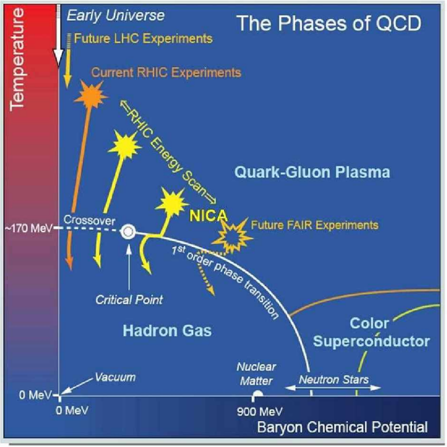
Effect of repulsive mean-field interactions among hadrons on susceptibilities of conserved charges

Somenath Pal (National Institute of Science Education and Research, Bhubaneswar)
In collaboration of Guruprasad Kadam, Hiranmaya Mishra, Abhijit Bhattacharyya

Importance of studying hot and dense hadronic matter



- Hot hadronic matter possibly existed approximately 10^{-6}s after the Big Bang.
- Dense hadronic matter can be found in the cores of neutron stars.
- In ultra relativistic heavy ion colliders like Relativistic Heavy Ion Collider (RHIC) at Brookhaven National Laboratory (BNL) and Large Hadron Collider (LHC) at CERN, quark-gluon plasma can be created, which, subsequently hadronizes.

Hadron Resonance Gas Model

The grand canonical partition function of a hadron resonance gas

$$\ln \mathcal{Z}^{id} = \sum_i \ln \mathcal{Z}_i^{id}$$

where the sum is over all the hadrons. id refers to ideal, i.e., noninteracting HRG. For particle i

$$\ln \mathcal{Z}_i^{id} = \pm \frac{Vg_i}{2\pi^2} \int_0^\infty p^2 dp \ln\{1 \pm exp[-(E_i - \mu_i)/T]\}$$

V is the volume of the system
g_i=degeneracy of ith particle
p=momentum of particle
E_i=energy of ith particle=$\sqrt{p^2 + m_i^2}$
μ_i=chemical potential of ith particle=$B_i\mu_B + Q_i\mu_C + S_i\mu_S$ B_i=Baryon number, Q_i=Charge, S_i=Strangeness
(+)sign is for fermions and (-)sign is for bosons.

Hadron Resonance Gas Model Contd..

- HRG model is a low temperature statistical thermal model used to describe a dilute gas of hadrons.
- It is based on S-matrix formulation of statistical mechanics.
- In the relativistic virial expansion, the interactions among the particles are manifested in the form of two particle scattering phase shifts.
- Attractive Interactions are taken care of by taking the resonance particles as stable particles.
- Repulsive interactions are not included in Hadron Resonance gas model.
- This model cannot describe the deconfined phase of QCD.

Repulsive interaction through mean-field approach

Repulsive interactions can be treated in mean field approach where the single particle energies ϵ_a get shifted by the mean field repulsive interaction

$$\varepsilon_a = \sqrt{p^2 + m_a^2} + U(n) = E_a + U(n)$$

n=total number density
U=potential energy for repulsive interaction

If V(r) is arbitrary interhadron potential then after spacial integration
U(n)=Kn

K is a constant depending on interhadron potential.

Mean-field approach

The total hadron number density is

$$n(T,\mu) = \sum_a n_a = n_B + n_{\bar{B}} + n_M$$

n_a=number density of a$^{\text{th}}$ hadronic species.
n_B=total number density of baryons
$n_{\bar{B}}$=total number density of anti-baryons
n_M=total number density of mesons

$$n_B = \sum_{a\in B} \int d\Gamma_a \, \frac{1}{e^{\frac{(E_a - \mu_{\text{eff},B})}{T}} + 1}$$

where $\mu_{\text{eff},B} = c_i\mu_i - K_B n_B$ and $c_i = (B_i, Q_i, S_i), \mu_i = (\mu_B, \mu_Q, \mu_S)$.

$$n_{\bar{B}} = \sum_{a\in \bar{B}} \int d\Gamma_a \, \frac{1}{e^{\frac{(E_a - \mu_{\text{eff},\bar{B}})}{T}} + 1}$$

where $\mu_{\text{eff},\bar{B}} = \bar{c}_i\mu_i - K_B n_{\bar{B}}$.

$$n_M = \sum_{a\in M} \int d\Gamma_a \, \frac{1}{e^{\frac{(E_a - \mu_{\text{eff},M})}{T}} - 1}$$

where $\mu_{\text{eff},M} = c_i\mu_i - K_M n_M$

Mean-field approach

$$P_{B\{\bar{B}\}}(T,\mu) = T\sum_{a\in B\{\bar{B}\}}\int d\Gamma_a \ln\left[1+e^{-(\frac{E_a-\mu_{\rm eff}\{\bar{\mu}_{\rm eff}\}}{T})}\right] - \phi_{B\{\bar{B}\}}(n_{B\{\bar{B}\}})$$

$$P_M(T,\mu) = -T\sum_{a\in M}\int d\Gamma_a \ln\left[1-e^{\frac{(E_a-\mu_{{\rm eff},M})}{T}}\right] - \phi_M(n_M)$$

where,

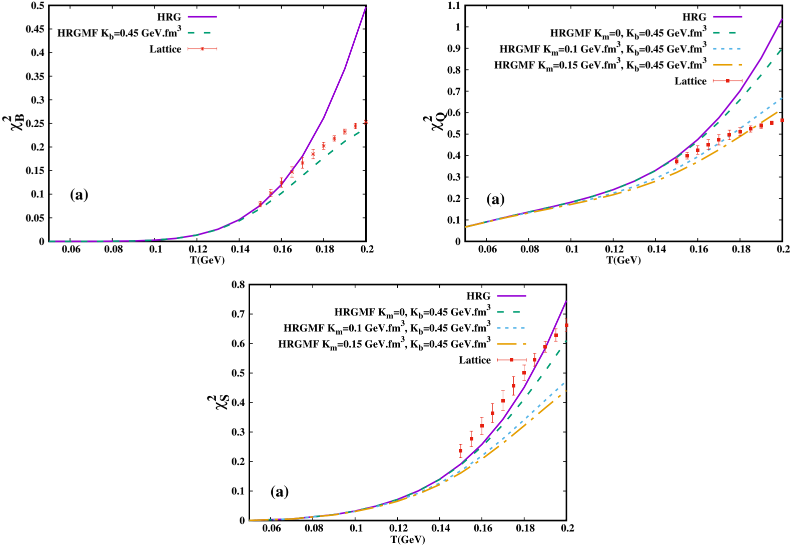
$$\phi_B(n_{B\{\bar{B}\}}) = -\frac{1}{2}K_B n^2_{B\{\bar{B}\}}$$

and

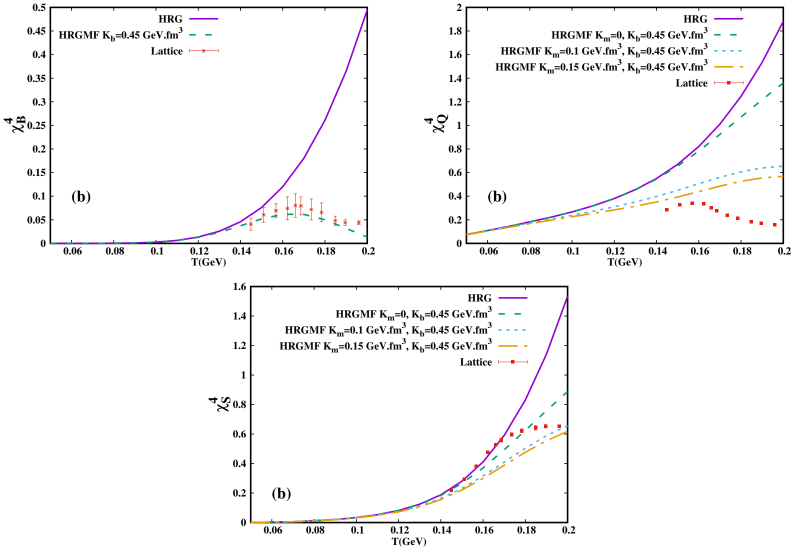
$$\phi_M(n_M) = -\frac{1}{2}K_M n_M^2$$

$\phi_B(n_{B\{\bar{B}\}})$ and $\phi_M(n_M)$ are terms to avoid double counting the potential.

Susceptibilities of second order in HRG mean-field model



Susceptibilities of fourth order in HRG mean-field model



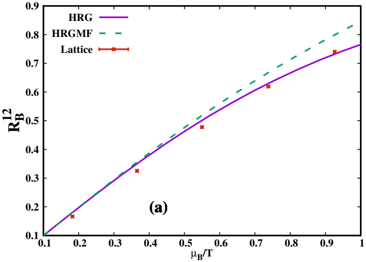
Constraining chemical potentials at $n_Q = 0.4 n_B$ and $n_S = 0$

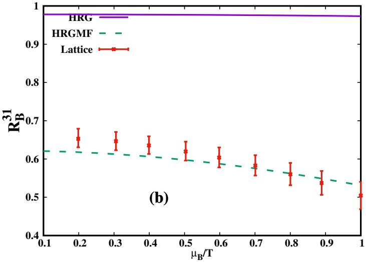
$\hat{\mu}_i = \mu_i / T$;   T=158 MeV
$\hat{\mu}_Q \simeq q_1(T)\hat{\mu}_B + q_3(T)\hat{\mu}_B^3$
$\hat{\mu}_S \simeq s_1(T)\hat{\mu}_B + s_3(T)\hat{\mu}_B^3$
$R_B^{12} = \chi_B^1(T, \mu_B)/\chi_B^2(T, \mu_B)$
$R_B^{31}(T, \mu_B) = \frac{\chi_B^3(T,\mu_B)}{\chi_B^1(T,\mu_B)}$
$R_B^{42} = \frac{\chi_B^4(T,\mu_B)}{\chi_B^2(T,\mu_B)}$
where

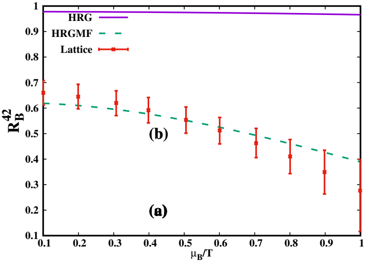
$$q_k(T) = \frac{q_{k,0n} + q_{k,1n}\bar{t} + q_{k,2n}\bar{t}^2 + q_{k,3n}\bar{t}^3 + q_{k,4n}\bar{t}^4}{1 + q_{k,1d}\bar{t} + q_{k,2d}\bar{t}^2 + q_{k,3d}\bar{t}^3 + q_{k,4d}\bar{t}^4} \quad (1)$$

PHYSICAL REVIEW D 95, 054504 (2017)

Ratios of Susceptibilities at finite μ_B in HRG mean-field model







Conclusions

- We have chosen three representative values of mesonic mean-field repulsive parameter to investigate the effect of repulsive interaction in HRG model.
- Introduction of repulsive mean-field interaction in HRG model improves the description of the baryon number and electric charge susceptibilities. But the description of the strange sector is worsened.
- The parameters chosen here leads to good agrement with the results of lattice QCD for non-zero electric charge and strangeness chemical potentials.