Heavy quark transport within viscous quark-gluon plasma

Adiba Shaikh

Indian Institute of Technology Bombay

In collaboration with

M. Kurian (McGill U)	V. Chandra (IIT GN)	S. K. Das (IIT Goa)	S. Dash, B. K. Nandi (IIT Bombay)

8th International Conference on Physics and Astrophysics of Quark Gluon Plasma (ICPAQGP-2023)

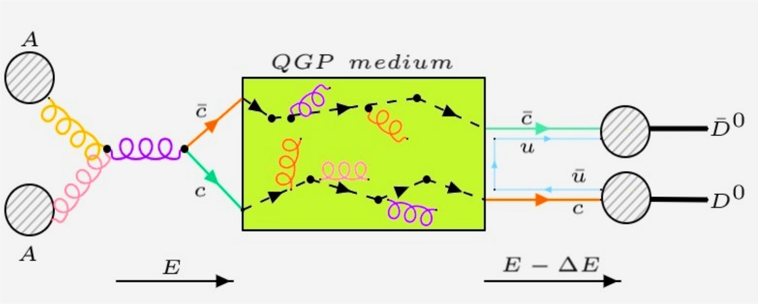
7-10 February 2023, Puri, Odisha, India

Variable Energy Cyclotron Centre, Kolkata, India

Introduction & Motivation

Heavy quarks (HQ) in Heavy-ion collision (HIC)

- $c\bar{c}/b\bar{b}$ pairs perturbatively created at early times in HIC experiments. $(\boldsymbol{m_{HQ}} >> \Lambda_{\boldsymbol{QCD}} \implies \alpha_{\boldsymbol{s}} << \mathbf{1})$
- Important probes to study QGP properties. $(\boldsymbol{m_{HQ}} >> \boldsymbol{T})$
- Charm quark moving in QGP loose its energy.
  1. Collision $(\mathbf{2} \to \mathbf{2})$: Elastic scattering with the medium constituents
  2. Radiation $(\mathbf{2} \to \mathbf{3})$: Medium induced gluon emission



HQ transport

Collisional energy loss

- Non-equilibrated heavy quark traversing equilibrated plasma.
- Brownian motion of heavy quark within QGP medium
- *Boltzmann transport equation* for the phase space density $f(\mathbf{x}, \mathbf{p}, t)$ of the heavy quark.
- Homogeneous plasma ($\partial f / \partial \mathbf{x} = \mathbf{0}$) with no external force ($\mathbf{F} = 0$)

B. Svetitsky, Phys. Rev. D, 37(9), 1988

$$\left(\frac{\partial}{\partial t} + \frac{\mathbf{p}}{E_p} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \frac{\partial}{\partial \mathbf{p}} \right) f(\mathbf{x}, \mathbf{p}, t) = \left(\frac{\partial f}{\partial t} \right)_{col} \implies \frac{\partial f(\mathbf{p}, t)}{\partial t} = \left(\frac{\partial f}{\partial t} \right)_{col}$$

- Landau's soft scattering approximation (small momentum transfer) $\implies$ *Fokker-Planck equation*

$$\frac{\partial f}{\partial t} \approx \frac{\partial}{\partial p_i} \left(A_i(\mathbf{p}) f + \frac{\partial}{\partial p_j} [B_{ij}(\mathbf{p})] f \right)$$

HQ transport

Collisional energy loss

- A_i and B_{ij} depends only on the initial momentum $(\mathbf{p})$
  $\rightarrow$ **Drag:**
$$A_i = p_i A(p^2)$$
$$\boxed{A(p^2) = \frac{p_i A_i}{p^2} = \langle \mathbf{1} \rangle - \frac{\langle \mathbf{p} \cdot \mathbf{p}' \rangle}{p^2}}$$
  $\rightarrow$ **Diffusion:** $B_{ij} \equiv \frac{1}{2}\langle (p-p')_i (p-p')_j \rangle$
$$B_{ij} = \left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_0(p^2) + \left(\frac{p_i p_j}{p^2}\right) B_1(p^2)$$

Transverse $\rightarrow$ $\boxed{B_0(p^2) = \frac{1}{2}\left(\delta_{ij} - \frac{p_i p_j}{p^2}\right) B_{ij} = \frac{1}{4}\left[\langle p'^2 \rangle - \frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2}\right]}$

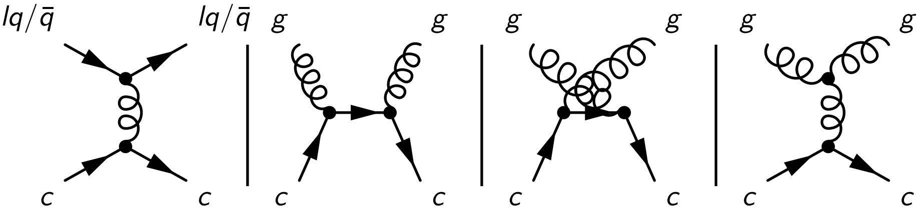
Longitudinal $\rightarrow$ $\boxed{B_1(p^2) = \left(\frac{p_i p_j}{p^2}\right) B_{ij} = \frac{1}{2}\left[\frac{\langle (\mathbf{p} \cdot \mathbf{p}')^2 \rangle}{p^2} - 2\langle \mathbf{p} \cdot \mathbf{p}' \rangle + p^2 \langle \mathbf{1} \rangle\right]}$

HQ transport

Collisional energy loss

- $HQ(p) \;+\; lq/l\bar{q}/g(q) \rightarrow HQ(p') \;+\; lq/l\bar{q}/g(q')$

$$\langle F(p)_{col} \rangle = \frac{1}{16(2\pi)^5 E_p \gamma_{HQ}} \int \frac{d^3q}{E_q} \int \frac{d^3q'}{E_{q'}} \int \frac{d^3p'}{E_{p'}} \sum |\mathcal{M}|^2_{2\to 2}\, \delta(p+q-p'-q') \times f(E_q)\, (1 \pm f(E'_q))\, F(p)$$



HQ transport coefficients

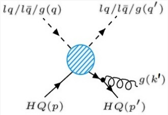
Radiative Energy Loss

- $HQ(p) \; + \; lq/l\bar{q}/g(q) \rightarrow HQ(p') \; + \; lq/l\bar{q}/g(q') \; + \; g(k')$

$$\langle F(p)_{rad}\rangle = \langle F(p)_{col}\rangle \times \int \frac{d^3k'}{(2\pi)^3 E_{k'}} \frac{12 g_s^2}{{k'_\perp}^2} \left(1 + \frac{m_{HQ}^2}{s} e^{2y_{k'}}\right)^{-2} \delta(p+q-p'-q'-k') \times (1+f(E_{k'}))\, \theta(E_p - E_{k'})\, \theta(\tau - \tau_f)$$

S. Mazumdar *et al.*, Phys. Rev. D 89, 014002 (2014)

- Soft gluon emission by the charm quark induced by the QGP medium (after scattering by light quarks, antiquarks and gluons).
- For soft gluon emission, $k' = (E_{k'}, \mathbf{k}'_\perp, k'_z) \rightarrow 0$,

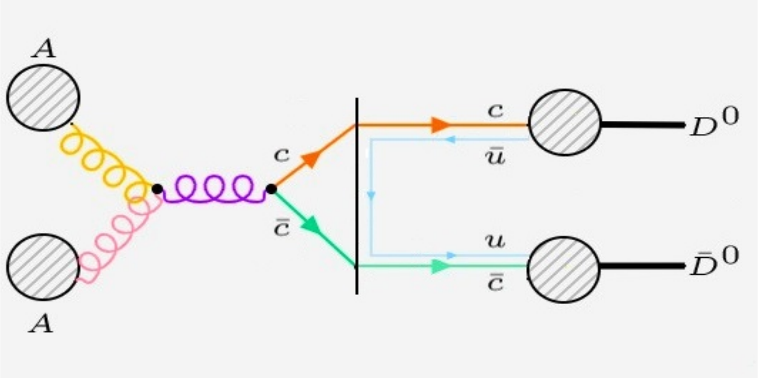


$$|\mathcal{M}|^2_{2\rightarrow 3} = |\mathcal{M}|^2_{2\rightarrow 2} * \frac{12 g_s^2}{{k'_\perp}^2} \left(1 + \frac{m_{HQ}^2}{s} e^{2y_{k'}}\right)^{-2}$$

R. Abir et al., Phys. Rev. D 85, 054012 (2012)

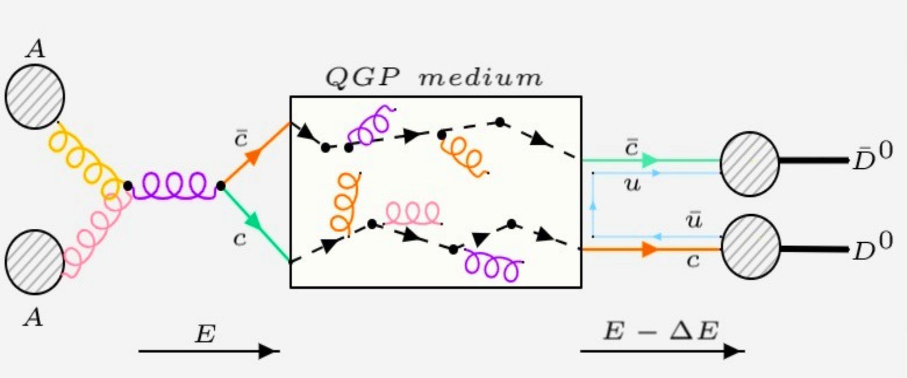
No QGP

Charm quark transport in vacuum



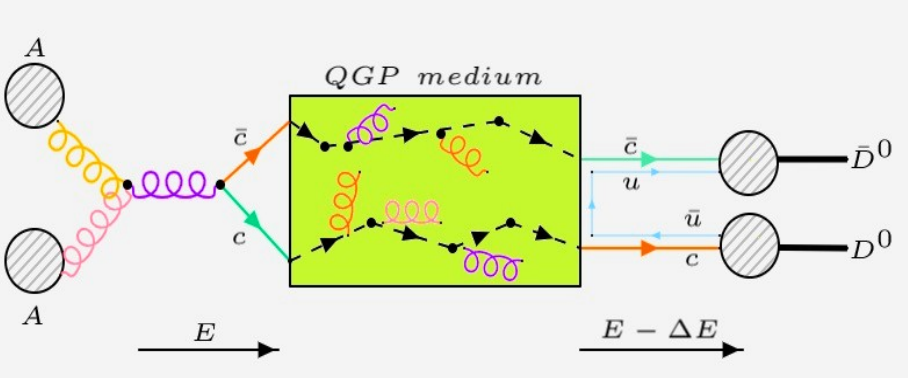
Ideal QGP

Charm quark transport in non-interacting medium



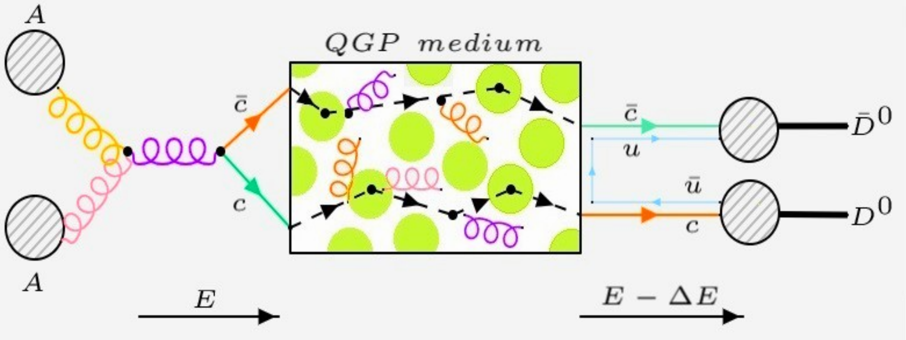
Deviation from Ideal case

Charm quark transport within thermally interacting QGP



Deviation from Ideal case

Charm quark transport with thermally interacting QGP (EQPM)



[i] Thermal medium interaction: EQPM

- **EQPM**: **E**ffective fugacity **Q**uasi-**P**article **M**odel (lattice QCD EoS based)

V. Chandra *et al.*, **Phys. Rev. C 76, 054909 (2007)**

- In-medium interactions of QGP encoded into particle: **quasiparticle**
- Introduction of temperature dependent effective fugacity z_k in the distribution functions of quasiparticle $k \equiv (lq, l\bar{q}, g)$.

$$f_k^0 = \frac{\mathbf{z_k}\ e^{-E_k/T}}{1 \pm \mathbf{z_k}\ e^{-E_k/T}}$$

- Quasiparticle dispersion relation: $\tilde{q}_k{}^\mu = q_k^\mu + \delta\omega_k\, u^\mu$
- Collective excitations of quasipartons: $\delta\omega_k = T^2\, \partial_T \{\ln(z_k)\}$
- Effective strong coupling constant $\alpha_{s(eff)}$ is introduced through EQPM based Debye mass.

$$\alpha_{s(eff)}(T) = \alpha_s(T) \frac{\left\{ \frac{2N_c}{\pi^2} \text{PolyLog}[2, z_g] - \frac{2N_f}{\pi^2} \text{PolyLog}[2, -z_q] \right\}}{\left\{ \frac{N_c}{3} - \frac{N_f}{6} \right\}}$$

S. Mitra *et al.*, **Phys. Rev. D 96, 094003 (2017)**

[ii] Viscous hydrodynamic corrections

- Leading order shear and bulk viscous corrections to the (anti)quark and gluon distribution function obtained by solving the effective kinetic theory.

S. Bhadury *et al.*, J. Phys. G 47 (2020) 8, 085108

- Energy-momentum tensor for the dissipative (viscous) hydrodynamics,

$$T^{\mu\nu} = \varepsilon u^{\mu} u^{\nu} - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$$

- Quasiparticle distribution function near local thermal equilibrium,

$$f_k = f_k^0 + \delta f_k \text{ where } \delta f_k / f_k^0 << 1$$

- Boost-invariant Bjorken (longitudinal) expansion of the fluid.
- LO viscous corrections to the quasiparticle thermal distribution function.
  - By solving the relativistic Boltzmann equation with RTA using Chapman-Enskog method,

$$\delta f_k = f_k^0(1 \pm f_k^0)\{\phi_k(bulk)^{(1)} + \phi_k(shear)^{(1)}\}$$

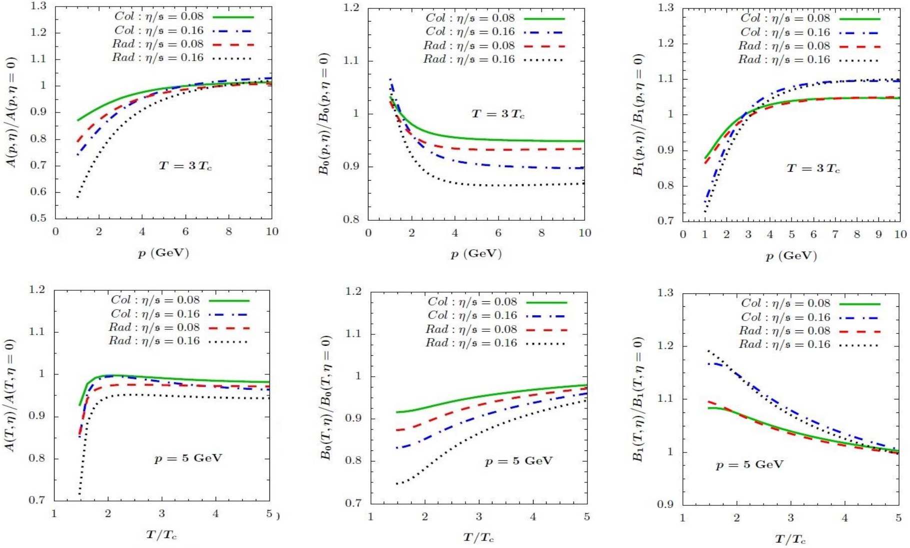
$$\phi_k(bulk)^{(1)} = \frac{s}{\beta_{\Pi}\omega_k T\tau}\left(\frac{\zeta}{s}\right)\left[\omega_k^2 c_s^2 - \frac{|\overrightarrow{q_k}|^2}{3} - \omega_k \delta\omega_k\right]$$

$$\phi_k(shear)^{(1)} = \frac{s}{\beta_{\pi}\omega_k T\tau}\left(\frac{\eta}{s}\right)\left[\frac{|\overrightarrow{q_k}|^2}{3} - (q_k)_z^2\right]$$

A. Shaikh *et al.*, Phys. Rev. D 104, 034017 (2021)

Results for **shear** viscous correction (i. Transport Coefficients)

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

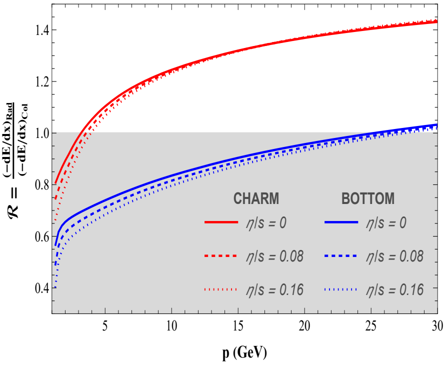


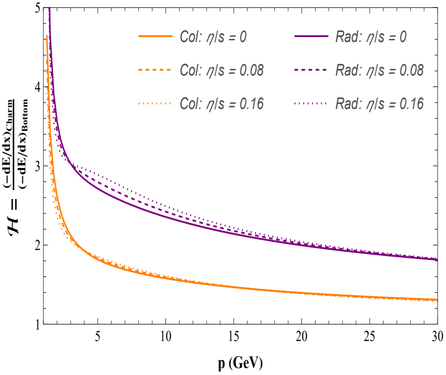
Results for **shear** viscous correction (ii. Energy Loss)

$$N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}$$

$$m_b = 4.2\,\text{GeV}$$

Differential energy loss: $-\dfrac{dE}{dx} = p\,A(p, T)$

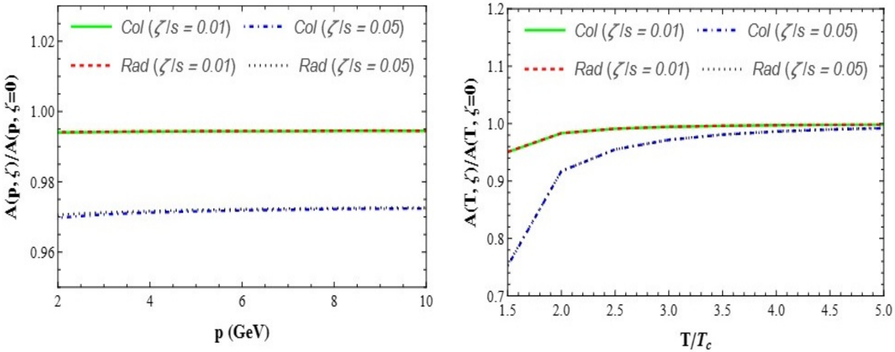




A. Shaikh, S. Dash, B. K. Nandi, [arXiv: 2302.02235]

Results for **bulk** viscous correction

$$\boxed{N_c = N_f = 3, \quad m_{lq} = \mu_{lq} = 0, \quad m_c = 1.3\,\text{GeV}, \quad T_c = 170\,\text{MeV}, \quad \tau = 0.25\,\text{fm}}$$



A.Shaikh *et al.*, **PoS CHARM2020 (2021) 060**

Second order shear viscous correction

$$\delta f(shear) = f^0(1 \pm f^0)\{\phi^{(1)} + \phi^{(2)}\}$$

General Case

$$\begin{aligned}
\phi^{(2)} = \frac{\beta}{\beta_\pi} \Bigg[& \frac{5}{14\beta_\pi (u\cdot p)} p^\alpha p^\beta \pi^\gamma_\alpha \pi_{\beta\gamma} \\
& - \frac{\tau_\pi}{(u\cdot p)} p^\alpha p^\beta \pi^\gamma_\alpha \omega_{\beta\gamma} - \frac{(u\cdot p)}{70\beta_\pi} \pi^{\alpha\beta}\pi_{\alpha\beta} \\
& + \frac{6\tau_\pi}{5} p^\alpha \dot{u}^\beta \pi_{\alpha\beta} - \frac{\tau_\pi}{5} p^\alpha (\nabla^\beta \pi_{\alpha\beta}) \\
& - \frac{\tau_\pi}{2(u\cdot p)^2} p^\alpha p^\beta p^\gamma (\nabla_\gamma \pi_{\alpha\beta}) \\
& + \frac{3\tau_\pi}{(u\cdot p)^2} p^\alpha p^\beta p^\gamma \pi_{\alpha\beta} \dot{u}_\gamma \\
& - \frac{\tau_\pi}{3(u\cdot p)} p^\alpha p^\beta \pi_{\alpha\beta} \theta \\
& + \frac{\beta + (u\cdot p)^{-1}}{4(u\cdot p)^2 \beta_\pi} (p^\alpha p^\beta \pi_{\alpha\beta})^2
\end{aligned}$$

For Bjorken (1D) expansion

$(\omega_{\mu\nu} = \dot{u}_\mu = 0)$

$$\begin{aligned}
\phi^{(2)} = \frac{s^2}{T\beta_\pi^2 \tau^2} \left(\frac{\eta}{s}\right)^2 \Bigg[& -\left(\frac{10}{63}\right) \frac{|\vec{q}|^2 + 3q_z^2}{E} \\
& - \left(\frac{4}{105}\right) E + \left(\frac{4}{15}\right) E - \left(\frac{4}{3}\right) \frac{q_z^2}{E} \\
& - \left(\frac{2}{3}\right) \frac{|\vec{q}|^2 - 3q_z^2}{3E} \\
& + \left(\frac{1}{T} + \frac{1}{E}\right) \left(\frac{|\vec{q}|^2 - 3q_z^2}{3E}\right)^2
\end{aligned}$$

C. Chattopadhyay *et al.*, Phys. Rev. C 91, 024917 (2015)

Conclusion

1. Heavy quark transport coefficients is studied in a **viscous QCD** medium with **collisional and radiative processes**.
2. The thermal medium interactions are incorporated using **EQPM** and the **first-order shear and bulk viscous corrections** are included in the distribution function of the quasiparticles.
3. **Shear viscous corrections** are substantial for slow-moving HQ ($p \approx 1 - 2$ GeV at $T = 3T_c$) where the increase in η/s decreases the drag coefficient (exception: B_0 vs p and B_1 vs T).
4. **Bulk viscous corrections** are prominent near transition temperature ($T \approx 1.5T_c$).
5. The **transition from collisional to radiative dominance** of energy loss mechanism for charm quark occurs at almost one order of magnitude less in initial momentum as compared with the bottom quark.
6. The effect of the **second-order viscous corrections** on the HQ transport coefficients is in progress.

Thank you !