Spin polarization and alignments from color fields in the glasma

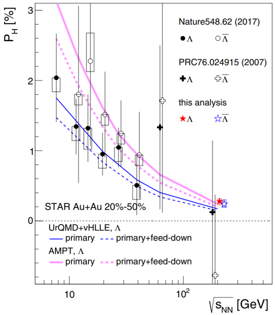
Avdhesh Kumar

In Collaboration with: **Prof. Di-Lun Yang and Prof. Berndt Mueller**

Based on
arXiv:2212.13354 [nucl-th].

International Conference on Physics and Astrophysics of Quark Gluon Plasma (ICPAQGP-2023), 7-10 February 2023, Puri, Odisha, India

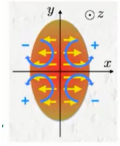
Global and Local Spin polarization of Λ hyperons in RHIC experiment



[J. Adam *et al.* (STAR), Phys. Rev. C 98, 014910(2018)]

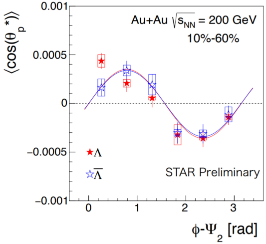
UrQMD+vHLLE: I. Karpenko, F. Becattini, EPJC 77, 213 (2017)

AMPT: H. Li, L. Pang, Q. Wang, and X. Xia, PRC 96, 054908 (2017)



[S.Voloshin, EPJ Web Conf. 171, 07002 (2018)]

[F. Becattini, I. Karpenko, PRL 120 (2018) no.1, 012302, [1707.07984]]

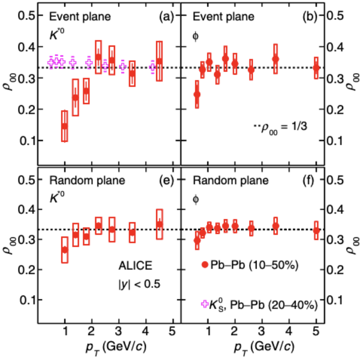


T. Niida, NPA 982 (2019) 511514 [1808.10482];

[J. Adam *et al.* (STAR), Phys. Rev. Lett. 123, 132301 (2019)]

Spin alignments of vector mesons in HICs

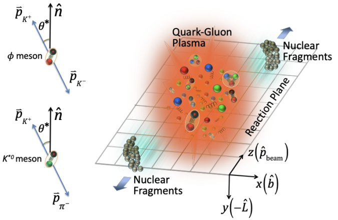
Deviation of ρ_{00} from 1/3 indicates net spin alignment



[S. Acharya et al. (ALICE), PHYSICAL REVIEW LETTERS 125, 012301 (2020)]

Spin alignment of vector mesons are measured by the ρ_{00}-component of spin density matrix ρ_{mn} with unit trace.

$$\frac{dN}{d\cos\theta^*} \propto \left[1 - \rho_{00} + \cos^2\theta^*(3\rho_{00} - 1)\right]$$



[A. Mohamed *et al.* (STAR), arXiv:2204.02302 [hep-ph]]

Progress from theory side

Modified Cooper Frye formula, $\mathcal{P}^{\mu}(p) = \frac{\int d\Sigma \cdot p \mathcal{J}_5^{\mu}(p,X)}{2m \int d\Sigma_{\mu} \mathcal{N}^{\mu}(p,X)}$.

In the global equilibrium: $\mathcal{P}^{\mu}(p) = -\frac{1}{8m}\epsilon^{\mu\rho\sigma\tau} p_{\tau} \frac{\int d\Sigma_{\lambda} p^{\lambda} f_p(1-f_p)\varpi_{\rho\sigma}}{\int d\Sigma_{\lambda} p^{\lambda} n_F}$;

$$f_p = (1 + \exp[\beta \cdot p - \mu Q/T])^{-1}, \quad \boxed{\varpi^{\mu\nu} = -\frac{1}{2}\left(\partial_{\mu}\beta_{\nu} - \partial_{\nu}\beta_{\mu}\right)}$$

[F. Becattini, V. Chandra, L. Del Zanna, E. Grossi, Ann. Phys. 338, 32 (2013), R. Fang, L. Pang, Q. Wang, X. Wang Phys. Rev. C 94, 024904 (2016)]

Describes the global polarization Λ-polarization.
Problem with explaining the local polarization $\longrightarrow$ spin sign problem

In Local equilibrium, thermal shear corrections defined in term of tensor $\xi_{\mu\nu} = \frac{1}{2}(\partial_{\mu}\beta_{\nu} + \partial_{\nu}\beta_{\mu})$ can play some role.

[F. Becattini el al., Phys. Lett. B 820 (2021) 136519, also arXiv:2103.14621 [nucl-th]]

[Shuai Y. F. Liu, Yi Yin, JHEP 07 (2021) 188, arXiv:2103.09200 [hep-ph]]

This potentially resolves the spin sign problem.

[F. Becattini, M. Buzzegoli, A. Palermo, G. Inghirami, and I. Karpenko, Phys. Rev. Lett. 127, 272302 (2021), arXiv:2103.14621]

[B. Fu, S. Y. F. Liu, L. Pang, H. Song, Y. Yin, Phys. Rev. Lett. 127, 142301 (2021)], arXiv: 2103:10403 [hep-ph]]

[Wojciech Florkowski, AK, Aleksas Mazeliauskas, Radoslaw Ryblewski, Phys. Rev. C 105, 064901 (2022); arXiv:2112.02799 [hep-ph]]

[Wojciech Florkowski, AK, Aleksas Mazeliauskas, Radoslaw Ryblewski, Phys. Rev. C 105, 064901 (2022); arXiv:2112.02799 [hep-ph]]

[Cong Yi, Shi Pu, Di-Lun Yang, Phys.Rev.C 104 (2021) 6, 064901; arXiv: 2106.00238 [hep-ph]]

Results are sensitive to EoS [S. K. Singh, J. Alam, arXiv: 2110.15604 [hep-ph]], freeze-out temperature, out-off equilibrium corrections should also be considered.

Progress from theory side

From spin-dependent coalescence model, 00-component of the spin density matrix for spin-1 vector mesons found to be [Z.-T. Liang and X.-N. Wang, PLB 629, 20 (2005)]

$$\boxed{\rho_{00} = \frac{1 - \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i}{3 + \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i}}$$

Vorticity contribution to global spin polarization of Λ–hyperons $\mathcal{P}_q^y \sim \mathcal{P}_\Lambda^y \sim 10^{-4}$ at 2.76 TeV. This implies that $\mathcal{P}_q^y \mathcal{P}_q^y = 10^{-8}$ that means $\rho_{00} \approx 1/3$.

[S. Acharya et al. (ALICE Collaboration), PHYSICAL REVIEW C 101, 044611 (2020)]

Some more studies,

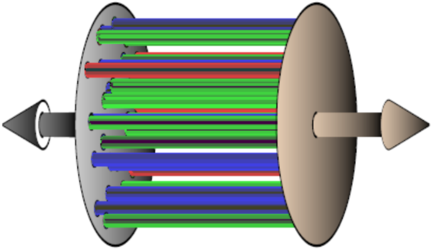
[Xin-Li Sheng, Lucia Oliva, and Qun Wang, Phys. Rev. D, 101(9):096005, 2020; Erratum: Phys. Rev. D, 105:099903, 2022]

[Xin-Li Sheng, Qun Wang, and Xin-Nian Wang, Phys. Rev. D, 102(5):056013, 2020]

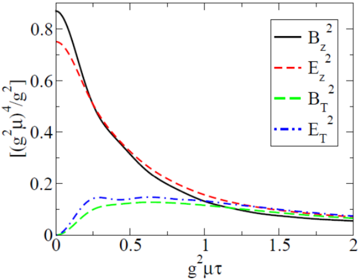
What is source of large deviations??

Does microscopic properties of the QGP play any role? Particularly, any role of gluons and color d.o.f.?

Color electromagnetic fields at early times in HICs



[review: F. Gelis, E. Iancu, J. Jalilian-Marian, R. Venugopalan, Ann.Rev.Nucl.Part.Sci.60:463-489,2010]



[T. Lappi, Phys.Lett.B 643 (2006) 11-16]

Color flux tubes in the glasma phase : longitudinal chromo-EM fields in early times.

Instabilities can further enhance the fields.

Possibilty: spins of quarks gets dynamically polarized from the color fields in the glasma phase.

Quantum Kinetic theory

QKT is based on [H.T. Elze, M. Gyulassy and D. Vasak, Transport Equations for the QCD Quark Wigner Operator, Nucl. Phys. B 276 (1986) 706]

$$\grave{S}^{<}(p,X) = \int \frac{d^4Y}{(2\pi)^4} e^{-\frac{ip\cdot Y}{\hbar}} \bar{\psi}\left(X+\frac{Y}{2}\right) U\left(X+\frac{Y}{2},X\right) \otimes U\left(X,X-\frac{Y}{2}\right) \psi\left(X-\frac{Y}{2}\right)$$

The dynamical evolution of $\grave{S}^{<}(p,X)$ is govern by the Kadanoff-Baym equation [H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett., B177, 402–408, 1986]

$$\boxed{\left[\gamma^{\mu}\left(\hat{\Pi}_{\mu} + i\frac{\hbar}{2}\hat{\nabla}_{\mu}\right) - m\right]\grave{S}^{<}(p,X) = 0.}$$

$$\begin{aligned}
\hat{\Pi}_{\mu}\grave{S}^{<} &= p_{\mu}\grave{S}^{<} + \frac{i\hbar}{8}[F_{\nu\mu}, \partial_p^{\nu}\grave{S}^{<}]_c + O(\hbar^2) \\
\hat{\nabla}_{\mu}\grave{S}^{<} &= D_{\mu}\grave{S}^{<} + \frac{1}{2}\{F_{\nu\mu}, \partial_p^{\nu}\grave{S}^{<}\}_c - \frac{i\hbar}{24}[(\partial_p\cdot DF_{\nu\mu}), \partial_p^{\nu}\grave{S}^{<}]_c + O(\hbar^2)
\end{aligned}$$

$D_{\mu}O = \partial_{\mu}O + i[A_{\mu}, O]_c$, $A_{\mu} = t^a A^a_{\mu}$, $F_{\nu\mu} = t^a F^a_{\nu\mu}$, and $t^a = \frac{\lambda^a}{2}$ with λ^a being the Gell-Mann matrices. Moreover, $\{,\}_c$ and $[,]_c$ denote the anti-commutation and commutation relations in color space.

Quantum Kinetic Theory

- Clifford algebra decomposition[H. T. Elze, M. Gyulassy, and D. Vasak, Phys. Lett., B177, 402–408, 1986]

$$\grave{S}^{<}(p,X)=\left(\mathcal{F}(p,X)+i\gamma^{5}\mathcal{P}(p,X)+\gamma^{\mu}\mathcal{V}_{\mu}(p,X)+\gamma^{5}\gamma^{\mu}\mathcal{A}_{\mu}(p,X)+\frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}(p,X)\right)$$

- 10 different equations can be derived for the different coefficient functions.
- Adopting the power counting scheme: $\mathcal{V}^{\mu}\approx O(\hbar^{0})$, $\mathcal{A}^{\mu}\approx O(\hbar)$ and perturbatively solving the equations up to order $\hbar$ we can find
- Only two relevant components: $\mathcal{V}_{\mu}$ and $\mathcal{A}_{\mu}$

$$\boxed{\mathcal{V}^{\mu}=2\pi\delta\left(p^{2}-m^{2}\right)p^{\mu}f_{V}}$$

$$\boxed{\mathcal{A}^{\mu}=2\pi\left[\delta(p^{2}-m^{2})\tilde{a}^{\mu}+\frac{\hbar}{2}p_{\nu}\delta'(p^{2}-m^{2})\{\tilde{F}^{\mu\nu},f_{V}\}_{\mathrm{c}}\right]}$$

- Dynamical variable in $\mathcal{V}_{\mu}/\mathcal{A}_{\mu}$: $\underbrace{f_{V}(p,X)}_{\text{Vector charge distribution}}$ and $\underbrace{\tilde{a}^{\mu}(p,X)}_{\text{Spin four vector}}$

In $m\to 0$, $\tilde{a}^{\mu}(p,X)=p^{\mu}f_{A}(p,X)$

- Finally one can derive dynamical equations for $f_{V}(p,X)$ and $\tilde{a}^{\mu}(p,X)$ (SKE and AKE)

Quantum Kinetic Theory

- Both the SKE and AKE are matrices in color space. Only color singlet components of $f_V(p,X)$ and $\tilde{a}^\mu(p,X)$ directly contribute to physical observables.
- Color decomposition: $O = O^s I + O^a t^a, \qquad J_5^\mu(X) = 4\int \frac{d^4p}{(2\pi)^4}\mathrm{tr_c}(\mathcal{A}^\mu)$
- Color decomposition of SKE and AKE leads to coupled dynamical equations for the color singlet and color octet components of vector and axial charge distributions: f_V^s, f_V^a, $\tilde{a}^{s\mu}$, $\tilde{a}^{a\mu}$
- After solving one can obtain f_V^a and $\tilde{a}^{a\mu}$ in terms of f_V^s, $\tilde{a}^{s\mu}$ whose dynamics is govern by the following equations

SKE: $$\delta(p^2-m^2)\Big(p\cdot\partial f_V^s(p,X) - \partial_p^\kappa D_\kappa[f_V^s]\Big) = 0$$

AKE: $$\delta(p^2-m^2)\Big(p\cdot\partial \tilde{a}^{s\mu}(p,X) - \partial_p^\kappa D_\kappa[\tilde{a}^{s\mu}] + \hbar\partial_p^\kappa\left(A_\kappa^\mu[f_V^s]\right)\Big) = 0$$

$$D_\kappa[O] = \bar{C}_2 \int_{k,X'}^{p,X} p^\lambda p^\rho \left(F^a_{\kappa\lambda}(X)F^a_{\alpha\rho}(X')\right)\partial_p^\alpha O(p,X')$$

$$A_\kappa^\mu[O] = \frac{\bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma}\int_{k,X'}^{p,X} p^\lambda p_\rho\Big(\partial_{X'\sigma}\left(F^a_{\kappa\lambda}(X)F^a_{\alpha\nu}(X')\right) + \partial_{X\sigma}\left(F^a_{\kappa\nu}(X)F^a_{\alpha\lambda}(X')\right)\Big)\partial_p^\alpha O(p,X')$$

$$\int_{k,X'}^{p,X} \equiv \int d^4k \int \frac{d^4X'}{(2\pi)^4} e^{ik\cdot(X'-X)}\left(\pi\delta(p\cdot k) + iPV(1/p\cdot k)\right), \qquad \bar{C}_2 = 1/(2N_c) \text{ with } N_c \text{ the number of colors}$$

Spin polarization of quarks and spin alignment of vector mesons

Modified Cooper-Frye Formula:

$$\langle \mathcal{P}^\mu(\boldsymbol{p})\rangle = \frac{\int d\Sigma \cdot p \langle \mathcal{J}_5^{s\mu}(\boldsymbol{p}, X)\rangle}{2m \int d\Sigma_\mu \langle \mathcal{N}^{s\mu}(\boldsymbol{p}, X)\rangle}$$

$$\langle \mathcal{P}_q^\mu(\boldsymbol{p})\mathcal{P}_{\bar{q}}^\mu(\boldsymbol{p})\rangle = \frac{\int d\Sigma_X \cdot p \int d\Sigma_Y \cdot p \langle \mathcal{J}_5^{s\mu}(\boldsymbol{p}, X)\mathcal{J}_5^{s\mu}(\boldsymbol{p}, Y)\rangle}{4m^2(\int d\Sigma_X \cdot \mathcal{N}^s(\boldsymbol{p}, X))^2}$$

$$\rho_{00} = \frac{1 - \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i\rangle}{3 + \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i\rangle}$$

$$\mathcal{N}^{s\mu}(\boldsymbol{p}, X) = 4N_c\left(p^\mu f_V^s\right)_{p_0=\epsilon_{\boldsymbol{p}}},$$

$$\mathcal{J}_5^{s\mu}(\boldsymbol{p}, X) = 4N_c\left(\boxed{\tilde{a}^{s\mu}} + \hbar \bar{C}_2 \boxed{\mathcal{A}_Q^\mu}\right)_{p_0=\epsilon_{\boldsymbol{p}}},$$

$\epsilon_{\boldsymbol{p}} = \sqrt{|\boldsymbol{p}|^2 + m^2}$ is the onshell energy.

Non-dynamical source term: $\mathcal{A}_Q^\mu|_{p_0=\epsilon_{\boldsymbol{p}}} = \mathcal{A}_{Q1}^\mu + \mathcal{A}_{Q2}^\mu$

$$\mathcal{A}_{Q1}^\mu = \left[\frac{\partial_{p\kappa}}{2} \int_{k,X'}^{p,X} p^\beta \left(\tilde{F}^{a\mu\kappa}(X)F_{\alpha\beta}^a(X')\right)\partial_p^\alpha f_V^s(p, X')\right]_{p_0=\epsilon_{\boldsymbol{p}}}$$

$$\mathcal{A}_{Q2}^\mu = \frac{1}{2\epsilon_{\boldsymbol{p}}^2}\left(p_{\perp\kappa} - \epsilon_{\boldsymbol{p}}^2 \partial_{p_{\perp}\kappa}\right)\left[\int_{k,X'}^{p,X} p^\beta \left(\tilde{F}^{a\mu\kappa}(X)F_{\alpha\beta}^a(X')\right)\partial_p^\alpha f_V^s(p, X')\right]_{p_0=\epsilon_{\boldsymbol{p}}},$$

$V_{\perp\mu}$ represents the spatial component of an arbitrary four vector V_μ.

Spin polarization of quarks and spin alignment of vector mesons

From the AKE: $\delta(p^2 - m^2)\Big(p\cdot\partial\tilde{a}^{s\mu}(p,X) - \partial_p^\kappa D_\kappa[\tilde{a}^{s\mu}] + \hbar\partial_p^\kappa\left(A^\mu_\kappa[f^{s}_V]\right)\Big) = 0$

• At weak coupling, given no initial polarization, $\tilde{a}^{s\mu}$ has to be induced by the dynamical source term $\hbar\partial_p^\kappa\left(A^\mu_\kappa[f^{s}_V]\right)$, which yields $\tilde{a}^{s\mu}\sim\mathcal{O}(g^2)$ and the diffusion term $\partial_p^\kappa D_\kappa[\tilde{a}^{s\mu}]$ is accordingly of $\mathcal{O}(g^4)$. Unless we consider the evolution for sufficiently long time, one may assume $p\cdot\partial\tilde{a}^{s\mu}(p,X)\gg\partial_p^\kappa D_\kappa[\tilde{a}^{s\mu}]$.

In this case solution is:

$$\tilde{a}^{s\mu}(p,X) = -\hbar\int_{k,X'}^{p,X}\partial_p^\kappa\left(A^\mu_\kappa[f^{s}_V](X')\right)$$

$$= -\frac{\hbar\bar{C}_2}{2}\epsilon^{\mu\nu\rho\sigma}\int_{k,X'}^{p,X}\partial_p^\kappa\int_{k',X''}^{p,X'}p^\lambda p_\rho\Big(\partial_{X''\sigma}\left(F^a_{\kappa\lambda}(X')F^a_{\alpha\nu}(X'')\right) + \partial_{X'\sigma}\left(F^a_{\kappa\nu}(X')F^a_{\alpha\lambda}(X'')\right)\Big)\partial_p^\alpha f^{s}_V(p,X'')$$

• Non-dynamical source term depends on the color-field correlator only at the late time when spin freezes out. Dynamical contribution comes by integrating the color-field correlator over a whole period before the spin freeze-out.

• Non-dynamical source term can be dropped as compared to dynamical source term.

Spin polarization of quarks and spin alignment of vector mesons

Thus the spin polarization of quarks can be evaluated via

$\langle \mathcal{P}_q^i \rangle \sim \langle \tilde{a}^{si}(p, X) \rangle$

Spin alignment results from the out of plane spin correlation

$\langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle \sim \langle \tilde{a}^{si}(p, X) \tilde{a}^{si}(p, Y) \rangle$

In the experimental set up, i is assigned as the spin quantization axis which is the direction of of angular momentum (y-direction).

Thus we have to evaluate, $\langle \mathcal{P}_q^y \mathcal{P}_{\bar{q}}^y \rangle \sim \langle \tilde{a}^{sy}(p, X) \tilde{a}^{sy}(p, Y) \rangle$

- Finally one has to evaluate the correlations of QCD field strength tensors

For practical applications, one has to convert the field strengths into chromo-electric and -magnetic fields, which are explicitly given by

$$F^a_{\mu\nu} = -\epsilon_{\mu\nu\alpha\beta} B^{a\alpha} \bar{n}^\beta + E^a_{[\mu} \bar{n}_{\nu]}, \quad \tilde{F}^{a\mu\nu} = B^{a[\mu} \bar{n}^{\nu]} + \epsilon^{\mu\nu\alpha\beta} E^a_\alpha \bar{n}_\beta,$$

where $\bar{n}^\mu = (1, \mathbf{0})$ denotes the temporal direction.

Color-field correlations in the glasma

We shall focus on the central rapidity regime, $\eta \to 0$. In this case the field correlations can be obtained following Ref.[P. Guerrero-Rodriguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)]

$$\begin{aligned}
\langle E_T^{ai}(X) B_T^{a'j}(X') \rangle &= 0, \quad \langle E^{az}(X) B^{a'z}(X') \rangle = 0, \\
\langle E_T^{ai}(X) E^{a'z}(X') \rangle &= 0, \quad \langle B_T^{ai}(X) B^{a'z}(X') \rangle = 0
\end{aligned}$$

The non-vanishing correlators

$$\langle E_T^{ai}(X') E_T^{a'j}(X'') \rangle = -\bar{N}_c \delta^{aa'} \epsilon^{in} \epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_-(u_\perp, v_\perp) \frac{q^n l^m}{ql} J_1(qX_0') J_1(lX_0'') \theta(X_0') \theta(X_0''),$$

$$\langle B_T^{ai}(X') B_T^{a'j}(X'') \rangle = -\bar{N}_c \delta^{aa'} \epsilon^{in} \epsilon^{jm} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_+(u_\perp, v_\perp) \frac{q^n l^m}{ql} J_1(qX_0') J_1(lX_0'') \theta(X_0') \theta(X_0''),$$

$$\langle E_T^{ai}(X') B^{a'z}(X'') \rangle = -i\bar{N}_c \delta^{aa'} \epsilon^{in} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_-(u_\perp, v_\perp) \frac{q^n}{q} J_1(qX_0') J_0(lX_0'') \theta(X_0') \theta(X_0''),$$

$$\langle B_T^{ai}(X') E^{a'z}(X'') \rangle = -i\bar{N}_c \delta^{aa'} \epsilon^{in} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_+(u_\perp, v_\perp) \frac{q^n}{q} J_1(qX_0') J_0(lX_0'') \theta(X_0') \theta(X_0''),$$

$$\langle E^{az}(X') E^{a'z}(X'') \rangle = \bar{N}_c \delta^{aa'} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_+(u_\perp, v_\perp) J_0(qX_0') J_0(lX_0'') \theta(X_0') \theta(X_0''),$$

$$\langle B^{az}(X') B^{a'z}(X'') \rangle = \bar{N}_c \delta^{aa'} \int_{\perp;q,u}^{X'} \int_{\perp;l,v}^{X''} \Omega_-(u_\perp, v_\perp) J_0(qX_0') J_0(lX_0'') \theta(X_0') \theta(X_0'').$$

- Where, $\bar{N}_c \equiv \frac{1}{2} g^2 N_c$, $\Omega_\mp(u_\perp, v_\perp) = [G_1(u_\perp, v_\perp) G_2(u_\perp, v_\perp) \mp h_1(u_\perp, v_\perp) h_2(u_\perp, v_\perp)]$
$\int_{\perp;q,u}^{X'} \equiv \int \frac{d^2 q_\perp}{(2\pi)^2} \int d^2 u_\perp \, e^{iq_\perp (X'-u)_\perp}$

Analyis with GBW kind of distribution

- We take the small-momentum limit $\hat{p}_\perp^\mu \equiv p_\perp^\mu/p_0 \ll 1$ for $p_0 = \epsilon_{\boldsymbol{p}} \equiv \sqrt{\boldsymbol{p}^2 + m^2}$ being onshell and adopt the Golec–Biernat Wusthof [GBW] distribution where $h_{1,2} = 0$ [P. Guerrero-Rodriguez and T. Lappi, Phys. Rev. D 104, 014011 (2021)]

$$\Omega_\pm(u_\perp, v_\perp) = \Omega(u_\perp, v_\perp) = \frac{Q_s^4}{g^4 N_c^2}\left(\frac{1-e^{-Q_s^2|u_\perp - v_\perp|^2/4}}{Q_s^2|u_\perp - v_\perp|^2/4}\right)^2 = \frac{Q_s^4}{g^4 N_c^2}\left(\frac{1-e^{-Q_s^2|s_\perp - t_\perp - r_\perp|^2/4}}{Q_s^2|s_\perp - t_\perp - r_\perp|^2/4}\right)^2.$$

- $r_\perp \equiv X_\perp - Y_\perp, \quad s_\perp = X_\perp - u_\perp, \quad t_\perp = Y_\perp - v_\perp.$

In this case, one can carry out the multidimensional integration's involved in the expressions. It can be shown $\langle \tilde{a}^{sx}(p,X)\rangle = 0,\ \langle \tilde{a}^{sy}(p,X)\rangle = 0,\ \langle \tilde{a}^{sz}(p,X)\rangle = 0$

- While one finds

$$\langle \tilde{a}^{sy}(p,X)\tilde{a}^{sy}(p,Y)\rangle \approx \left(\frac{\bar{C}_2}{2}p_0\left(\partial_{p0} f_V(p_0)\right)\right)^2 \frac{g^4 N_c^2(N_c^2-1)}{4(2\pi)^4 p_0^4}\,\frac{Q_s^8}{g^8 N_c^4}\,\frac{\hat{\mathcal{I}}(Q_s X_0, Q_s|r_\perp|, \theta_r)}{Q_s^2}$$

$$\hat{\mathcal{I}}(Q_s X_0, Q_s|r_\perp|, \theta_r) = \hat{\mathcal{I}}_1(Q_s X_0, Q_s|r_\perp|, \theta_r) + \hat{\mathcal{I}}_2(Q_s X_0, Q_s|r_\perp|, \theta_r) + \hat{\mathcal{I}}_3(Q_s X_0, Q_s|r_\perp|, \theta_r)$$

where $\theta_r \equiv \cos^{-1}(r_\perp^x/|r_\perp|)$ and $\hat{\mathcal{I}}'s$ are the eight-dimensional integrals. The first prefactor associated with $1/p_0^4$ comes during the simplification of I_1, I_2 and I_3 while $Q_s^8/(g^8 N_c^4)$ stems from the square of GBW distribution and the extra $1/Q_s^2$ is introduced to make $\hat{\mathcal{I}}(Q_s X_0, Q_s|r_\perp|, \theta_r)$ dimensionless.

Results

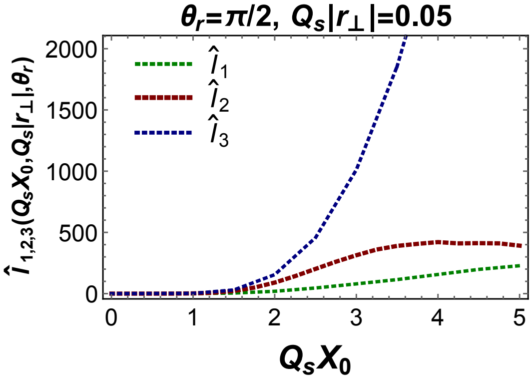


Figure: Numerical results for $\hat{\mathcal{I}}_{1,2,3}(Q_s X_0, 0.05, \pi/2)$. For computational convenience, we approximate $\hat{\mathcal{I}}_{1,2,3}(Q_s X_0, 0.05, \theta_r) \approx \hat{\mathcal{I}}_{1,2,3}(Q_s X_0^{\rm th}, 0, \theta_r)$.

Results

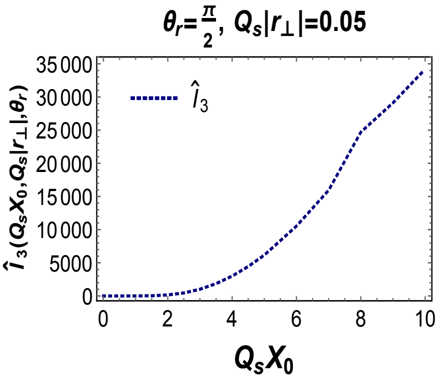


Figure: The Q_sX_0 dependence of $\hat{I}_3$ up to $Q_sX_0 = 10$.

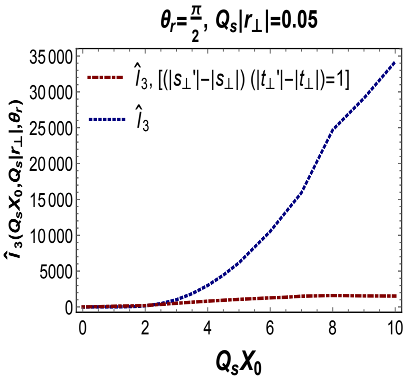


Figure: The origin of the rapid growth for $\hat{I}_3$.

Results

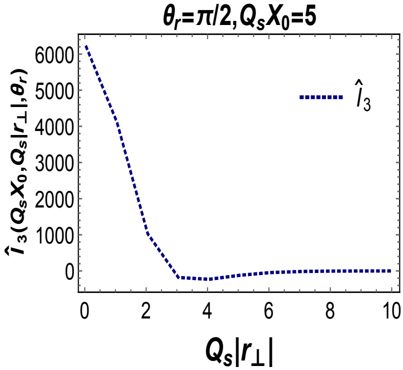


Figure: The $|r_\perp|$ dependence of $\hat{I}_3$.

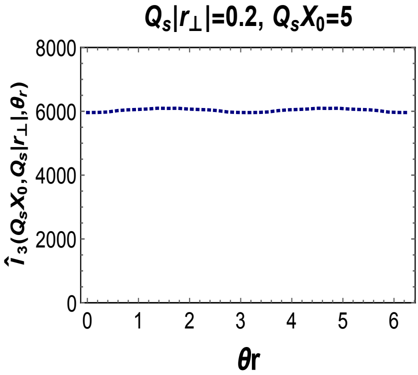


Figure: The θ_r dependence of $\hat{I}_3$ at small $Q_s|r_\perp|$.

Results

If freeze-out takes place at a constant value of the proper time, we can obtain,

$$d\Sigma_X \cdot p \approx X_0 \sqrt{m^2 + |p_\perp^2|} d^2 X_\perp d\eta$$

Finally can get,

$$\langle \mathcal{P}_q^i(\mathbf{p}) \mathcal{P}_{\bar{q}}^i(\mathbf{p}) \rangle \approx \frac{\int d^2 X_\perp \int d^2 Y_\perp \langle \tilde{a}^i(\boldsymbol{p}, X) \tilde{a}^i(\boldsymbol{p}, Y) \rangle}{4m^2 \left(\int d^2 X_\perp f_V(p_0) \right)^2} \Bigg|_{p_0 = \epsilon_{\boldsymbol{p}}} .$$

Change of coordinates,

$$\int d^2 X_\perp \int d^2 Y_\perp = \int d^2 r_\perp \int d^2 R_\perp, \quad R_\perp \equiv (X_\perp + Y_\perp)/2, \quad r_\perp = (X_\perp - Y_\perp)$$

Consequently, the out-of-plane spin correlation may be approximated as

$$\langle \mathcal{P}_q^y(\mathrm{p}) \mathcal{P}_{\bar{q}}^y(\mathrm{p}) \rangle \approx \left(\frac{\bar{C}_2 p_0 \left(\partial_{p_0} f_V(p_0) \right)}{2} \right)^2 \frac{(N_c^2 - 1)}{4(2\pi)^4 p_0^4 N_c^2} \frac{Q_s^6}{g^4} \frac{\int^{\mathrm{gl}} d^2 r_\perp \int^{\mathrm{gl}} d^2 R_\perp \hat{\mathcal{I}}(Q_s X_0, Q_s |r_\perp|, \theta_r)}{4m^2 \mathrm{A}_{\mathrm{T}}^2 f_{\mathrm{eq}}(p_0)^2} \Bigg|_{p_0 = \epsilon_{\boldsymbol{p}}}$$

$\int^{\mathrm{gl}} d^2 r_\perp \int^{\mathrm{gl}} d^2 R_\perp \rightarrow$ transverse plane of glasma

$\mathrm{A}_{\mathrm{T}} \rightarrow$ transverse area of the QGP, $f_{\mathrm{eq}}(p_0) = 1/(e^{p_0/T} + 1)$ corresponds to the thermal distribution function with T being the freeze-out temperature and m may be approximated as the constituent quark mass.

Results

From the numerical plot we have seen that dominant contribution is around $|r_\perp| \sim 0$. In this case we may postulate $\mathcal{I}(Q_s X_0, Q_s|r_\perp|, \theta_r) \sim \mathcal{I}(Q_s X_0, 0, 0) e^{-|r_\perp|^2 Q_s^2}$, Therefore, we may approximate

$$\int^{\mathrm{gl}} d^2 r_\perp \hat{\mathcal{I}}(Q_s X_0, Q_s|r_\perp|, \theta_r) \sim \pi Q_s^{-2} \hat{\mathcal{I}}(Q_s X_0, 0, 0)$$

Taking $\Lambda \sim Q_s \gg m \gg |\boldsymbol{p}|$ such that $\partial_{p0} f_V \approx -1/(4Q_s)$ and $\int^{\mathrm{gl}} d^2 R_\perp \approx \mathrm{A_N}$ with $\mathrm{A_N}$ being the transverse area of nuclei, we could further approximate

$$\boxed{\langle \mathcal{P}_q^y(\mathbf{p}) \mathcal{P}_{\bar{q}}^y(\mathbf{p}) \rangle \sim \frac{(N_c^2 - 1)(e^{m/T} + 1)^2}{N_c^4 (16\pi)^4 g^4} \frac{\pi Q_s^2 \mathrm{A_N}}{m^4 \mathrm{A_T^2}} \hat{\mathcal{I}}(Q_s X_0^{\mathrm{th}}, 0, 0)}$$

- Despite the actual value of $\hat{\mathcal{I}}(Q_s X_0^{\mathrm{th}}, 0, 0)$, the correlation could be enhanced by the factor Q_s^2/g^4 at weak coupling from higher collision energy.

Results

We then take $Q_s = 3$ GeV, $\alpha_s = g^2/(4\pi) \approx 1/3$, $N_c = 3$, $A_{\mathrm{N}} \sim A_{\mathrm{T}} \approx 100$ fm^2, $m \approx 500$ MeV as the constituent quark mass for strange quarks, and $T \approx 150$ MeV as the freeze-out temperature for numerical estimation.

By setting $X_0^{\mathrm{th}} \approx 0.5$ fm in the natural unit and correspondingly $Q_s X_0^{\mathrm{th}} \approx 7.5$, which yields $\hat{\mathcal{I}}(7.5, 0, 0) \approx \hat{\mathcal{I}}_3(7.5, 0, \pi/2) \approx 15000$ [can be noticed from the plot], we then obtain $\langle \mathcal{P}_q^y(\mathbf{p}) \mathcal{P}_{\bar{q}}^y(\mathbf{p}) \rangle \approx 0.004$.

Which is much larger than vorticity contribution ($\sim 10^{-6}$) at RHIC and ($\sim 10^{-8}$) LHC energies [F. Becattini, I. Karpenko, M. Lisa, I. Upsal, and S. Voloshin, Phys. Rev. C95, 054902 (2017), L. Adamczyk et al. (STAR), Nature 548, 62 (2017), S. Acharya et al. (ALICE), Phys. Rev. C 101, 044611 (2020)].

Summary and Outlook

- Using the quantum kinetic theory description for massive fermions with background color fields we derived the expressions for dynamical spin polarization and correlations generated from color fields.
- Using GBW kind of gluon distributions it turns out the dynamical spin polarization of quarks is zero but we have a non-vanishing spin correlation which is much higher than the vorticity contribution.

Outlook:

Spin alignment of vector mesons is determined by [Z.-T. Liang and X.-N. Wang, Phys. Lett. B 629, 20 (2005), Y.-G. Yang, R.-H. Fang, Q. Wang, and X.-N. Wang, Phys. Rev. C 97, 034917 (2018)]

$$\rho_{00} = \frac{1 - \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}{3 + \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}$$

Formula was derived based on the assumption that the spins of a quark and of an antiquark are fully polarized along the quantization axis. In a more generic case, the above formula gets modified Ref.[X.-L. Sheng, L. Oliva, Z.-T. Liang, Q. Wang, and X.-N. Wang (2022), 2206.05868]

$$\rho_{00} \approx \frac{1 + \sum_{j=x,y,z} \langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle - 2 \langle \mathcal{P}_q^i \mathcal{P}_{\bar{q}}^i \rangle}{3 + \sum_{j=x,y,z} \langle \mathcal{P}_q^j \mathcal{P}_{\bar{q}}^j \rangle}$$

- By symmetry of the color fields from glasma, $\langle \mathcal{P}_q^x(\mathbf{p}) \mathcal{P}_{\bar{q}}^x(\mathbf{p}) \rangle \sim \langle \mathcal{P}_q^y(\mathbf{p}) \mathcal{P}_{\bar{q}}^y(\mathbf{p}) \rangle$ at small momenta, while $\langle \mathcal{P}_q^z(\mathbf{p}) \mathcal{P}_{\bar{q}}^z(\mathbf{p}) \rangle$ has to be evaluated separately
- We have performed the calculations at small transverse momentum. It will be nice to perform the calculations to a more general case.

THANK YOU FOR YOUR ATTENTION

Integrals

$$\hat{\mathcal{I}}_1 \approx \int_{s,t,s',t'}^{\perp} \frac{\Theta(X_0 - |s_\perp|)\Theta(Y_0 - |t_\perp|)\Theta(|s_\perp| - |s'_\perp|)\Theta(|t_\perp| - |t'_\perp|)}{|s_\perp||t_\perp||s'_\perp||t'_\perp|}$$
$$\times\left[\hat{s}^x_\perp \hat{t}^x_\perp \hat{s}'^y_\perp \hat{t}'^y_\perp + \hat{s}^y_\perp \hat{t}^y_\perp \hat{s}'^x_\perp \hat{t}'^x_\perp - 2\hat{s}^x_\perp \hat{t}^y_\perp \hat{s}'^y_\perp \hat{t}'^x_\perp\right]\partial_{ux}\partial_{vx}\Omega_+(u_\perp, v_\perp)\Omega_-(u'_\perp, v'_\perp)$$

which could be evaluated numerically with given $\Omega_\pm$.

$$\hat{\mathcal{I}}_2 \approx \int_{s,t,s',t'}^{\perp} \frac{\Theta(X_0 - |s_\perp|)\Theta(Y_0 - |t_\perp|)\Theta(|s_\perp| - |s'_\perp|)\Theta(|t'_\perp| - |t_\perp|)}{|s_\perp||t_\perp||s'_\perp||t'_\perp|}$$
$$\times\left(|t_\perp| - |t'_\perp|\right)\hat{s}^{[x}_\perp \hat{s}'^{y]}_\perp\left(\hat{t}'^y_\perp \partial_{vx} - \hat{t}'^x_\perp \partial_{vy}\right)\partial_{ux}\partial_{vx}\Omega_-(v'_\perp, u'_\perp)\Omega_+(u_\perp, v_\perp)$$

and

$$\hat{\mathcal{I}}_3 \approx -\int_{s,t,s',t'}^{\perp} \frac{\Theta(X_0 - |s_\perp|)\Theta(Y_0 - |t_\perp|)\Theta(|s_\perp| - |s'_\perp|)\Theta(|t_\perp| - |t'_\perp|)}{|s_\perp||t_\perp||s'_\perp||t'_\perp|}$$
$$\times\left(|s'_\perp| - |s_\perp|\right)\left(|t'_\perp| - |t_\perp|\right)\left[\hat{s}^y_\perp \hat{t}^y_\perp \partial^2_{u'x}\partial^2_{v'x} - 2\hat{s}^y_\perp \hat{t}^x_\perp \partial^2_{u'x}\partial_{v'y}\partial_{v'x}\right.$$
$$\left.+\hat{s}^x_\perp \hat{t}^x_\perp \partial_{u'x}\partial_{u'y}\partial_{v'x}\partial_{v'y}\right]\Omega_-(u_\perp, v_\perp)\Omega_+(u'_\perp, v'_\perp)$$

Axial Current

The axial current for quarks

$$J_5^\mu(X) = 4\int \frac{d^4p}{(2\pi)^4}\mathrm{tr_c}(\mathcal{A}^\mu) = 4N_c\int \frac{d^4p}{(2\pi)^4}(2\pi\delta(p^2-m^2))\big(\tilde{a}^{s\mu} + \hbar\bar{C}_2\mathcal{A}_Q^\mu\big)$$
$$= \int \frac{d^4p}{(2\pi)^4}(2\pi\delta(p^2-m^2))\mathcal{J}_5^\mu(p,X)$$

with

$$\mathcal{J}_5^\mu(p,X) = 4N_c\big(\tilde{a}^{s\mu} + \hbar\bar{C}_2\mathcal{A}_Q^\mu\big)$$

The quatity $\mathcal{N}^\mu(p,X)$ is

$$\mathcal{N}^\mu(p,X) = 4N_c p^\mu f_V^s(p,X)$$

• In the small-momentum limit $\hat{p}^{\mu}_{\perp} \equiv p^{\mu}_{\perp}/p_0 \ll 1$ for $p_0 = \epsilon_{\boldsymbol{p}} \equiv \sqrt{\boldsymbol{p}^2 + m^2}$ being onshell. It turn out

$$\langle \tilde{a}^{sy}(p,X)\rangle \approx -\left(\frac{\bar{C}_2}{2}p_0\left(\partial_{p0}f_V(p_0)\right)\right)\int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\Big[\partial_{X''0}\left\langle E^a_{[1}(X')E^a_{3]}(X'')\right\rangle + (X''_0 - X'_0)\Big(\partial^2_{X''1}\left\langle E^a_1(X')E^a_3(X'')\right\rangle + \partial_{X''2}\partial_{X''1}\left\langle E^a_2(X')E^a_3(X'')\right\rangle\Big)\Big]$$

From the above defined field correlator, concludes $\langle \tilde{a}^{sy}(p,X)\rangle = 0$. By symmetry argument it can also shown $\langle \tilde{a}^{sx}(p,X)\rangle = 0$

• Not clear for the case of $\langle \tilde{a}^{sz}(p,X)\rangle$.

$$\boxed{\langle \tilde{a}^{sz}(p,X)\rangle \approx -\left(\frac{\bar{C}_2}{2}p_0\left(\partial_{p0}f_V(p_0)\right)\right)\left(\mathcal{A}_a + \mathcal{A}_b + \mathcal{A}_c\right)}$$

$$\mathcal{A}_a = \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\partial_{X''0}\langle E^a_{[2}(X')E^a_{1]}(X'')\rangle$$

$$\mathcal{A}_b = -\int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\left(\partial^1_{X'}\langle B^a_{[3}(X')E^a_{1]}(X'')\rangle + \partial^2_{X'}\langle B^a_{[3}(X')E^a_{2]}(X'')\rangle\right)$$

$$\mathcal{A}_c = \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}(X''_0 - X'_0)\left(\partial^1_{X''}\partial_{X''[1}\langle E^a_1(X')E^a_{2]}(X'')\rangle + \partial^2_{X''}\partial_{X''[1}\langle E^a_2(X')E^a_{2]}(X'')\rangle\right)$$

- Although, $\langle \tilde{a}^{sy}(p,X)\rangle = 0$, the quantity $\langle \tilde{a}^{sy}(p,X)\tilde{a}^{sy}(p,Y)\rangle$ which contribute to spin alignment of will be non-zero. It turns out that

$$\boxed{\langle \tilde{a}^{sy}(p,X)\tilde{a}^{sy}(p,Y)\rangle \approx \left(\frac{\bar{C}_2}{2}p_0\left(\partial_{p0}f_V(p_0)\right)\right)^2\left(\mathcal{I}_1+\mathcal{I}_2+\mathcal{I}_3\right)},$$

$$\mathcal{I}_1 = \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\int_{\bar{k},Y'}^{p,Y}\int_{\bar{k}',Y''}^{p,Y'}\partial_{X'1}\partial_{Y'1}\left\langle B^{a[2}(X')E^{a1]}(X'')B^{b[2}(Y')E^{b1]}(Y'')\right\rangle$$

$$\begin{aligned}\mathcal{I}_2 \quad = \quad & -2\int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\int_{\bar{k},Y'}^{p,Y}\int_{\bar{k}',Y''}^{p,Y'}(Y_0''-Y_0')\Big(\partial_{X'1}\partial^2_{Y''1}\left\langle B^{a[2}(X')E^{a1]}(X'')E_1^b(Y')E_3^b(Y'')\right\rangle\\ & +\partial_{X'1}\partial_{Y''2}\partial_{Y''1}\left\langle B^{a[2}(X')E^{a1]}(X'')E_2^b(Y')E_3^b(Y'')\right\rangle\Big)\end{aligned}$$

$$\begin{aligned}\mathcal{I}_3 \quad = \quad & \int_{k,X'}^{p,X}\int_{k',X''}^{p,X'}\int_{\bar{k},Y'}^{p,Y}\int_{\bar{k}',Y''}^{p,Y'}(X_0''-X_0')(Y_0''-Y_0')\Big(\partial^2_{X''1}\partial^2_{Y''1}\left\langle E_1^a(X')E_3^a(X'')E_1^b(Y')E_3^b(Y'')\right\rangle\\ & +2\partial^2_{X''1}\partial_{Y''2}\partial_{Y''1}\left\langle E_1^a(X')E_3^a(X'')E_2^b(Y')E_3^b(Y'')\right\rangle\\ & +\partial_{X''2}\partial_{X''1}\partial_{Y''2}\partial_{Y''1}\left\langle E_2^a(X')E_3^a(X'')E_2^b(Y')E_3^b(Y'')\right\rangle\Big)\end{aligned}$$

- It is found that the non-vanishing spin correlation stems from the transverse spatial derivatives upon color-field correlators.

Similarly to Wick-theorem like decomposition four field correlations can be written in terms of two field correlations using the following formula

$$\begin{aligned}\left\langle \alpha_1^a(X')\alpha_2^a(X'')\alpha_3^b(Y')\alpha_4^b(Y'')\right\rangle &= \left\langle \alpha_1^a(X')\alpha_2^a(X'')\right\rangle \left\langle \alpha_3^b(Y')\alpha_4^b(Y'')\right\rangle \\ &+ \left\langle \alpha_1^a(X')\alpha_3^b(Y')\right\rangle \left\langle \alpha_2^a(X'')\alpha_4^b(Y'')\right\rangle \\ &+ \left\langle \alpha_1^a(X')\alpha_4^b(Y'')\right\rangle \left\langle \alpha_2^a(X'')\alpha_3^b(Y')\right\rangle\end{aligned}$$