Unpinning of Superfluid Vortices through (quasi) Neutron-Vortex Scattering and Pulsar Glitches

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Based on: *Glitches due to quasineutron-vortex scattering in the superfluid inner crust of a pulsar*; Biswanath Layek, Deepthi Godaba Venkata and Pradeepkumar Yadav; Physical Review D 107, 023004 (2023).

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Plan

Introduction: Pulsar Glitches

The pulsar-based time-scale is comparable to that of atomic clocks.

Pulsar Glitches: Sudden increase of rotational frequency[1].

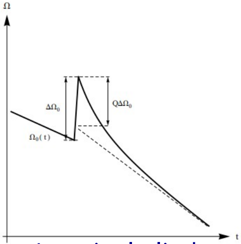

Figure: A typical glitch pattern of a pulsar.

Credit: https://minerva-access.unimelb.edu.au/handle/11343/36537.

- About 666 glitches have been observed in 208 pulsars.
- For example:
  - 30 glitches have been observed in Crab pulsar.
  - 24 glitches have been observed in Vela pulsar.
- Typical interglitch time is of few years.
- Glitch sizes $(\frac{\Delta\Omega}{\Omega}) \sim 10^{-11} - 10^{-6}$.

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[1] Updated catalogue of pulsar glitches : *h*ttp://www.jb.man.ac.uk/pulsar/glitches/gTable.html

Crustquake Model of Glitches

- Oblateness, $\epsilon(t) = \frac{I_{zz} - I_{xx}}{I_0}$
- Glitch size, $\frac{\Delta\Omega}{\Omega} = -\frac{\Delta I}{I_0} = |\Delta\epsilon|$
  [*M. Ruderman, Nature (1969)*].
- But, the interglitch time is also proportional to $\Delta\epsilon$
  [*Baym and Pines, Annals of Physics (1971)*].

  Therefore, a larger glitch needs a longer waiting time, contrary to the observations.

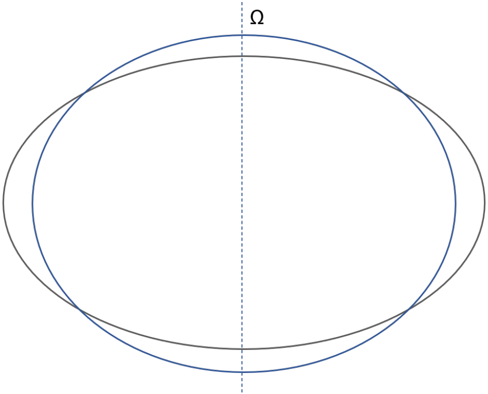

Figure: Demonstration of change in oblateness

Structure of Neutron Star

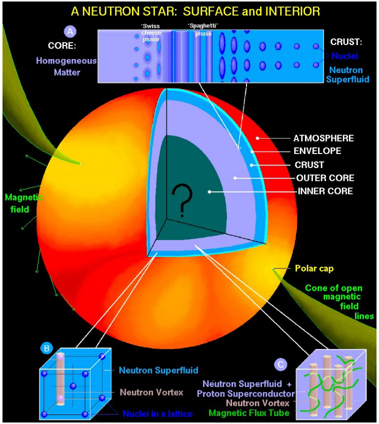

Figure: Structure of a neutron star.

Credit: Dany P Page

- Free neutrons in the inner crust are in a superfluid state.
- Vortex areal density $n_v = 4m_n\Omega/h \simeq 10^7 \mathrm{m}^{-2}(\Omega/\mathrm{s}^{-1})$.
- Vortices get pinned to irregularities, lattice points in NS's case.
- The superfluid's angular momentum, $I_f\Omega_p$ remains unchanged when pinned.
- Rest of the star corotates, with angular momentum $I_c\Omega_c(t)$, and slows down over time.

Vortex Unpinning Model

- Anderson suggested unpinning the vortices pinned by the magnetic flux tubes to nuclear sites in the inner crust's lattices as the cause of glitches [*Anderson & Itoh, Nature (1975)*].
- Assume at $t = 0$ vortices are pinned and $\Omega_c(t = 0) = \Omega_p$. A fraction of these vortices $\left(\frac{N_v}{N_{vt}}\right)$ get unpinned at $t = t_p$, resulting in pulsar glitches [*Layek & Yadav, MNRAS (2020)*].

$$\frac{\Delta\Omega}{\Omega} = \left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{N_v}{N_{vt}}\right). \tag{1}$$

Here, $\left(\frac{I_f}{I_c}\right)$ is the MI ratio of the bulk superfluid component and the rest of the star, and, $\tau = -(\Omega/2\dot{\Omega})$, is the characteristic age of the pulsar.

- The unpinning model needs a trigger mechanism.

Vortex Unpinning Through Neutron-Vortex Scattering

- The total energy of a deformed pulsar [*Baym and Pines, Annals of Physics (1971)*],

$$E = E_0 + \frac{L^2}{2I} + A\epsilon^2 + B(\epsilon - \epsilon_0)^2 \tag{2}$$

- Strain energy, $\Delta E = B\Delta\epsilon$ released through crustquake partially absorbed in cylindrical shell
- Thermally breaks some fraction of neutron Cooper pairs

excited neutron ($\sim E_f$) + *pinned vortex* ($-E_P$) $\rightarrow$ *de-excited neutron* ($E_f - E_p$) + *free vortex.*

- The unpinned vortex moves outwards.
- Unpins other vortices through knock-on process, initiating an avalanche.

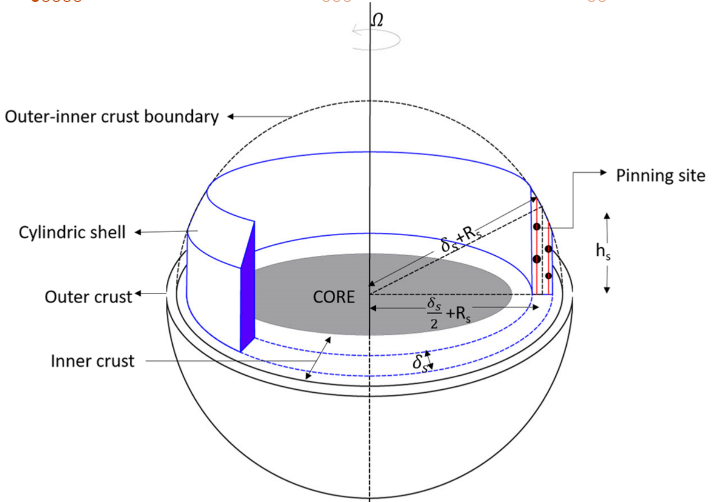

Figure: The vortex lines terminating on **the outer-inner crust boundary** defines the (average) height of the the affected pinning site (cylindrical shell).

Mathematical Tools

From the Vortex Unpinning model,

$$\frac{\Delta\Omega}{\Omega} = \left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{N_v}{N_{vt}}\right) = \left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{A_s}{2\pi R \Delta R}\right). \tag{3}$$

Mathematical Tool for finding the Volume of Affected Region

By energy balance

$$B\Delta\epsilon = N_e\Delta_f = \frac{\Delta_f^2}{E_f} n_f V_s, \tag{4}$$

Where,

B ($\sim 10^{48}$ erg) is a constant related to the modulus of rigidity of the crust,
$\Delta\epsilon = 10^{-8}$ for a typical one-year inter-glitch time,
$n_f \rightarrow$ Number density of the bulk superfluid neutrons,
$\Delta_f \rightarrow$ Energy gap parameter,
$E_f \rightarrow$ Fermi energy,
$N_e \rightarrow$ Number of excited neutrons produced from Cooper-pair breaking.

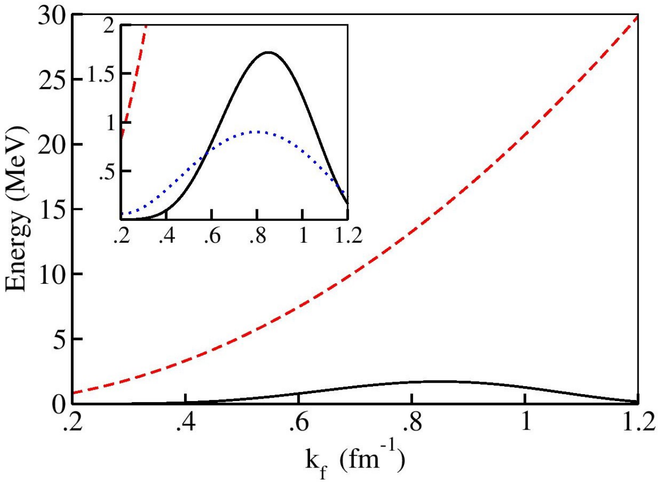

Figure: Excitation energy E_f ,the gap parameter $\Delta_f(k_f)$ and **pinning energy per site** E_p as a function of Fermi momentum

THERMALLY EFFECTED REGION	CYLINDRICAL SHELL
VOLUME	$h_s \simeq (R_s\delta_s)^{1/2}$ $A_s = 2\pi(R_s\delta_s)$ $V_s = 2\pi(R_s\delta_s)^{3/2}$
$N_v = A_s n_v$	$\frac{V_s n_v}{h_s} = \frac{B\Delta\epsilon E_f n_v}{n_f \Delta_f^2 (R_s\delta_s)^{1/2}}$
$\frac{\Delta\Omega}{\Omega}$	$\left(\frac{I_f}{I_c}\right)\left(\frac{t_p}{2\tau}\right)\left(\frac{R_s}{R}\right)\left(\frac{\delta_s}{\Delta R}\right) \simeq 10^{-6}\left(\frac{\delta_s}{\Delta R}\right)$

Results

Table: Fermi momentum (k_f), the distance of the cylindrical shell from the center (R_s), thickness of the shell (δ_s), the number of unpinned vortices (N_v), and the order of magnitude of the glitch size ($\Delta\Omega/\Omega$) for the Vela pulsar through local unpinning by excited neutrons.

k_f (fm^{-1})	R_s (in km)	δ_s (in meter)	N_v	$(\frac{\Delta\Omega}{\Omega})$
0.2	10.3	0.87	3.9×10^{13}	8.7×10^{-10}
0.8	10.2	0.01	4.3×10^{11}	1×10^{-11}
1.2	9.9	0.05	2.0×10^{12}	5×10^{-11}

Results

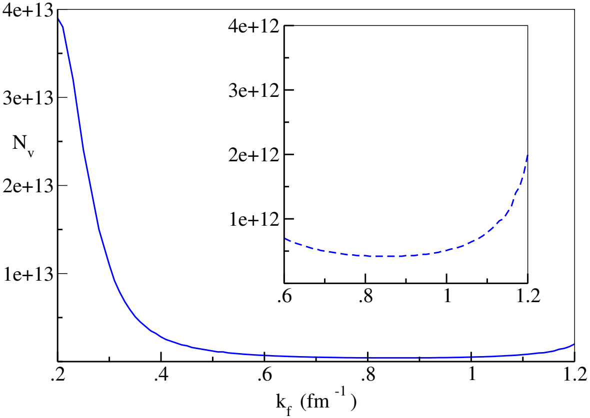

Figure: The number of unpinned vortices (N_v) versus the Fermi momentum k_f.

Result

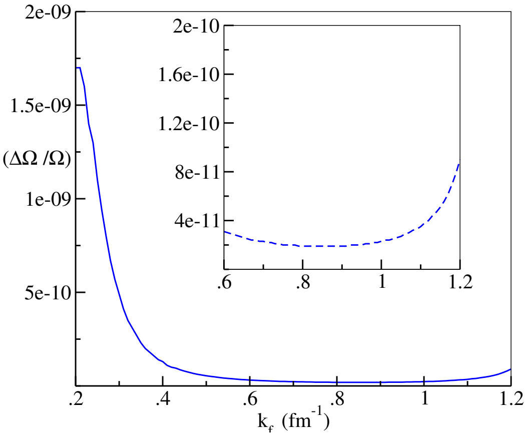

Figure: The typical glitch size for a Vela-like pulsar caused by the local unpinning.

Vortex avalanche through proximity knock-on

- Instantaneous release of ($\sim 10^{18}$) is necessary for explaining ($\frac{\Delta\Omega}{\Omega} \sim 10^{-6}$).
- Proximity knock-on:
  When intervortex distance, $d_v \to \eta d_v$ ($\eta < 1$),the unpinning probability for a single vortex can be written as

$$\lambda \tau_{tr} \propto e^{\frac{E_p}{\eta d_v}} \tag{5}$$

- Multiple, $N^i_{tr} = \delta_s \sqrt{n_v}$, triggers (in 1D) since a large number of vortices are unpinned from cylinderical shell region.

$$\lambda_{tr} \tau_{tr} \propto N_{tr} e^{\frac{E_p}{\eta d_v}} \tag{6}$$

where, N_{tr} is the cumulative number of triggers.

Vortex Avalanche

Table: A Few values of the Fermi momentum k_f at which the unpinning probability for a single vortex is enhanced due to multiple triggers.

k_f (fm^{-1})	$\lambda\tau_{tr}$	$\lambda_{tr}\tau_{tr}(N_{tr} = N^i_{tr})$
0.52	$\sim 10^{-3}$	~ 1.0
1.14	$\sim 10^{-3}$	~ 1.0

So, the glitch size for the Vela pulsar will be modified as

$$\left(\frac{\Delta\Omega}{\Omega}\right) \simeq 10^{-6}\left(\frac{\delta_a}{\Delta R}\right). \tag{7}$$

Here, δ_a is the width of the region of avalanche.

Conclusion

- Crustquake's strain energy is assumed to causes pair-breaking quasi-neutron excitations unpin a large number of vortices result in pulsar glitches.
- We find that Vela-like pulsars can results in glitches of size $\sim 10^{-11} - 10^{-9}$.
- We also explored the possibility of a vortex avalanche triggered by the movement of the unpinned vortices. An estimate of the glitch size caused by an avalanche shows a favourable result.
- The time scales associated with various events are compatible with glitch observations.

Thank You!

Time Scales

- The energy deposition and vortex unpinning (n-n scattering time scale $\tau_{nn} \simeq 10^{-5}$ s only) is almost instantaneous [*Layek & Yadav, MNRAS (2020)*].
- The time $t_v \simeq v_r/\Delta R \sim 0.1$ s taken by the vortex to reach the outer crust.
- For avalanche, the whole process is expected to be completed within $\sim 10^{-6}$ s.

Thus, the time interval between the crustquake's glitches and the unpinning vortex is of order tenth of a second, i.e., they seem to overlapwith the current resolution [*Ashton et al., Nature Astronomy (2019)*].

In the case of vortex avalanche, the source of the larger glitch (i.e., vortex unpinning) will be easily identifiable.