Kinetic theoretical formulation of Relativistic Spin-hydrodynamics

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Introduction :

- Non-central heavy-ion collisions produce large global angular momentum leading to spin polarization of hadrons.

  [STAR Collaboration, *Nature 548, 62 (2017)*]

- Theoretical models assuming equilibration of spin degrees of freedom, explains the global spin polarization.

  [F. Becattini et. al. *PRC 77, 024906 (2008), PRC 88, 034905 (2013), Ann. Phys. 338, 32 (2013)*]

- But the same models do not explain longitudinal spin polarization.

  [W. Florkowski et. al., *PRC 99, 044910 (2019)*, F. Becattini et. al., *EPJC 79, 741 (2019)*]

- Plausible source of discrepancy may be due to non-equilibrated spin.

The Problem :

The main problem we wish to address is :

- To search for a resolution of the ‘spin sign puzzle’ in longitudinal polarization.
  - $\rightarrow$ Formulate Dissipative Spin-hydrodynamics.

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(Formulation through Kubo formalism : Poster on '*Spin-hydro*' → Sourav Dey.)

(Addition of magnetic field : Talk on '*Spin-magnetohydro*' → Amaresh Jaiswal.)

Section Outline :

Relativistic Spin-hydrodynamics :

Summary and Outlook :

Relativistic Spin-hydrodynamics :

- Inspired by the success of Relativistic Hydrodynamics (RH) in explaining the multitude of properties of QGP evolution, development of a framework of RH with spin was started.

  [F. Becattini et. al. *Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213*]

  [W. Florkowski et. al., *PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017*]

  [D. Montenegro *et al*, *PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016*]

How to include internal degrees of freedom in a macroscopic theory?

[J. Weyssenhoff, A. Raabe, *Acta Phys. Pool. 9 (1947) 7*]

Relativistic Spin-hydrodynamics :

- Origin of spin is purely quantum mechanical.

- Any theory with spin should be built up from Quantum Field Theory (QFT).

- To derive a hydrodynamical description of a spin-polarized fluid starting from QFT, it was proved that a spin-polarization tensor ($\omega^{\mu\nu}$) must be introduced.

  [F. Becattini et. al. *PLB 789 (2019) 419-425*]

  — Relevant Poster → Sourav Dey.

- It has been argued that, at global equilibrium, the spin-polarization tensor should be same as the thermal vorticity.

  [F. Becattini et. al. *Annals Phys. 338 (2013) 32-49, PRC 95 (2017) 5, 054902, EPJC 77 (2017) 4, 213*]

  [N. Weickgenannt et. al. *PRL 127 (2021) 5, 052301*]

$$\boxed{\omega^{\mu\nu}|_{\text{geq}} \propto \varpi^{\mu\nu} = \left(\partial^\mu \beta^\nu - \partial^\nu \beta^\mu\right)/2}$$

$\beta^\mu = u^\mu/T$ is the inverse temperature four-vector.

Relativistic Spin-hydrodynamics :

- A theory of ideal spin-hydrodynamics was formulated for fluids in equilibrium.

  [W. Florkowski et al, *PRC 97 (2018) 4, 041901, PRD 97 (2018) 11, 116017*]

  [D. Montenegro et al, *PRD 96 (2017) 5, 056012, PRD 96 (2017) 7, 076016*]

- But, we want description of fluid with non-thermalized spin, where the relation, $\omega^{\mu\nu}|_{\text{geq}} \propto \varpi^{\mu\nu}$ may not hold.

- Thus, we need to understand, how the out-of-equilibrium system and hence $\omega^{\mu\nu}$ evolves.

Relativistic Spin-hydrodynamics :

- We first note that spin-polarization originates from the rotation of fluid.

- Hence, we will have to deal with three conserved currents :

$$\partial_\mu N^\mu = 0, \qquad \partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda J^{\lambda,\mu\nu} = 0$$

where, $J = L + S$. Also, $L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$.

- Talk on '*Relativistic Hydrodynamics*', $\rightarrow$ W. Florkowski.

- For symmetric $T^{\mu\nu}$ we have, $\boxed{\partial_\lambda S^{\lambda,\mu\nu} = 0}$

$$N^\mu = N^\mu_{\text{eq}} + \delta N^\mu, \qquad T^{\mu\nu} = T^{\mu\nu}_{\text{eq}} + \delta T^{\mu\nu}, \qquad S^{\lambda,\mu\nu} = S^{\lambda,\mu\nu}_{\text{eq}} + \delta S^{\lambda,\mu\nu}$$

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- The dissipative parts require microscopic description $\rightarrow$ Kinetic Theory.

Kinetic Theory with Spin :

- To import spin in kinetic theory (KT), we start from the Wigner function ($\mathcal{W}_{\alpha\beta}$), that bridges the gap between QFT and KT.

- For spin-1/2 particles we set up kinetic equation of $\mathcal{W}_{\alpha\beta}$ using Dirac equation,

$$\left[\gamma \cdot \left(p + \frac{i}{2}\partial\right) - m\right]\mathcal{W}_{\alpha\beta} = \mathcal{C}\left[\mathcal{W}_{\alpha\beta}\right]$$

[Xin-Li Sheng, *PhD Thesis (2019)*, N. Weickgenannt et al, *PRL 127 (2021) 5, 052301*, *PRD 100, 056018 (2019)*.]

- The Wigner function can be decomposed as,

$$\mathcal{W}_{\alpha\beta} = \frac{1}{4}\left(\mathcal{F} + i\gamma^5\mathcal{P} + \gamma^\mu\mathcal{V}_\mu + \gamma^5\gamma^\mu\mathcal{A}_\mu + \frac{1}{2}\Sigma^{\mu\nu}\mathcal{S}_{\mu\nu}\right)_{\alpha\beta}$$

$\mathcal{F} \to$ scalar component,

$\mathcal{P} \to$ pseudoscalar component,

$\mathcal{V}_\mu \to$ vector component,

$\mathcal{A}_\mu \to$ axial vector component,

$\mathcal{S}_{\mu\nu} \to$ tensor component.

where, the γ-matrices are the 4×4 Dirac γ-matrices and, $\Sigma^{\mu\nu} = i\gamma^{[\mu}\gamma^{\nu]}$.

Kinetic Theory with Spin :

- For spin-hydrodynamics it suffices to consider only $\mathcal{F}$ and $\mathcal{A}_\mu$ components.

  [Xin-Li Sheng, *PhD Thesis (2019)*]

	Scalar Component	***Axial Component***
Kin. Eq.	$k^\mu \partial_\mu \mathcal{F}(x,k) = \mathcal{C}_\mathcal{F}$	$k^\mu \partial_\mu \mathcal{A}^\nu(x,k) = \mathcal{C}^\nu_\mathcal{A}$
RTA	$C_\mathcal{F} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{F}_{\rm eq}(x,k) - \mathcal{F}(x,k)\Big]$	$C^\nu_\mathcal{A} = \frac{(k\cdot u)}{\tau_{\rm eq}}\Big[\mathcal{A}^\nu_{\rm eq}(x,k) - \mathcal{A}^\nu(x,k)\Big]$
Dist. fn.	$\mathcal{F}^\pm(x,k) = 2m\int_{p,s} f^\pm(x,p,s)\ \delta^{(4)}(k\mp p)$	$\mathcal{A}^\mu_\pm(x,k) = 2m\int_{p,s} s^\mu f^\pm(x,p,s)\ \delta^{(4)}(k\mp p)$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, *PLB 814 (2021) 136096*, *PRD 103 (2021) 1, 014030*]

Momentum measure $\rightarrow \quad \int_p(\cdots) \rightarrow \int \mathrm{dP}(\cdots), \quad \int \mathrm{dP} = d^3p/\,(2\pi)^3\, p^0.$

Spin measure $\rightarrow \quad \int_s(\cdots) \rightarrow \int \mathrm{dS}(\cdots), \quad \int \mathrm{dS} = (m/\pi\mathfrak{s})\int d^4s\,\delta(s\cdot s + \mathfrak{s}^2).$

Relativistic Kinetic Equation :

- We take the equilibrium (extended) phase-space distribution function to be :

$$f^{\pm}_{\text{eq}}(x,p,s) = e^{-\beta(u\cdot p)\pm\xi}\left(1+\frac{1}{2}\omega_{\mu\nu}s^{\mu\nu}\right)+\mathcal{O}(\omega^2)$$

[F. Becatinni et al., *Annals Phys. 338 (2013) 32-49*, W. Florkowski et al., *PRD 97 (2018) 11, 116017*]

- Near local equilibrium $f(x,p,s)$ is expanded using Chapman-Enskog :

$$f^{\pm}(x,p,s) = f^{\pm}_{\text{eq}}(x,p,s) + \delta f^{\pm}(x,p,s).$$

[de Groot, van Leewan, van Weert, *'Relativistic Kinetic Theory - Principle and Applications (1980)'.*]

- δf is the non-equilibrium correction and is obtained from the Boltzmann equation,

$$p^{\mu}\partial_{\mu}f^{\pm}(x,p,s) = -\frac{(u\cdot p)}{\tau_{\text{eq}}}\delta f^{\pm}(x,p,s)$$

[Anderson, Witting, *Physica 74 (3) (1974) 466–488*.]

- The conserved currents are expressed in kinetic theory as,

$$N^{\mu}=\int_{p,s} p^{\mu}\left(f^{+}-f^{-}\right); \qquad T^{\mu\nu}=\int_{p,s} p^{\mu}p^{\nu}\left(f^{+}+f^{-}\right); \qquad S^{\lambda,\mu\nu}=\int_{p,s} p^{\lambda}s^{\mu\nu}\left(f^{+}+f^{-}\right)$$

Dissipative Currents :

- The non-equilibrium parts give the transport coefficients:

$$\delta N^{\mu} = \tau_{\rm eq}\beta_n(\nabla^{\mu}\xi),$$
$$\delta T^{\mu\nu} = \tau_{\rm eq}\big[-\beta_{\Pi}\,\Delta^{\mu\nu}\,\theta + 2\,\beta_{\pi}\,\sigma^{\mu\nu}\big],$$
$$\delta S^{\lambda,\mu\nu} = \tau_{\rm eq}\Big[B_{\Pi}^{\lambda,\mu\nu}\theta + B_{n}^{\phi\lambda,\mu\nu}(\nabla_{\phi}\xi) + B_{\pi}^{\alpha\beta\lambda,\mu\nu}\sigma_{\alpha\beta} + B_{\Sigma}^{\rho\gamma\phi\lambda,\mu\nu}(\nabla_{\rho}\omega_{\gamma\phi})\Big]$$

- By choosing the Landau frame and matching conditions we found the following relations:

$$\dot{\xi} = \xi_{\theta}\,\theta, \qquad \dot{\beta} = \beta_{\theta}\,\theta, \qquad \beta\dot{u}_{\mu} = -\nabla_{\mu}\beta + \frac{n_0\ \tanh\xi}{(\mathcal{E}+\mathcal{P})}\,(\nabla_{\mu}\xi)$$
$$\dot{\omega}^{\mu\nu} = \mathcal{D}_{\Pi}^{\mu\nu}\theta + \mathcal{D}_{\rm n}^{\mu\nu\alpha}\,(\nabla_{\alpha}\xi) + \mathcal{D}_{\pi}^{\mu\nu\alpha\beta}\sigma_{\alpha\beta} + \mathcal{D}_{\Sigma}^{\lambda\mu\nu\alpha\beta\gamma}\,(\nabla_{\alpha}\omega_{\beta\gamma}),$$

[S.B., W. Florkowski, A. Jaiswal, A. Kumar and, R. Ryblewski, *PLB 814 (2021) 136096*, *PRD 103, 014030 (2021)*]

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Relativistic Spin-hydrodynamics :

Summary and Outlook :

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- **Summary :**
  1. Viscous effects may be necessary for the explanation of LSP.
  2. We found the dissipation of particle number and energy-momentum remain same.
  3. We found the dissipation of spin may depend on multiple hydrodynamic gradients.
  4. Evolution of spin-polarization tensor affected by multiple hydrodynamic gradients.

- **Outlook :**
  1. Formulation of a causal spin-hydrodynamics is required.
  2. A spin-hydrodynamics with non-local collisions is necessary.
  3. A spin-hydrodynamics for spin-1 particles needs to be formulated.
  4. Need to study phenomenological consequences of the theory.

Thank you.