Charm Fluctuations in (2+1)-flavor QCD at finite Temperature

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Strong-interaction matter under extreme conditions

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Motivation

- Strong interaction matter undergoes a chiral crossover at $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV. [HotQCD Collaboration, 2019]
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- Signals of exotic charm states such as tetraquarks can shed light on how quarks arrange themselves inside the bound states.

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- For Baryons the argument of $\cosh$ changes to
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- $\hat{\mu}_\mathrm{X} = \mu/\mathrm{T}$, $\mathrm{X} \in \{\mathrm{B}, \mathrm{Q}, \mathrm{S}, \mathrm{C}\}$.

Pressure Calculation using HRG

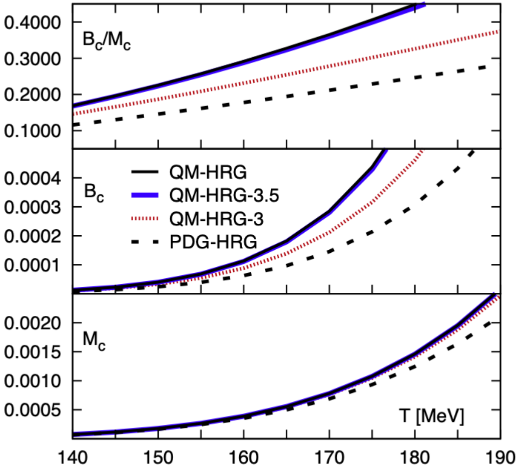

Figure: [HotQCD Collaboration, 2014]

Generalized Susceptibilities of Conserved Charges

$$\mathrm{M_C(T, \overrightarrow{\mu}) = \frac{1}{2\pi^2} \sum_i g_i \left(\frac{m_i}{T}\right)^2 K_2(m_i/T) \cosh(Q_i \hat{\mu}_Q + S_i \hat{\mu}_S + C_i \hat{\mu}_C)} \ .$$

- $\mathrm{K_2(x)} \sim \sqrt{\pi/2x}\, e^{-x}\, [1 + \mathbb{O}(x^{-1})]$. If $\mathrm{m_i} \gg \mathrm{T}$, then contribution to $\mathrm{P_C}$ will be exponentially suppressed.
- Λ_c^+ mass ~ 2286 MeV, Ξ_{cc}^{++} mass ~ 3621 MeV. At $\mathrm{T_{pc}}$, contribution to $\mathrm{B_C}$ from Ξ_{cc}^{++} will be suppressed by a factor of 10^{-3} in relation to Λ_c^+.

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- Dimensionless generalized susceptibilities of conserved charges:

$$\chi_{\mathrm{klmn}}^{\mathrm{BQSC}} = \left. \frac{\partial^{(\mathrm{k+l+m+n})} \ [\mathrm{P} \ (\hat{\mu}_\mathrm{B}, \hat{\mu}_\mathrm{Q}, \hat{\mu}_\mathrm{S}, \hat{\mu}_\mathrm{C}) \ /\mathrm{T}^4]}{\partial \hat{\mu}_\mathrm{B}^\mathrm{k} \ \partial \hat{\mu}_\mathrm{Q}^\mathrm{l} \ \partial \hat{\mu}_\mathrm{S}^\mathrm{m} \ \partial \hat{\mu}_\mathrm{C}^\mathrm{n}} \right|_{\vec{\mu}=0} \ .$$

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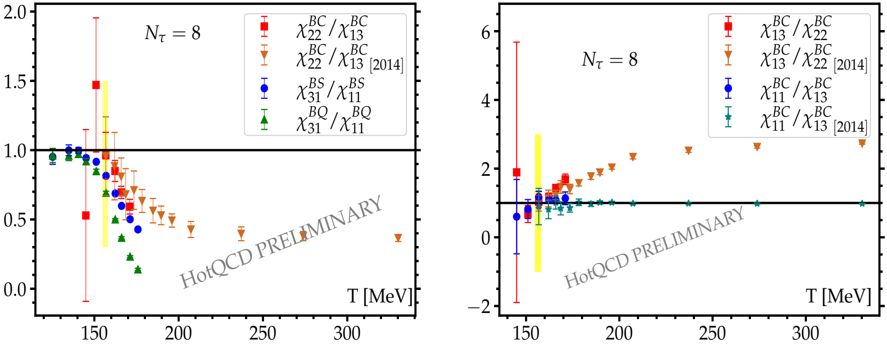
- At present, we have gone upto fourth order in calculating various cumulants.

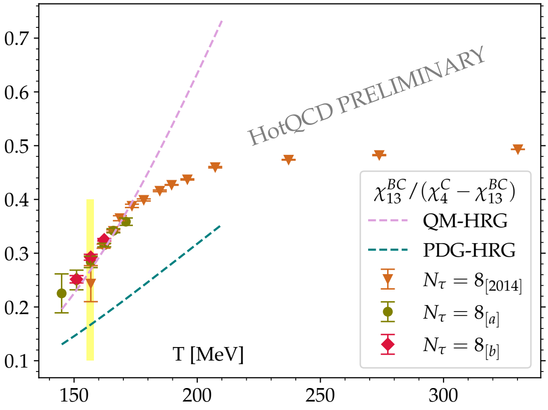
Simulation Details

- Partition function of QCD with 2 light, 1 strange and 1 charm quark flavors is :
$$\mathcal{Z} = \int \mathcal{D}[\mathrm{U}]\{\mathrm{det}\ \mathrm{D}(\mathrm{m_l})\}^{2/4}\{\mathrm{det}\ \mathrm{D}(\mathrm{m_s})\}^{1/4}\{\mathrm{det}\ \mathrm{D}(\mathrm{m_c})\}^{1/4}\mathrm{e}^{-\mathrm{S_g}}.$$
This can be used to calculate susceptibilities in the BQSC basis.
- We used (2+1)-flavor HISQ configurations generated by HotQCD collaboration for $\mathrm{m_s}/\mathrm{m_l} = 27$ and $\mathrm{N}_\tau = 8$.
- We treated charm-quark sector in the quenched approximation.
- We made use of 500 random vectors to calculate various traces per configuration, except for Tr $\left(\mathrm{D}^{-1}\frac{\partial \mathrm{D}}{\partial \mu}\right)$, for which we used 2000 random vectors.

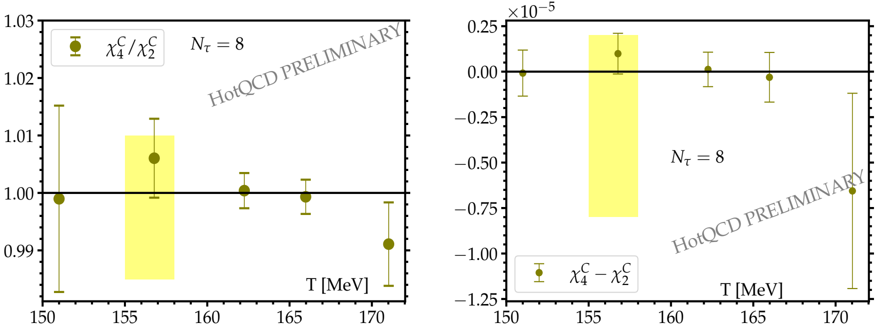
Results: Melting of Open-Charm States



Results: Missing States



$|\mathrm{C}| > 1$ states



We also want to explore the multiple charm sector. Ratio on left indicates that the contribution to partial pressure from $|\mathrm{C}| > 1$ sector is indeed very small and difference on right quantifies it. For HRG,

$$\chi_4^{\mathrm{C}} - \chi_2^{\mathrm{C}} = 12\mathrm{B}_{\mathrm{C},2} + 72\mathrm{B}_{\mathrm{C},3}.$$

Conclusions & Outlook

- Deviations from HRG in the open-charm sector near $\mathrm{T_{pc}} = 156.5 \pm 1.5$ MeV.
- Analysis shows that there are missing states in the PDG record.
- Analyse $\mathrm{N}_\tau = 12, 16$ to quantify the cut-off effects.
- Understand the dependence of cumulants on the hadron mass spectrum.