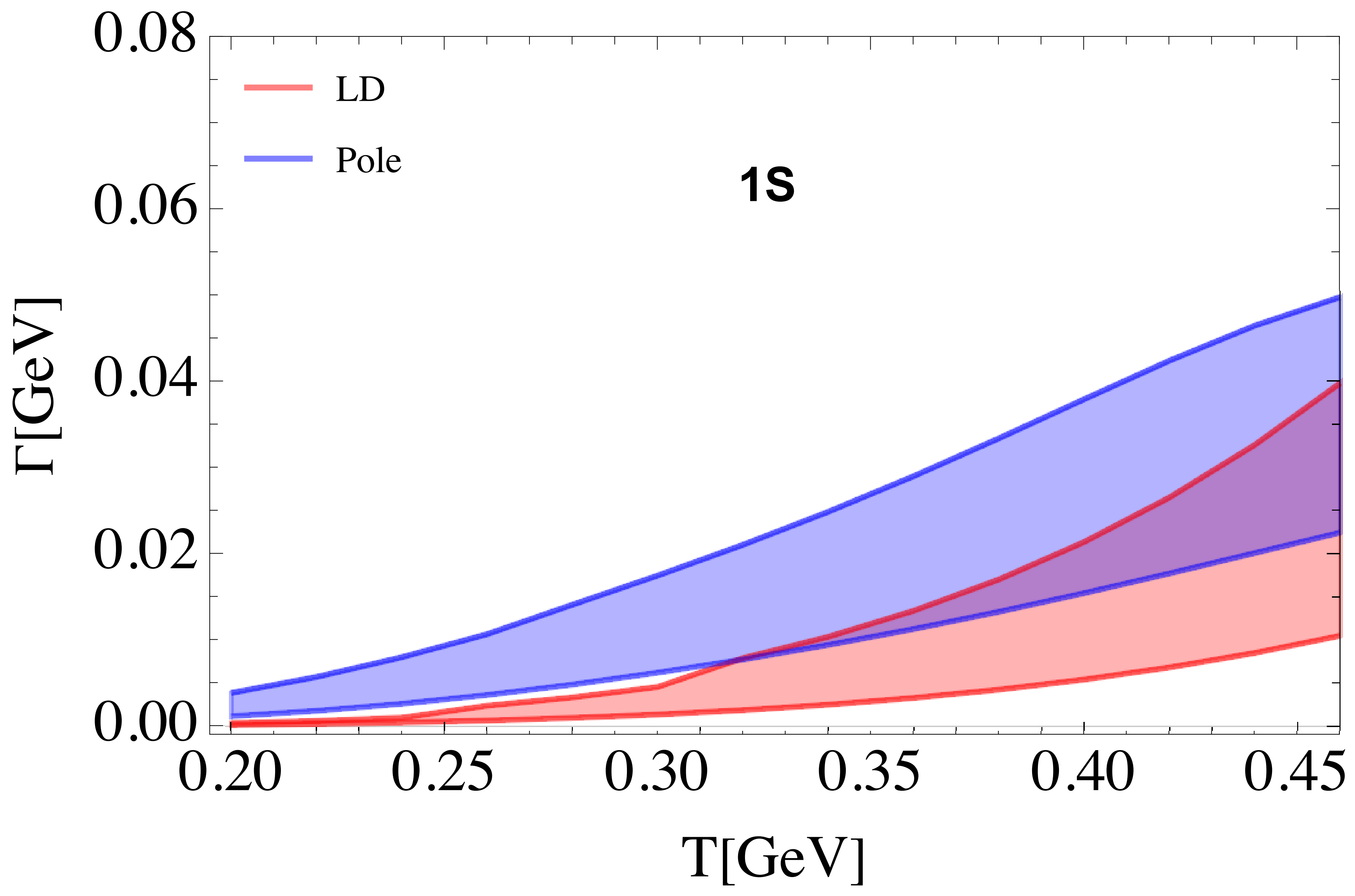
- For octet we consider two limiting cases:
  Octet states are not screened → $V_o(r,T) = \frac{C_F\alpha}{2Nr} + V_\infty$
  Octet states are completely screened → $V_o(r,T) = V_\infty$

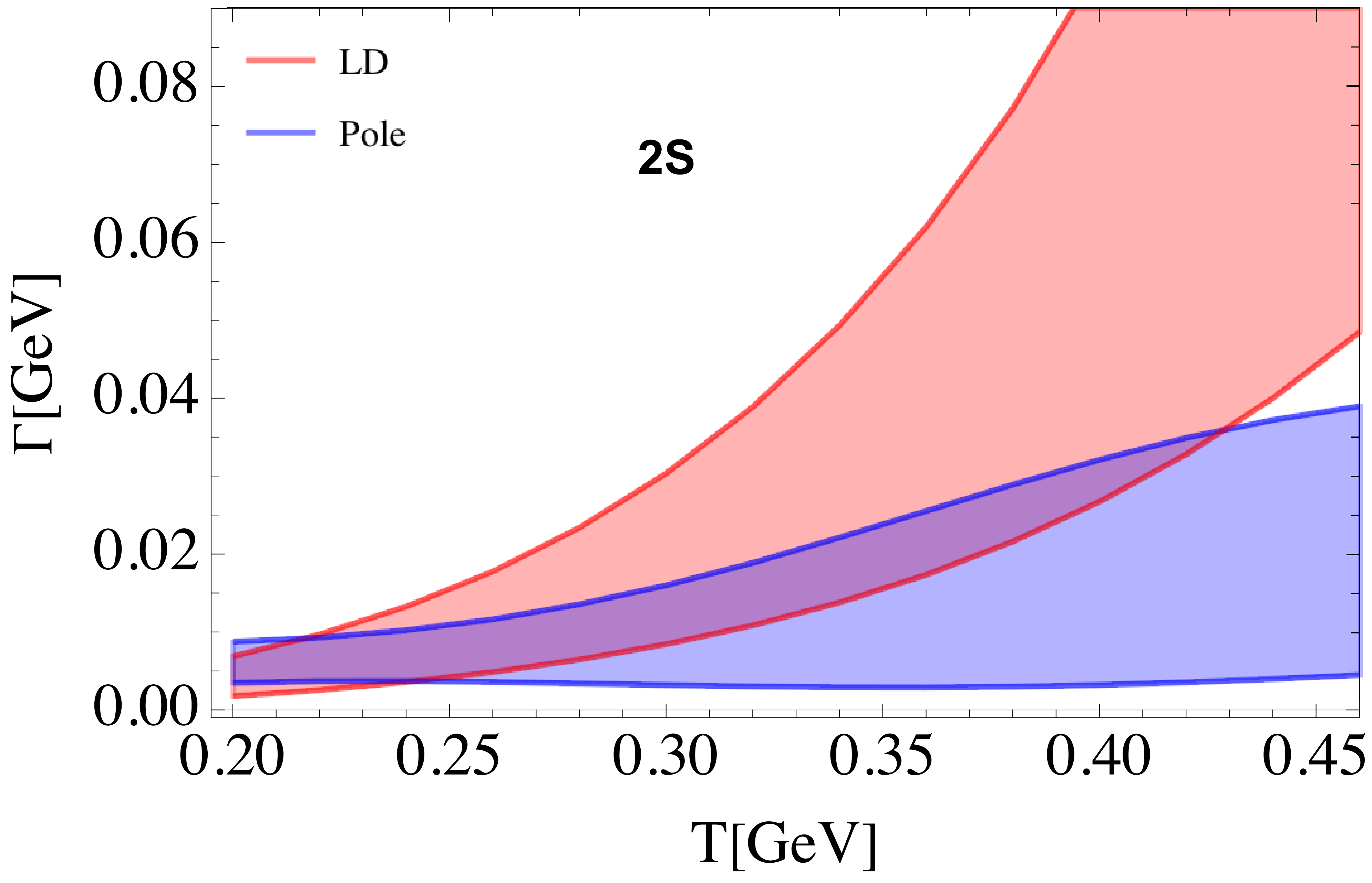
- Octet state wave function

$$|o\rangle = 4\pi R_l(pr)\sum_m Y_m^{*l}(\hat{r})Y_m^l(\hat{p}) \qquad |o\rangle = 4\pi j_l(pr)\sum_m Y_m^{*l}(\hat{r})Y_m^l(\hat{p})$$

Decay width (gluo-dissociation vs scattering)

- $E_b = 0.6$ GeV for 1S state and $E_b = 0.2$ GeV for 2S state
- For 1S both gluo-dissociation and Landau damping give similar contribution
- For 2S gluo-dissociation dominates at low temperature and Landau damping takes over at high temperature





Decay width (Imaginary potential)

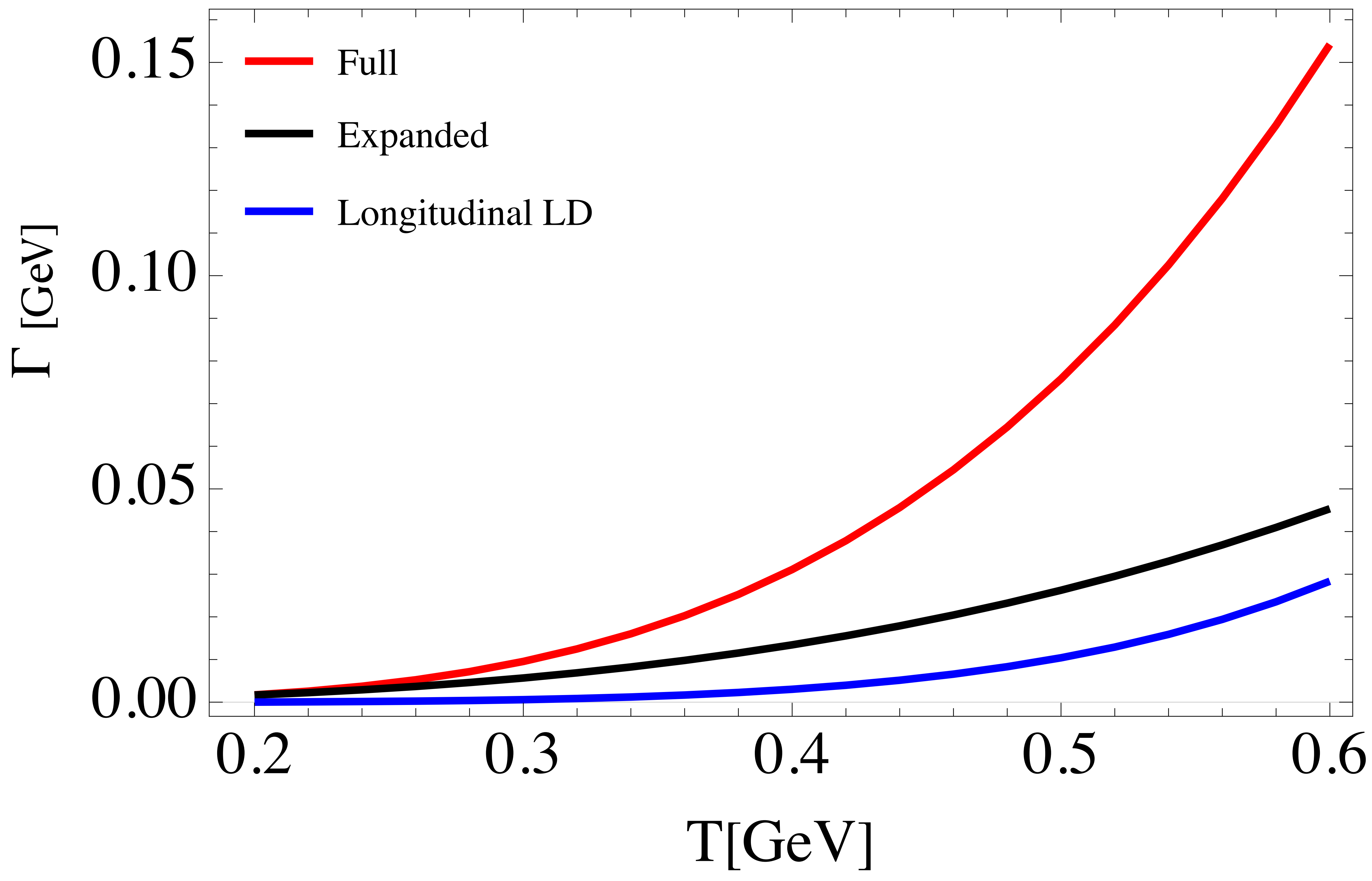
- Decay width with imaginary potential

$$\Gamma = 2\langle\phi|\Im V(r,T)|\phi\rangle$$

M. Laine et. al (2007)

$$\Im V(r,T) = \frac{g^2 C_F T}{2\pi}\int_0^\infty \frac{dz z}{(1+z^2)^2}\left(1-\frac{\sin(z m_D r)}{z m_D r}\right)$$

$$\phi(r) = \frac{1}{\sqrt{\pi a^3}} e^{-\frac{r}{a}}$$



- Assumes the binding energy of the species is zero

- Ignores the kinetic energy of the corresponding octet state

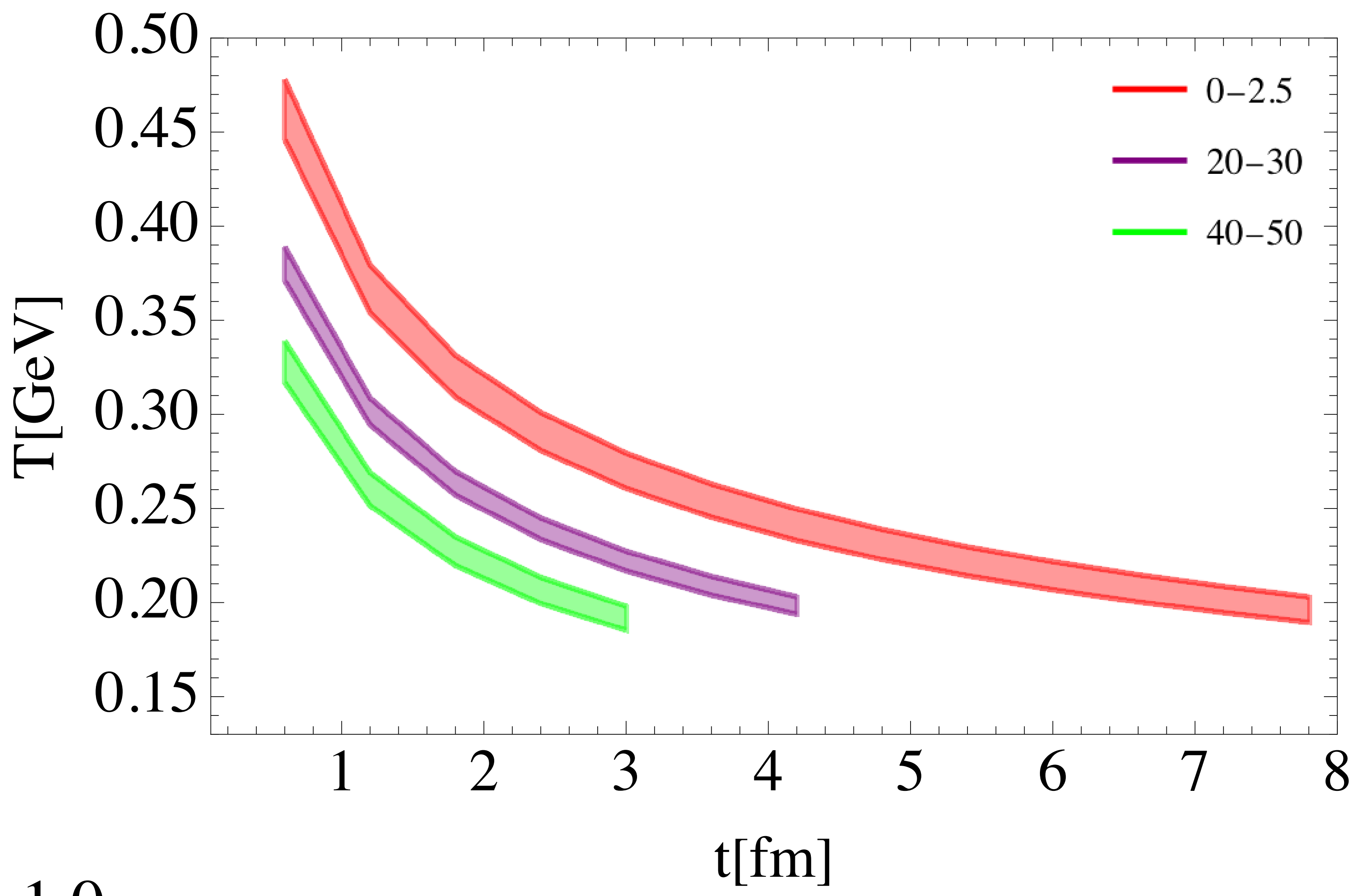
- Imaginary potential over-predicts the decay width of the singlet state

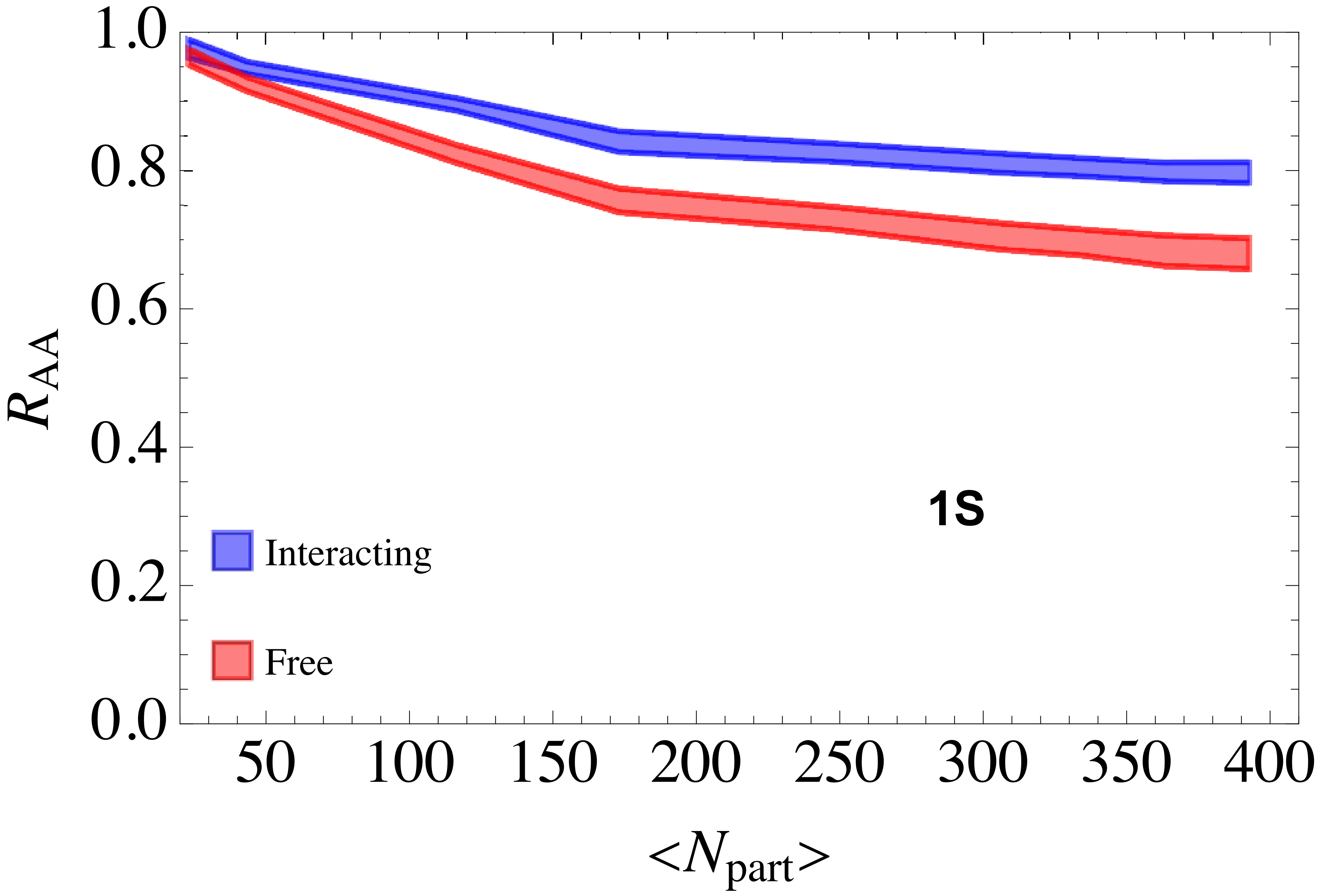
R_{AA} of 1S and 2S states

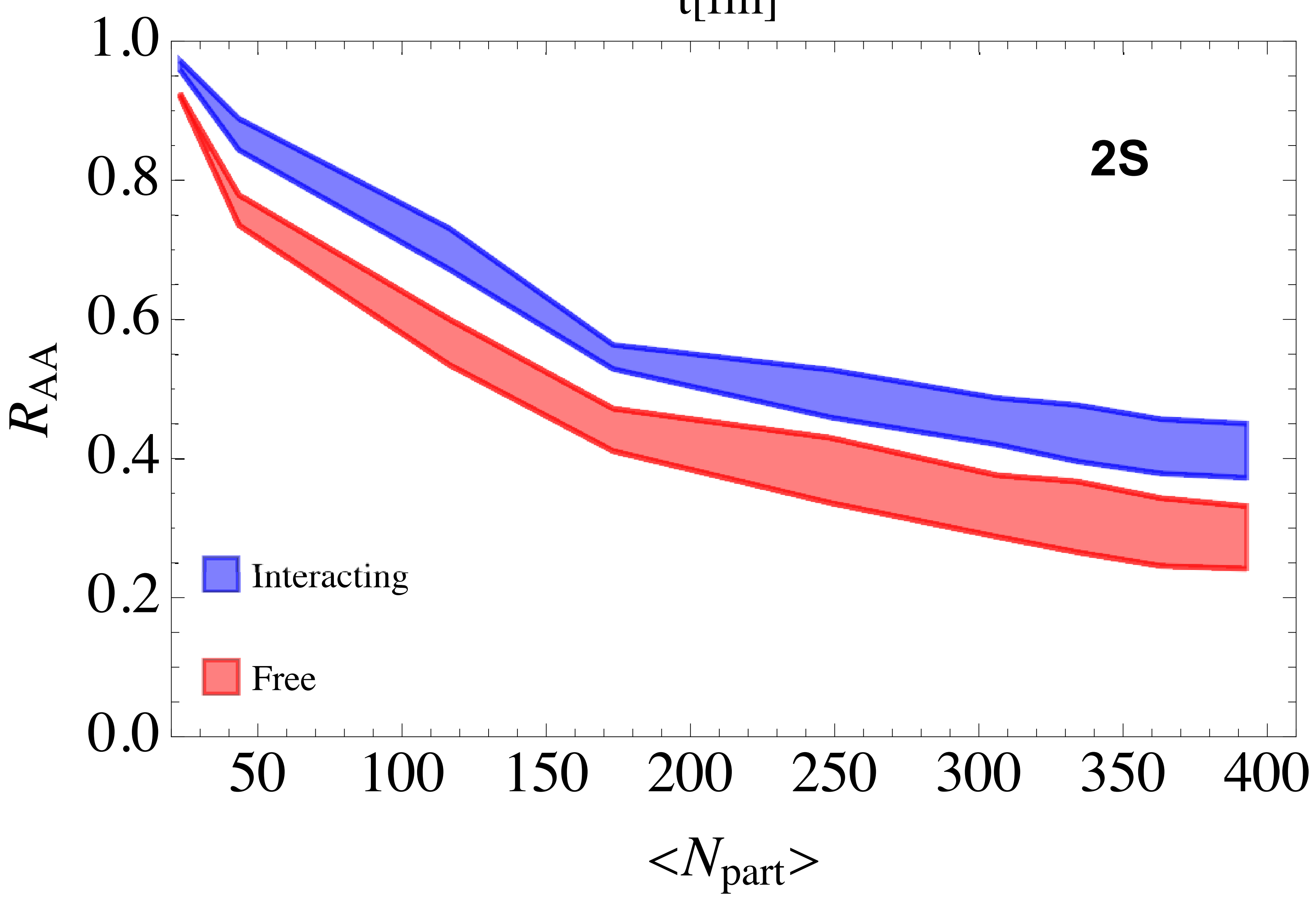
- Suppression: **A. Islam, M. Strickland (2020)**

$$N = N_0 e^{-\int_{t_0}^{t_f} \sum_i \Gamma(t) dt} \qquad T(t) = T(t_0)\left(\frac{t_0}{t}\right)^{\frac{1}{3}}$$

$$N_f = N + \sum \alpha_i N_i^h \qquad t_0 = 0.6 \text{ fm}$$







Quarkonium as open system

- In order to deal with non-equilibrium evolution of the bound states one needs open quantum system framework
  - Relevant scales for quantum dynamics

τ_s → System intrinsic time scale $(1/E_b)$

τ_E → Environment time scale $(1/T)$

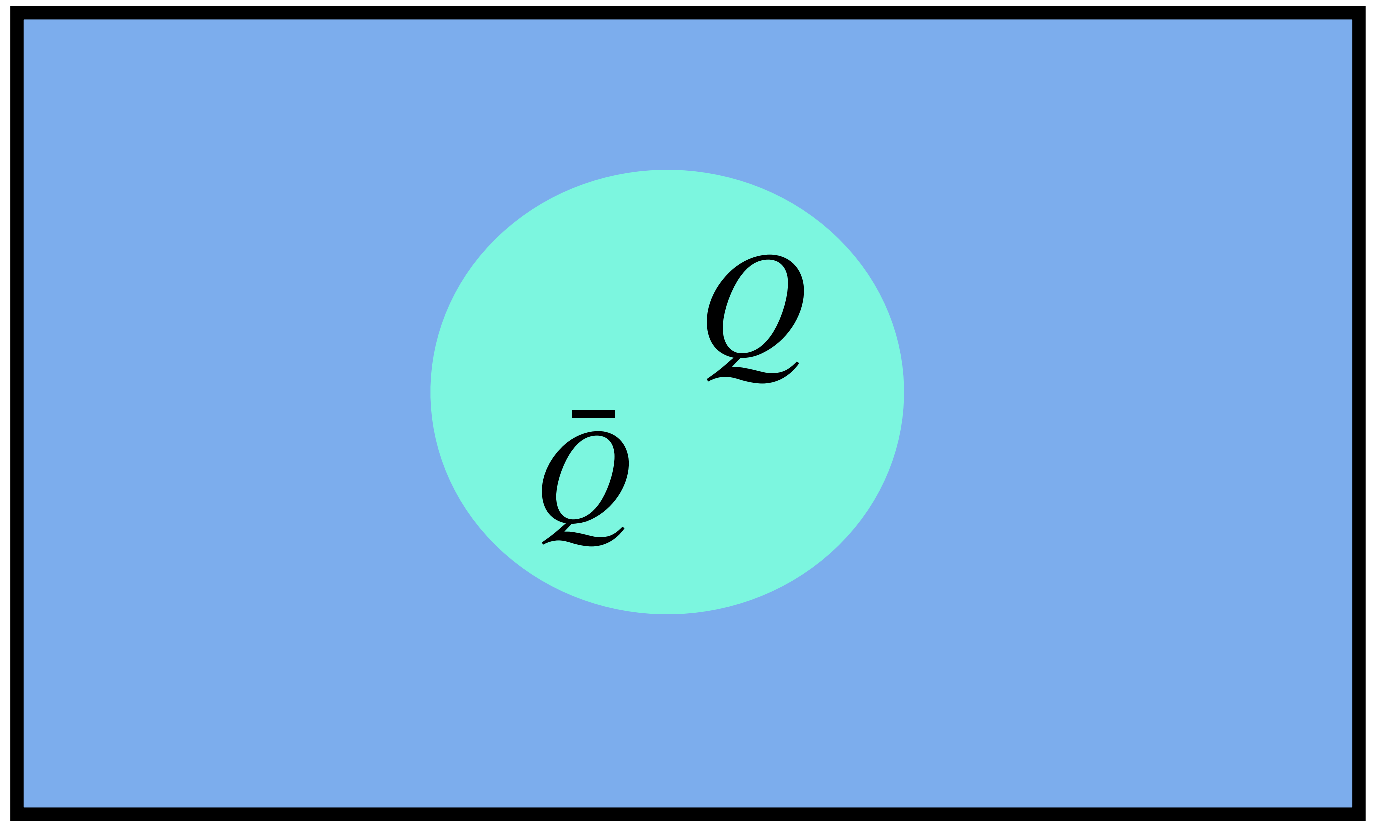
τ_R → Relaxation time scale $(\sim 1/g^2T)$

- Quantum evolution of the pair

$\tau_S \gg \tau_E$ → Quantum Browniann motion

$\tau_R \gg \tau_S$ → Quantum Optical limit

N.Brambilla et.al (2017, 2020)



X. Yao, T. Mehen (2019,2021)

Y. Akamatsu, (2022)

- Total density matrix

$\rho(t) = \rho_S(t) \otimes \rho_E(0)$ → Factorized form

$\tau_R \gg \tau_E$ → Markovian

$$\frac{d\rho_S(t)}{dt} = -i[H, \rho_S(t)] + \sum_i \gamma_i \left[L_i \rho_S(t) L_i^\dagger - \frac{1}{2} \left\{ L_i L_i^\dagger, \rho_S(t) \right\} \right]$$

- Quarkonium evolution is local in time

Density matrix evolution

Ongoing work with Vyshakh B. R. And R Sharma

- Hierarchies between τ_S and τ_E is not satisfied quiet well at least for 1S and 2S states

- Continuum transitions can not be explained within optical limit

- Non-local evolution of quarkonium state is important and therefore full structure of gluon spectral function should be used

- Density matrix evolution equation for single heavy quark

$$\frac{\partial \rho_s}{\partial t} = -\alpha^2 \mathrm{Tr}_E \int_0^t ds [H_I(t), [H_I(s), \rho_s(t) \otimes \rho_E(0)]] \qquad H = \frac{p^2}{2M} \otimes I_E + I_s \otimes H_E + g \int dx \delta(x - x_Q) \otimes A_0(x)$$

- With the trace over environment

$$\frac{\partial \rho_s}{\partial t} = -g^2 \int_0^t ds \int dxdy \left(\Gamma(x-y, t-s) \{ V_s(x,t), [\rho_s(t), V_s(y,s)] \} \right)$$

- In the Brownian limit, above equation reduces to Fokker-Planck equation after taking classical limit

Conclusions

- $E_b \ll T$ Hierarchy is not very well satisfied for 1S and 2S states

- Full structure of gluon spectral function is important for following the dynamics of quarkonia

- For 1S both gluo-dissociation and Landau damping give similar contribution

- Landau damping dominates for higher excited states

- Since $\tau_S \gg \tau_E$ hierarchy is not strictly valid for 1S and 2S, one need to consider memory effects for quarkonium evolution within the brownian limit