# Understanding the topological constituents of SU(3) gauge theory across the deconfinement transition

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# Outline

- Non-trivial holonomy, Calorons, Fermion zero modes.
- Motivation.
- Techniques used.
- Results.
- Further work.

# Introduction

- The $SU(N)$ gauge theory for $N \geq 3$ undergoes a first order phase transition.
- The Polyakov loop is an order parameter for confinement-deconfinement(C-D) transition

$$\vec{P}(\vec{x}) \equiv P[exp(\int_0^\beta dx_0 \ \vec{A_0})] \tag{1}$$

- The eigenvalues of the Polyakov loop is referred to as Holonomy. At spatial infinity, its holonomy, for $SU(3)$ can be represented as the diagonal matrix in the periodic gauge.

$$\lim_{|\vec{x}|\to\infty} P(\vec{x}) = diag\{e^{2\pi i \mu_0}, e^{2\pi i \mu_1}, e^{2\pi i \mu_2}\} \tag{2}$$

- Non-trivial holonomy implies the eigenvalue of the Polyakov loop at spatial infinity

$$\lim_{|\vec{x}|\to\infty} P(\vec{x}) \neq e^{i\frac{2\pi k}{N}} 1, \qquad k = 0, 1, \cdots, N-1 \tag{3}$$

- A $SU(N)$ instanton at finite temperature (or calorons) can be decomposed into $N$ constituent monopoles(or dyons), when the eigenvalues of Polyakov loop at spatial infinity is non-trivial [Kraan-van Baal, Lee-Lu(KvBLL)][T. C. Kraan and P. van Baal(1998),K. M. Lee and C. h. Lu(1998)].
- The solution was constructed through Nahm transformation that maps a $Q = 1$ solution of a $SU(N)$ gauge theory to a $Q = N$ solution in a compact $U(1)$ theory.
- Dual manifold in this case is basically a circle $S^1$, where the holonomy is located.
- For $SU(N)$ with $Q = 1$ Caloron there will be $N$ fractional substructure. $L, M_1, \cdots M_{N-1}$ dyons.

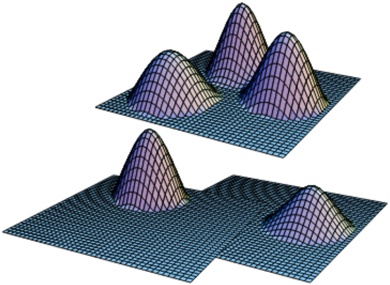
- For well-separated dyons, their actions are a fraction of the total instanton action given as $8\pi^2\nu_m/g^2$, where $\nu_m = \mu_{m+1} - \mu_m$, with $\mu_3 = 1 + \mu_0$, $\nu_m$'s represents the fractions of the circle on which the eigenvalues of the holonomy are located.
- If instanton-dyons are well separated, the local holonomy at the position of the *ith* dyon [D. Diakonov, (2009)] with $i = 1, 2, 3$ can be written as

$$P(x_i) = diag[e^{i2\pi\mu_{i-1}}, e^{i\pi\mu_i+\mu_{i+1}}, e^{i\pi\mu_i+\mu_{i+1}}] \tag{4}$$

- In the confined phase at $T \lesssim T_c$, where the average value of the Polyakov loop is close to zero, the instanton action is split evenly, between its constituent instanton-dyons, $\nu_0 = \nu_1 = \nu_2 = 1/3$ and corresponding $\mu_0 = 0,\ \mu_1 = 1/3,\ \mu_2 = 2/3.$

# Fermion Zero Modes

- Index theorem relates the zero modes of the Dirac operator with the topological constituents of the gauge field, i.e instantons.
- The location of the zero mode coincides with the topologically non-trivial gauge field background on which it is calculated.
- With the usual anti-periodic boundary condition that is with phase angle $\pi$, $(\psi(t+\beta) = e^{i\phi}\psi(t))$ zero mode is located at the L-dyon. By changing the phase angle one can also locate two M-dyons for e.g. M-dyon can be calculated for $\pm\pi/3$.



- Action density(top) plot for the SU(3) caloron on logarithmic scale. the bottom-left and bottom-right are the zero modes associated with the monopole with an appropriately chosen phase angle for the fermionic boundary condition[hep-lat/9907001].

- Overlap Dirac operator, $D = 1 - \gamma_5 sign(H_W)$ has been used as a probe to find the zero modes since it satisfies an exact index theorem, even with finite lattice spacing.
- If one calculates the local $Tr(P(x_i))/3$ at the location of the dyon, for the L-dyons the real part is given as $-1/3$ and the imaginary part is zero. For M-dyons corresponding to the phase angle, $\pm\pi/3$ the value of the local holonomy at their locations are $1/6 \pm i/\sqrt{12}$.

# Motivation

- Topological constituents of QCD are believed to play a role in driving confinement. In order to produce confinement these topological constituents needs to interact with the $P(\vec{x})$ and suppress its values.
- Instantons were previously shown to explain confinement in $2+1$ dimensions but in $3+1$ dimensions they do not couple with the Polyakov loop.
- Since the instanton-dyon solutions depend on the Polyakov loop, it might lead to confinement.
- In this work we focus on $SU(3)$ gauge theory, which is much cleaner since C-D transition is first order rather than a crossover in QCD. We aim to look at any correlations between the instanton-dyons and the Polyakov loop.

# Wilson flow

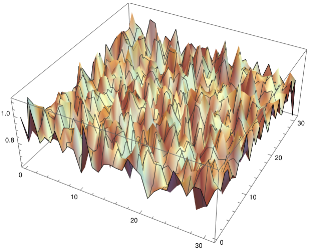
- The SU(3) gauge field configurations generated using Wilson action on a $32^3 \times 8$ lattice .
- The gauge ensembles contains UV fluctuations, which make it harder to look for the topological constituents.
- Gradient flow [M. Luscher,(2010) [arXiv:1006.4518]] [hep-lat]] is one such smearing technique which helps remove those UV fluctuations and smoothen the configuration to observe topological objects.
- The gradient flow equation is given as,

$$\dot{V}_t(x,\mu) = -g_0^2[\partial_{x,\mu} S_W(V_t)]V_t(x,\mu), \quad V_t(x,\mu)|_{t=0} = U(x,\mu) \tag{5}$$

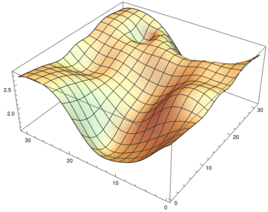
where $t$ is the flow time, $U(x,\mu)$ are the initial gauge links. The gauge action is simply,

$$S_W = \frac{2N}{g_0^2}\sum_{x,\mu>\nu}(1 - \frac{1}{N}Re(TrU(x)_{\mu\nu}))$$

# Wilson flow



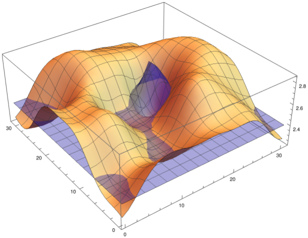

$t/a^2 = 0$




$t/a^2 = 1.0$

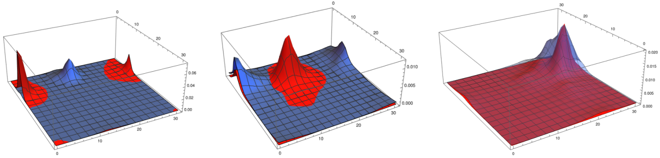
- We have considered $32^3 \times 8$ lattice for simulation at $T = 1.1T_c$.
- A typical configuration, the left one is with UV fluctuations, after flow the fluctuations are significantly smoothened out as shown in the right.

# Results




- This is a typical spatial distribution of the fermion zero mode density(scaled appropriately) for phase angle $\phi = \pi$(blue) superimposed with the real part of the Polyakov loop after gradient flow at $1.1T_c$.
- One can see the zero-mode peaked at the local minima of the Polyakov loop.

# Results



- The first two pictures shown are the well-separated zero modes with different phase angles $\pi$(red) and $-\pi/3$(blue). The third one is typical overlapped modes(properly scaled).
- Well-separated modes are very rare to find. With the increase in temperature, M-dyons widen further only the L-dyon corresponding to the phase angle $\pi$ remains.

# Further work

- Further improvement of statistics required to look for well-separated dyons and calculate the local Polyakov loop as a function from the zero-mode core.
- Further we need to study the same for $SU(2)$ gauge theory and show how the correlations between fermion zero modes and the Polyakov loop changes with color.
- Since $SU(2)$ phase transition is of $2nd$ order and for $N > 2$ transition is $1st$ order we expect to observe its effects in the correlations with the Polyakov loop.
- It will be also important to perform continuum and finite volume extrapolation in all such studies related to the topological objects.

Thank You.