STATISTICAL HADRONIZATION MODEL FOR LOW-ENERGY HEAVY-ION COLLISIONS

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THE HENRYK NIEWODNICZAŃSKI INSTITUTE OF NUCLEAR PHYSICS POLISH ACADEMY OF SCIENCES

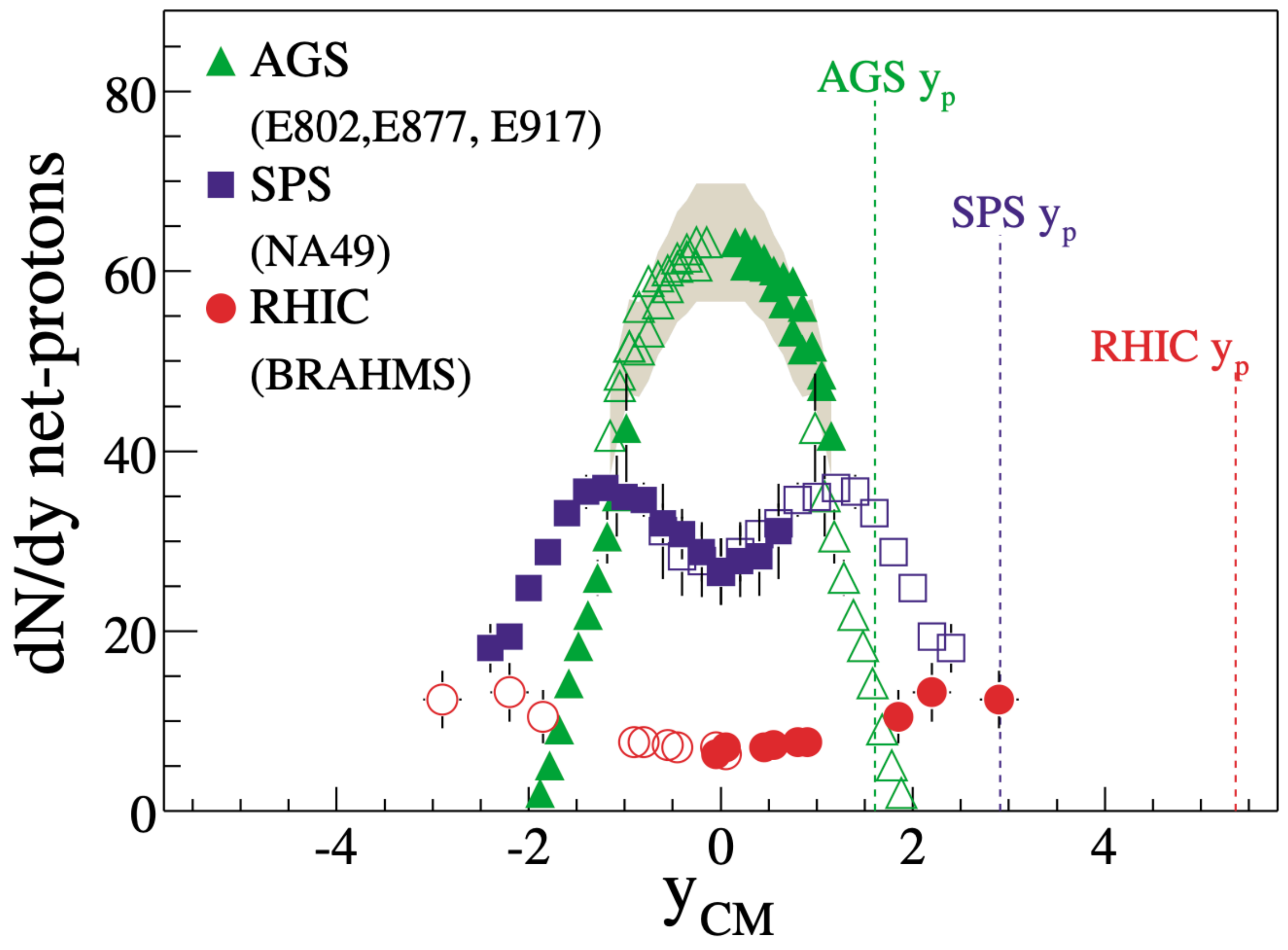
Introduction

Heavy-ion collisions at lower beam energies provide access to QCD matter at **high net-baryon densities.**

Basic description is obtained with transport models (UrQMD) and the emphasis is usually put on non-equilibrium features.

S. A. Bass et al., Prog. Part. Nucl. Phys. 41, 255 (1998),
O. Buss, et al, Phys. Rept. 512, 1 (2012),
H. Petersen, D. Oliinychenko, M. Mayer, J. Staudenmaier, and S. Ryu, Nucl. Phys. A 982, 399 (2019),
C. Hartnack, R. K. Puri, J. Aichelin, J. Konopka, S. A. Bass, H. Stoecker, and W. Greiner, Eur. Phys. J. A 1, 151 (1998)
W. Cassing and E. L. Bratkovskaya, Phys. Rept. 308, 65 (1999).

I. G. Bearden et al. (BRAHMS), PRL 93, 102301 (2004)



The problem of whether the fireball that is formed in the few-GeV energy regime is thermalized remains a matter of debate.

The study of **hadron yields** and **spectra** is crucial to answer this question.

Introduction

Thermal models of hadron production (based on the idea of **statistical hadronization**) have been very successful in **describing hadron yields in various collision processes.**

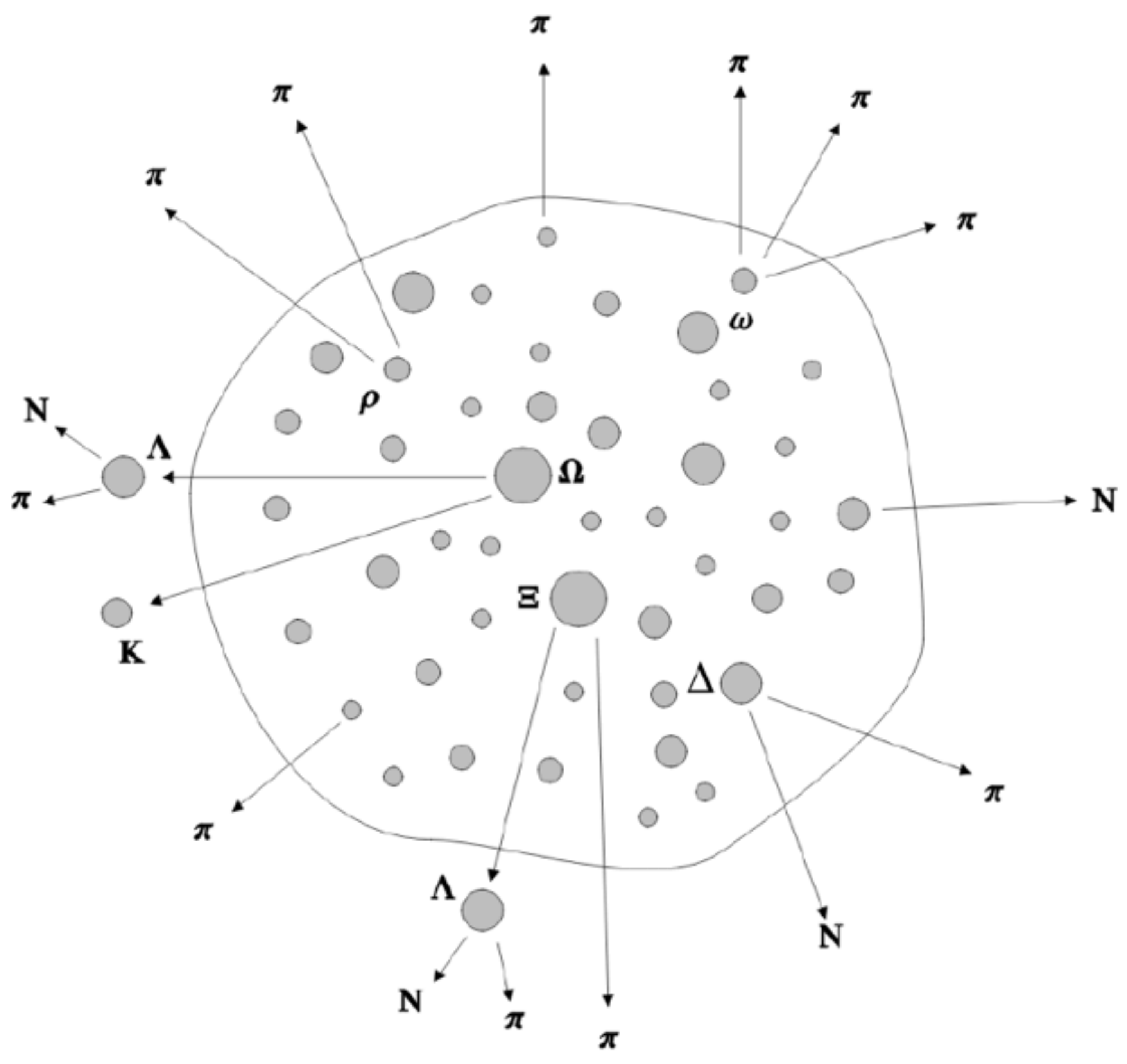
J. Cleymans, H. Satz, F. Becattini, M. Gazdzicki, J. Sollfrank, W. Florkowski, W. Broniowski, J. Letessier, J. Rafelski, R. Stock, M. I. Gorenstein, A. Andronic, P. Braun-Munzinger, K. Redlich, and J. Stachel …

The **hadron yields** can be explained over several orders of multiplicity by fixing just **a few thermodynamic parameters.**

Matter formed at the chemical freeze-out is treated as multicomponent **ideal hadron resonance gas.**

The role of hadronic resonanses is crucial in describing the data at high energies (~400 states are included).

At lower energies their role is diminished.



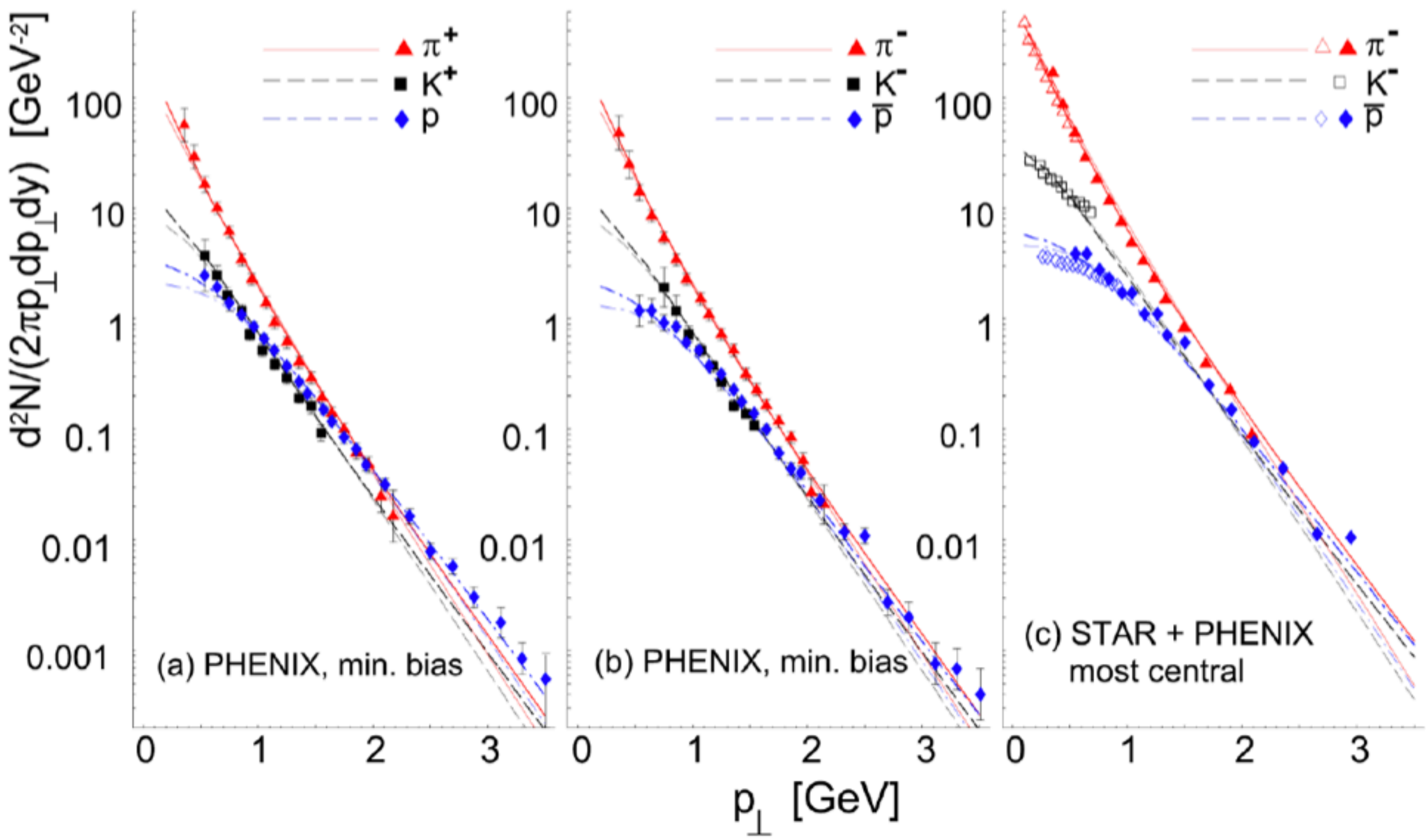
INTRODUCTION

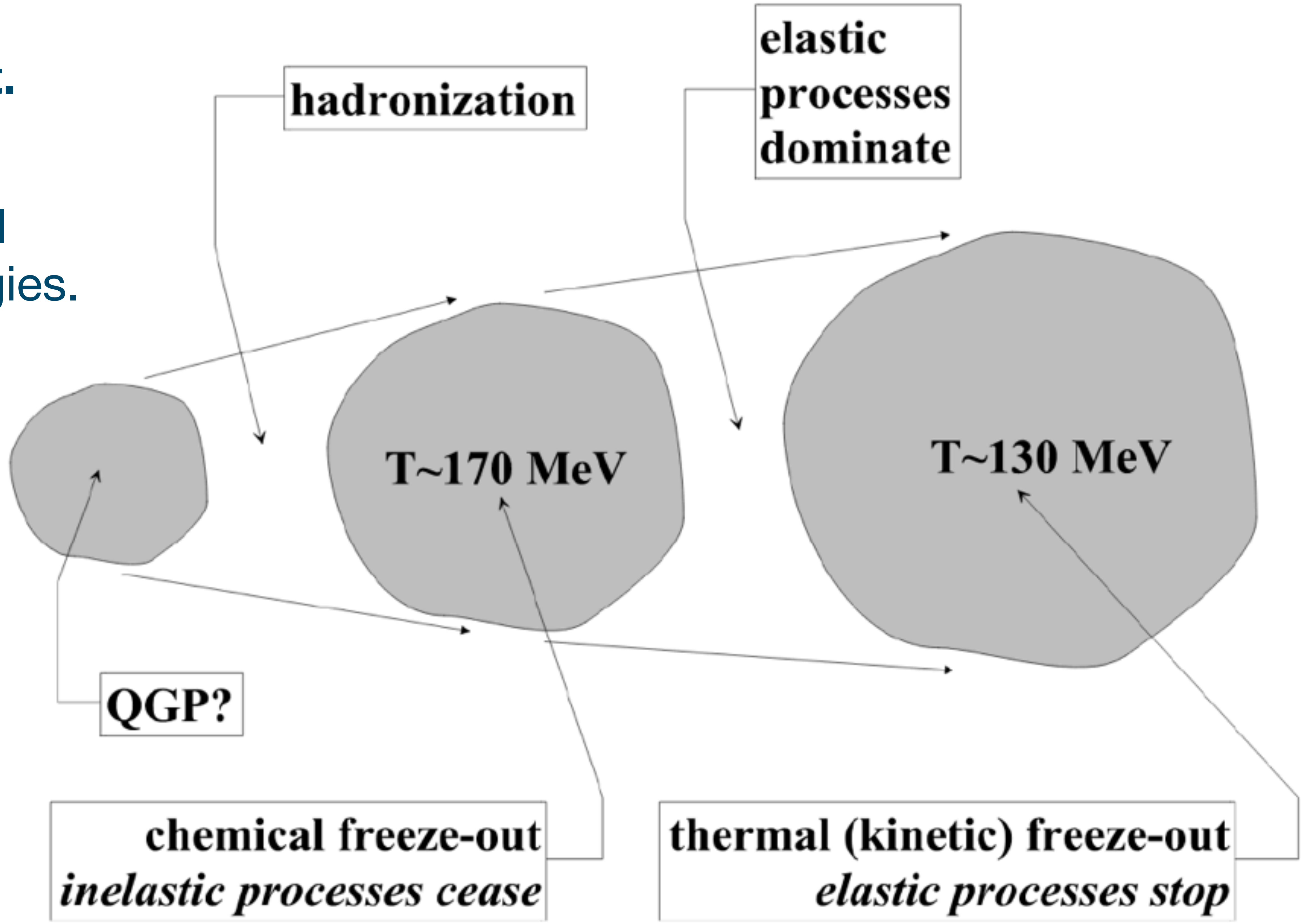
1) Studies of ratios of hadron yields define **chemical freeze-out.**

2) Studies of spectra of hadrons define **kinetic freeze-out.**

We adopt the **single-freeze-out scenario** where chemical and kinetic freeze-outs coincide - successful at high energies.

W. Broniowski, W. Florkowski, PRL 87, 272302 (2001)





BLAST-WAVE MODELS

Instead of determining freeze-out conditions from hydrodynamic simulations one can **model the freeze-out conditions (hypersurface and flow).**

In the original formulation of the **blast-wave model by Siemens and Rasmussen (SR)** the **freeze-out was spherical and the flow was radial.**

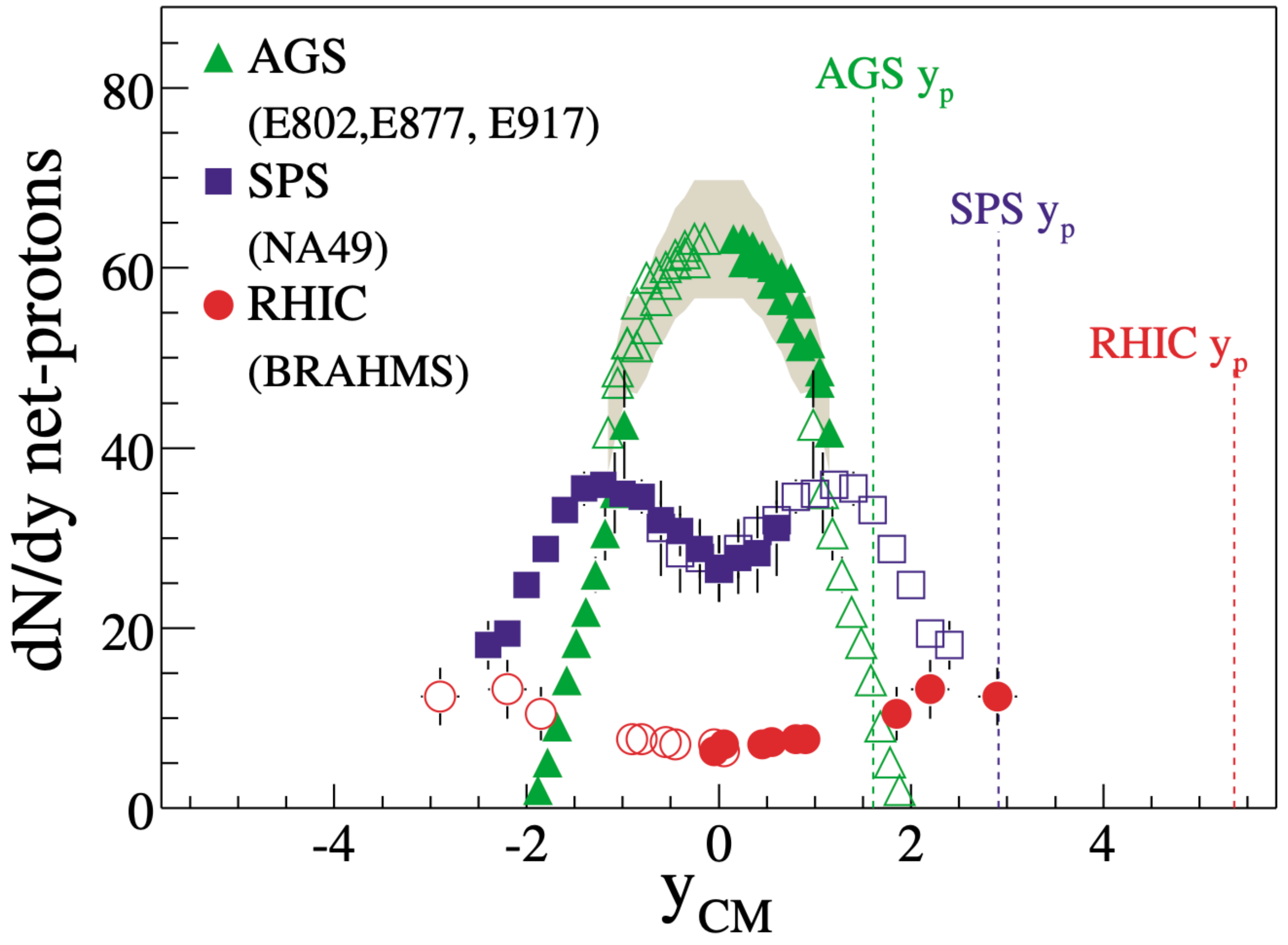
P. Siemens and O. Rasmussen, PRL 42, 880 (1979)

This approach was modified for higher energies (RHIC and LHC) assuming boost-invariance and cyllindrical symmetry.

E. Schnedermann, J. Sollfrank, U. Heinz, PRC 48, 2462 (1993)

We aim to re-examine RS model in the context of low-energy collision measurements performed by HADES where boost-invariance is not observed.

I. G. Bearden et al. (BRAHMS), PRL 93, 102301 (2004)



Cooper-Frye formula

Invariant momentum spectrum of particles emitted from an expanding source is given by

F. Cooper and G. Frye, PRD 10, 186 (1974).

$$E_p \frac{dN}{d^3p} = \int d^3\Sigma_\mu(x)\, p^\mu f(x,p)$$

$$E_p = \sqrt{m^2 + \boldsymbol{p}^2}.$$

Assuming a **spherically symmetric source** the freeze-out points are defined by the space-time coordinates

$$x^\mu = (t, \boldsymbol{x}) = (t(\zeta), r(\zeta)\boldsymbol{e}_r)$$

$$\boldsymbol{e}_r = (\cos\phi \sin\theta, \sin\phi\sin\theta, \cos\theta)$$

$$\zeta \longrightarrow (t(\zeta), r(\zeta))$$

$$d^3\Sigma_\mu = -\epsilon_{\mu\alpha\beta\gamma} \frac{\partial x^\alpha}{\partial a} \frac{\partial x^\beta}{\partial b} \frac{\partial x^\gamma}{\partial c}\, da\, db\, dc$$

$$\longrightarrow$$

$$d^3\Sigma_\mu = (r'(\zeta), t'(\zeta)\boldsymbol{e}_r)\, r^2(\zeta) \sin\theta\, d\theta\, d\phi\, d\zeta$$

We assume **sudden freezeout** $t(r) = \text{const}$

With the hadron four-momentum parametrized as $p^\mu = (E_p, p\,\boldsymbol{e}_p)$ $\quad \boldsymbol{e}_p = (\cos\varphi\sin\vartheta, \sin\varphi\sin\vartheta, \cos\vartheta)$

We get

$$d^3\Sigma(x)\cdot p = E_p \sin\theta\, d\theta\, d\phi\, r^2 dr$$

LOCAL THERMAL EQUILIBRIUM

Assume that the hadron system formed is very close to **local thermodynamic equilibrium**

$$f(x,p) = \frac{g_s}{(2\pi)^3}\left[\Upsilon^{-1}\exp\left(\frac{p\cdot u}{T}\right) - \chi\right]^{-1}$$

The fugacity is defined as

G. Torrieri, S. Steinke, W. Broniowski, W. Florkowski, J. Letessier, and J. Rafelski, CPC 167, 229 (2005).

$$\Upsilon = \gamma_q^{N_q+N_{\bar{q}}}\gamma_s^{N_s+N_{\bar{s}}}\exp\left(\frac{\mu}{T}\right) \qquad \mu = \sum_Q Q\mu_Q \qquad Q \in \{B, I_3, S\}$$

We allow for **strangeness undersaturation** (characteristic feature at low beam energies).

HUBBLE-LIKE RADIAL FLOW

We introduce a **spherically symmetric flow**

$$u^\mu = \gamma(r)(1, v(r)\boldsymbol{e}_r)$$

In the **original SR blast-wave model**, it was assumed that the thermodynamic parameters as well as the radial flow velocity are constant

$$(T = \text{const}, \mu = \text{const}, v = v_0 = \text{const})$$

We take **Hubble-like flow**

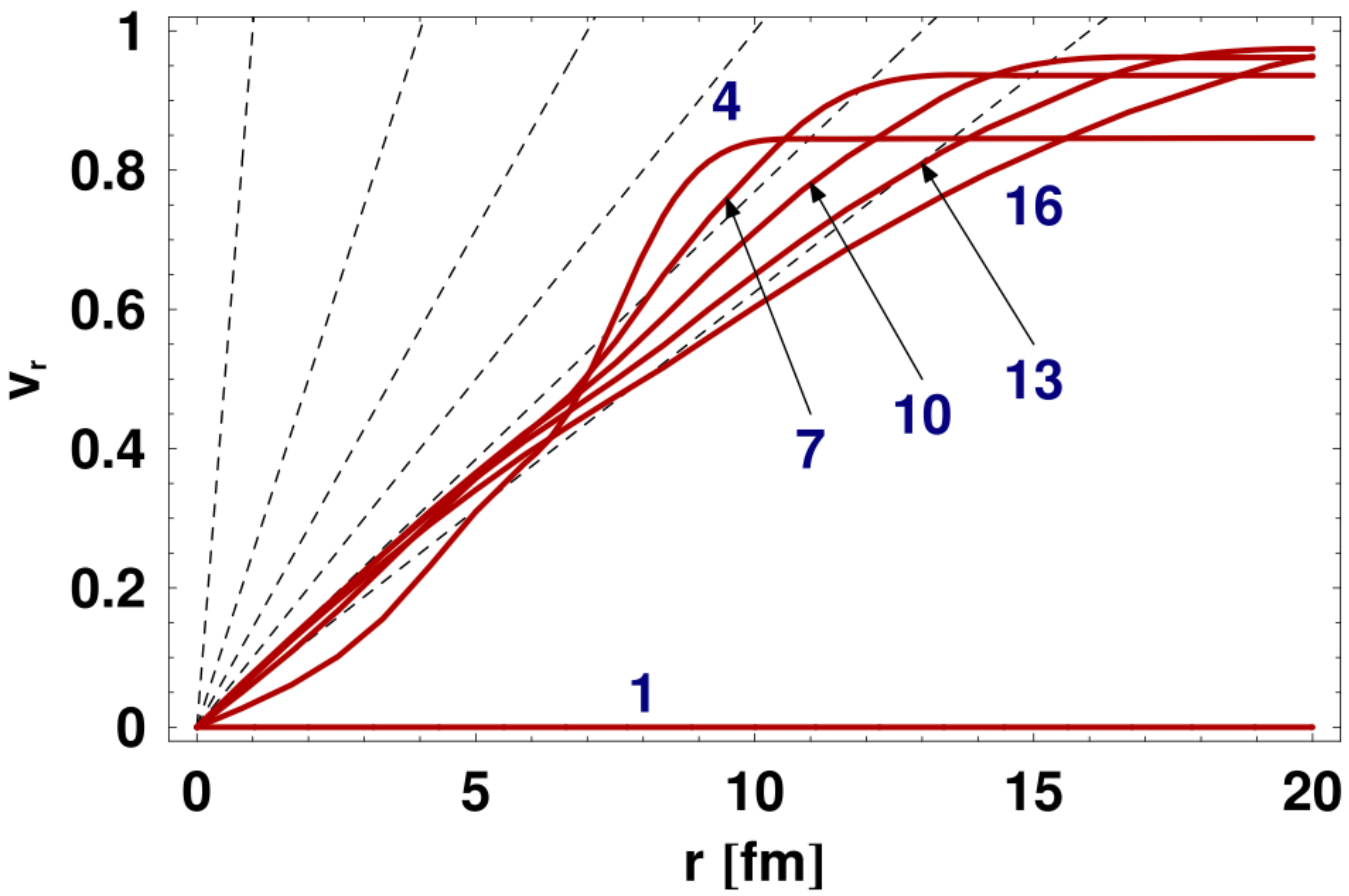
$$v(r) = \tanh(Hr)$$

The parameter **H plays a role of the Hubble constant** in the theory of expanding Universe.

As a result we get

$$p \cdot u = \gamma(E_p - pv\kappa) \qquad \kappa \equiv \boldsymbol{e}_p \cdot \boldsymbol{e}_r$$

M. Chojnacki, W. Florkowski, and T. Csorgo, PRC 71, 044902 (2005).



Condition of constant radial flow breaks requirement that the flow at the center of the system should vanish.

Results of hydrodynamic calculations indicate that the **radial flow linearly grows with radius** for small values of r.

THERMINATOR

Our freeze-out prescription is implemented in the **THERMINATOR** Monte Carlo hadron generator which allows for studies of hadron production taking place on **arbitrary freeze-out hypersurfaces** defined in the four-dimensional space-time.

A. Kisiel, T. Taluc, W. Broniowski, and W. Florkowski, Comput. Phys. Commun. 174, 669 (2006).
M. Chojnacki, A. Kisiel, W. Florkowski, and W. Broniowski, Comput. Phys. Commun. 183, 746 (2012).

THERMINATOR generates primordial particles at the freeze-out hypersurface.

Unstable particles are then allowed to decay contributing to feed-down.

The code includes contributions from decays of all heavier resonances — most of them are very small or negligible.

The largest contribution comes from decays of the lowest-lying baryonic resonance, i.e., Delta (1232).

THERMODYNAMIC PARAMETERS

We obtain thermodynamic model parameters from the ratios of experimental yields measured by HADES in Au+Au collisions at 2.4 GeV.

We assume that the **protons finally bound in the emitted deuterons, tritons, and ₃Helium nuclei were initially frozen out as unbound nucleons**; hence, they are included in the proton yield

$$N = \int d^3\Sigma_\mu(x) \int \frac{d^3p}{E_p} p^\mu f(x,p).$$

$$N = n(T,\Upsilon) \int d^3\Sigma_\mu(x) u^\mu(x) \equiv n(T,\Upsilon)\mathcal{V},$$

In the studies of the ratios of hadronic yields the invariant volume cancels (if the thermodynamic parameters are constant on the freeze-out hypersurface!).

$$T = 49.6 \pm 1 \text{ MeV}, \ \mu_B = 776 \pm 3 \text{ MeV},$$

$$\mu_{I_3} = -14.1 \pm 0.2 \text{ MeV}, \ \mu_S = 123.4 \pm 2 \text{ MeV},$$

$$\gamma_s = 0.16 \pm 0.02$$

J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, PRC 73, 034905 (2006).
P. Castorina, A. Iorio, D. Lanteri, H. Satz, and M. Spousta, PRC 101, 054902 (2020).

TABLE I. Particle multiplicities used in the determination of the freeze-out parameters. Protons bound in nuclei are taken into account as shown.

Particle	Multiplicity	Uncertainty	Ref.
p	77.6	$\pm$ 2.4	[29,31]
p (bound)	46.5	$\pm$1.5	[29,31]
π^+	9.3	$\pm$0.6	[32]
π^-	17.1	$\pm$1.1	[32]
K^+	5.98 10^{-2}	$\pm$6.79 10^{-3}	[33]
K^-	5.6 10^{-4}	$\pm$5.96 10^{-5}	[33]
Λ	8.22 10^{-2}	$^{+5.2}_{-9.2}$ 10^{-3}	[34]

$\sqrt{s_{NN}} = 2.4$ GeV full phase space for the 10% Au-Au collisions

[29] M. Szala (HADES), Light nuclei formation in heavy ion collisions measured with HADES
[31] M. Szala (HADES), Springer Proc.Phys. 250 (2020) 297-301
[32] J. Adamczewski-Musch et al., (HADES) EPJA 56 (2020) 10, 259
[33] J. Adamczewski-Musch et al. (HADES), PLB 778, 403 (2018).
[34] J. Adamczewski-Musch et al. (HADES), PLB 793, 457 (2019).

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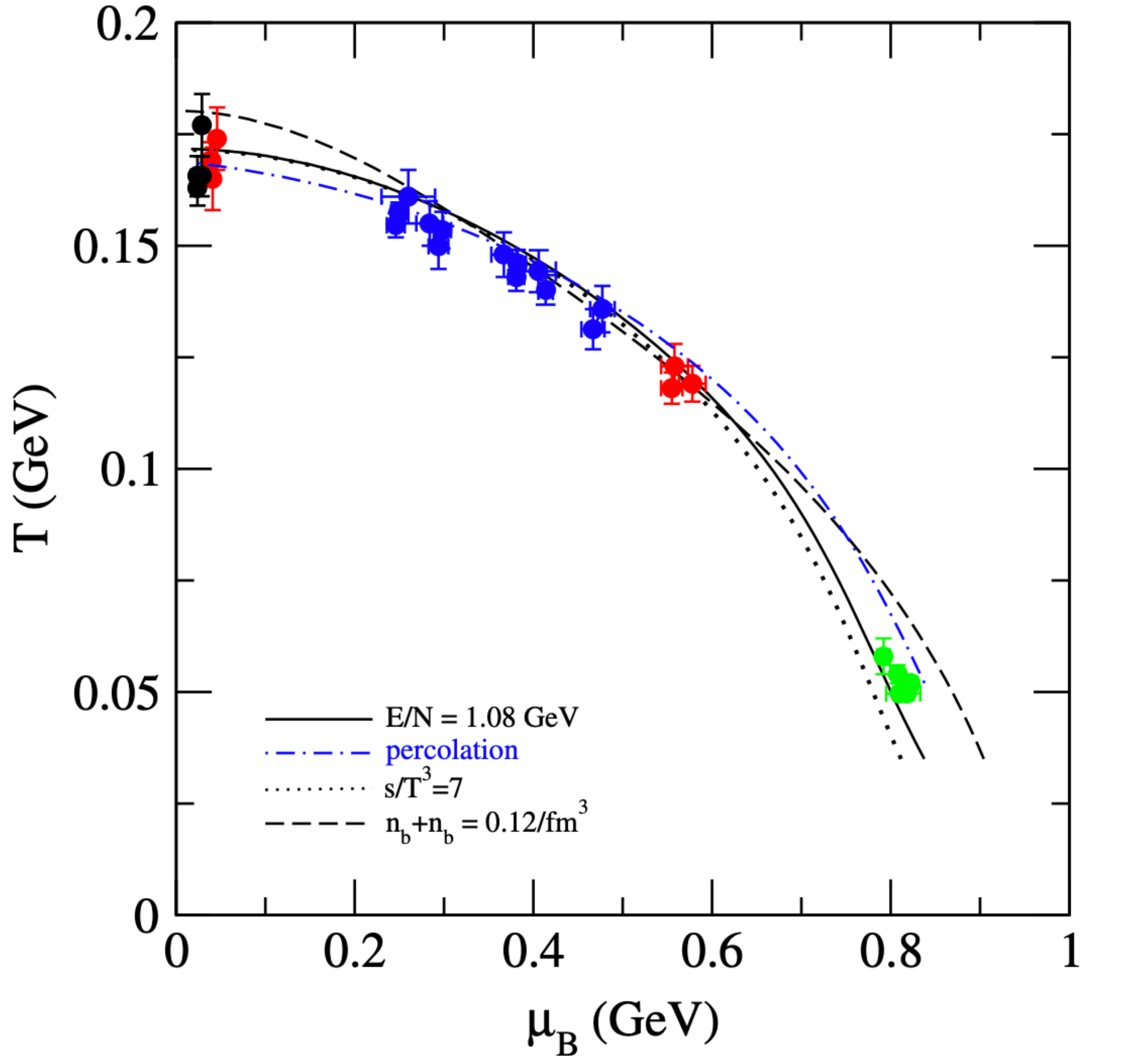
J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, PRC 73, 034905 (2006).



J. Cleymans, H. Oeschler, K. Redlich, and S. Wheaton, PRC 73, 034905 (2006).
P. Castorina, A. Iorio, D. Lanteri, H. Satz, and M. Spousta, PRC 101, 054902 (2020).

Transverse-momentum and rapidity spectra

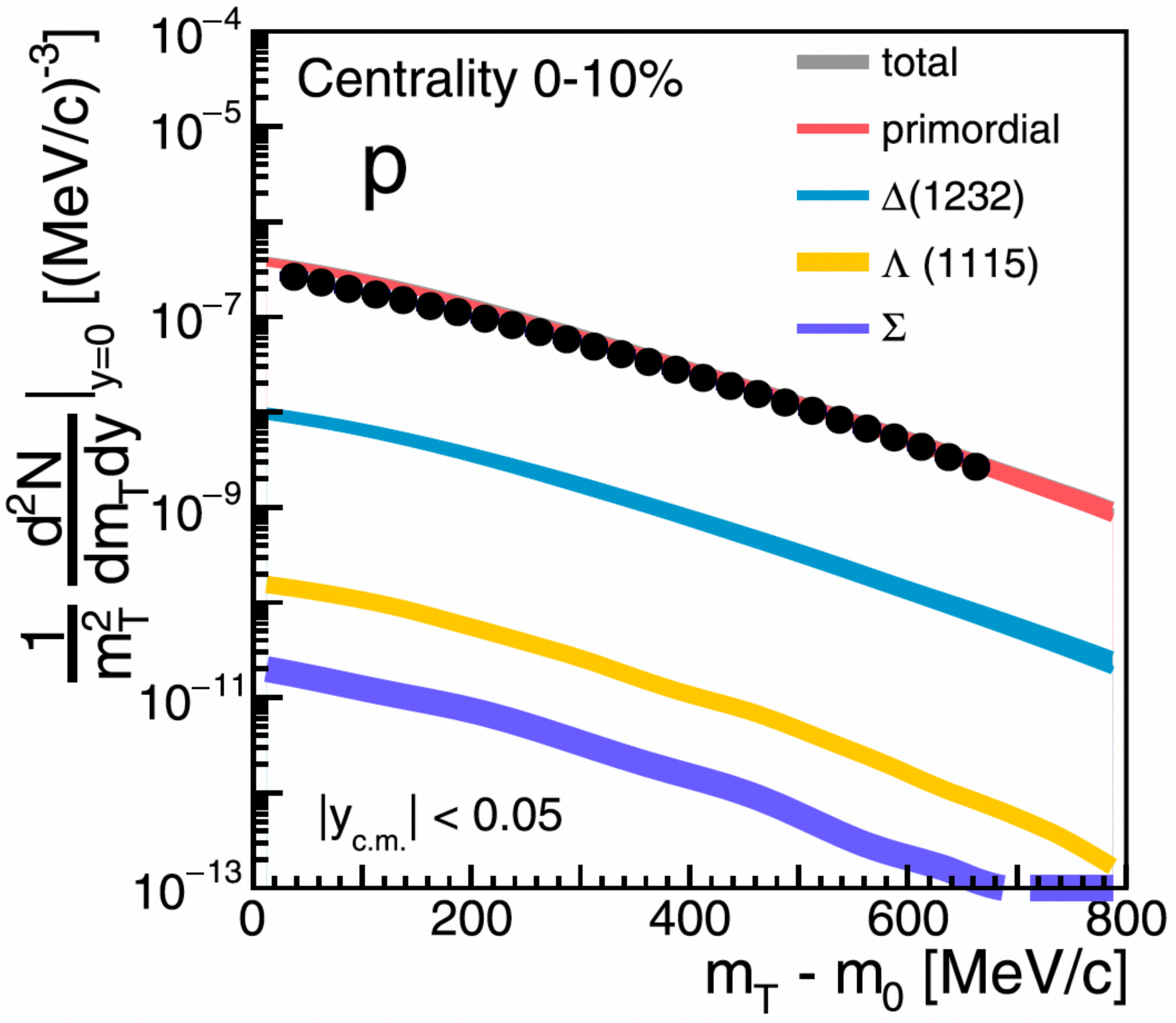
For a fixed value of **H**, the absolute normalization of the yields determines the value of **R**.
Hence we may treat **R** as a function of **H** and we are left with only one independent parameter **H.**

Value of H is obtained from the fit of the proton transverse-mass spectrum by minimization of the quadratic deviation

$$Q^2(H) = \sum_i \frac{(Q_{i,\mathrm{model}}(H) - Q_{i,\mathrm{exp}})^2}{Q^2_{i,\mathrm{exp}}}$$

R = 16.02 fm

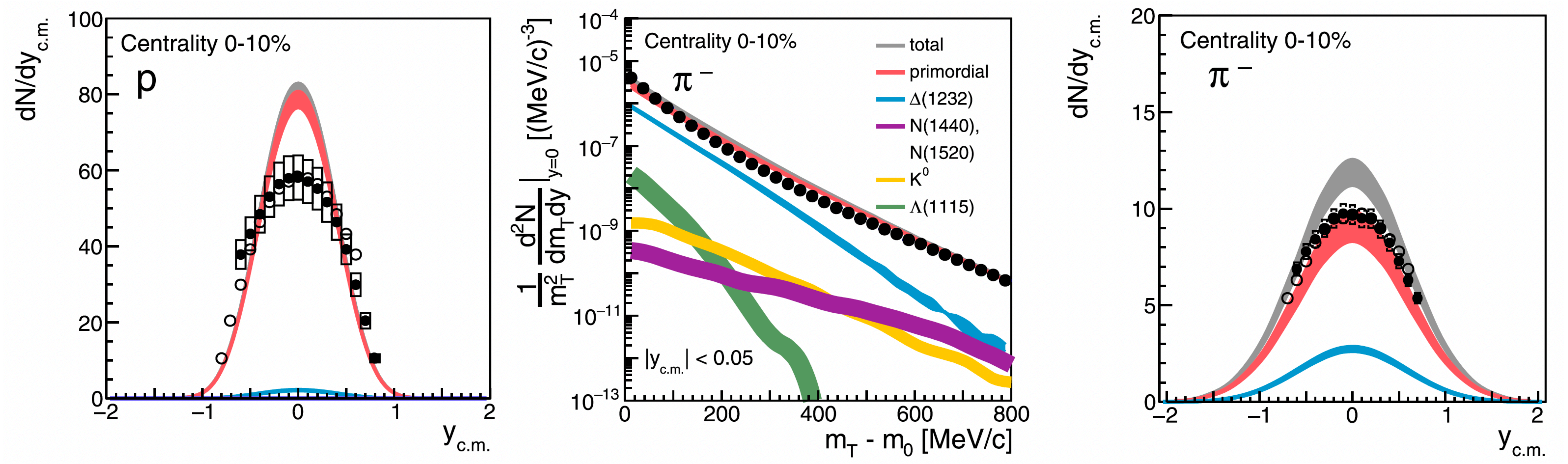
H = 0.04 1/fm



$(Q = 0.20)$

Transverse-momentum and rapidity spectra

Having determined the value of H, we can predict other model spectra.



$Q = 0.28$

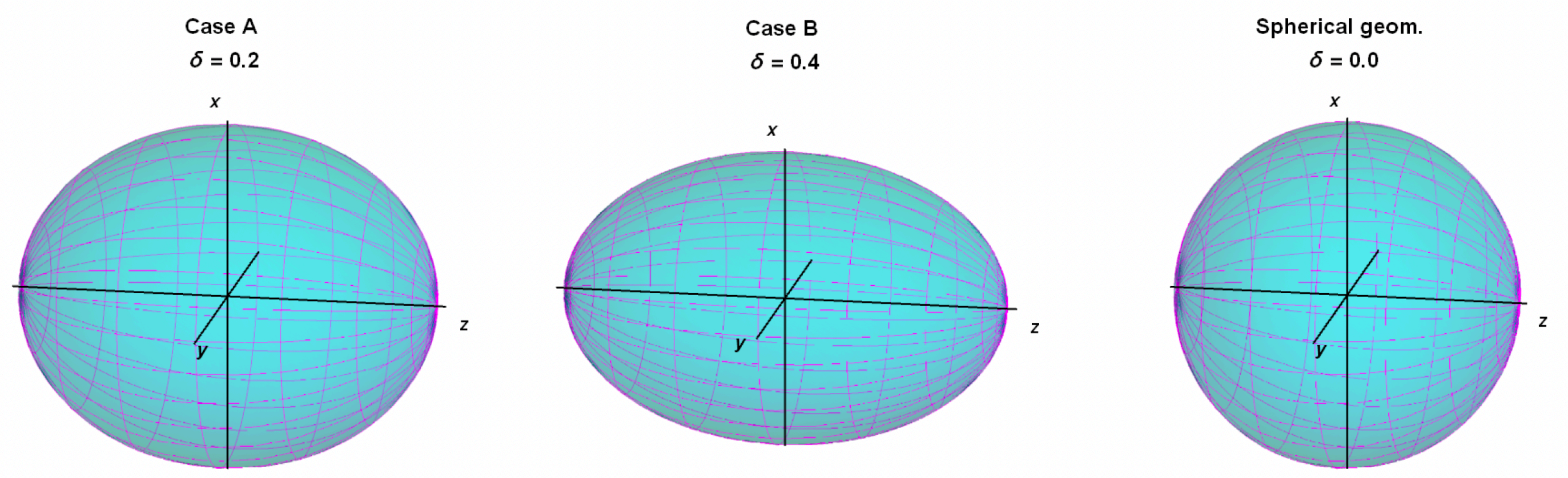
The fact that the rapidity distribution is equally well described (compared to the transverse-mass distribution) points out the **approximate spherical symmetry** of the produced system!

SPHEROIDAL EXTENSION

$$x^\mu = \left(t, r\sqrt{1-\epsilon}\sin\theta\,\hat{\mathbf{e}}_\rho, r\sqrt{1+\epsilon}\cos\theta\right)$$

$$u^\mu = \gamma(\zeta,\theta)\left(1, v(\zeta)\sqrt{1-\delta}\sin\theta\,\hat{\mathbf{e}}_\rho, v(\zeta)\sqrt{1+\delta}\cos\theta\right)$$

$$\hat{\mathbf{e}}_\rho = (\cos\phi, \sin\phi)$$



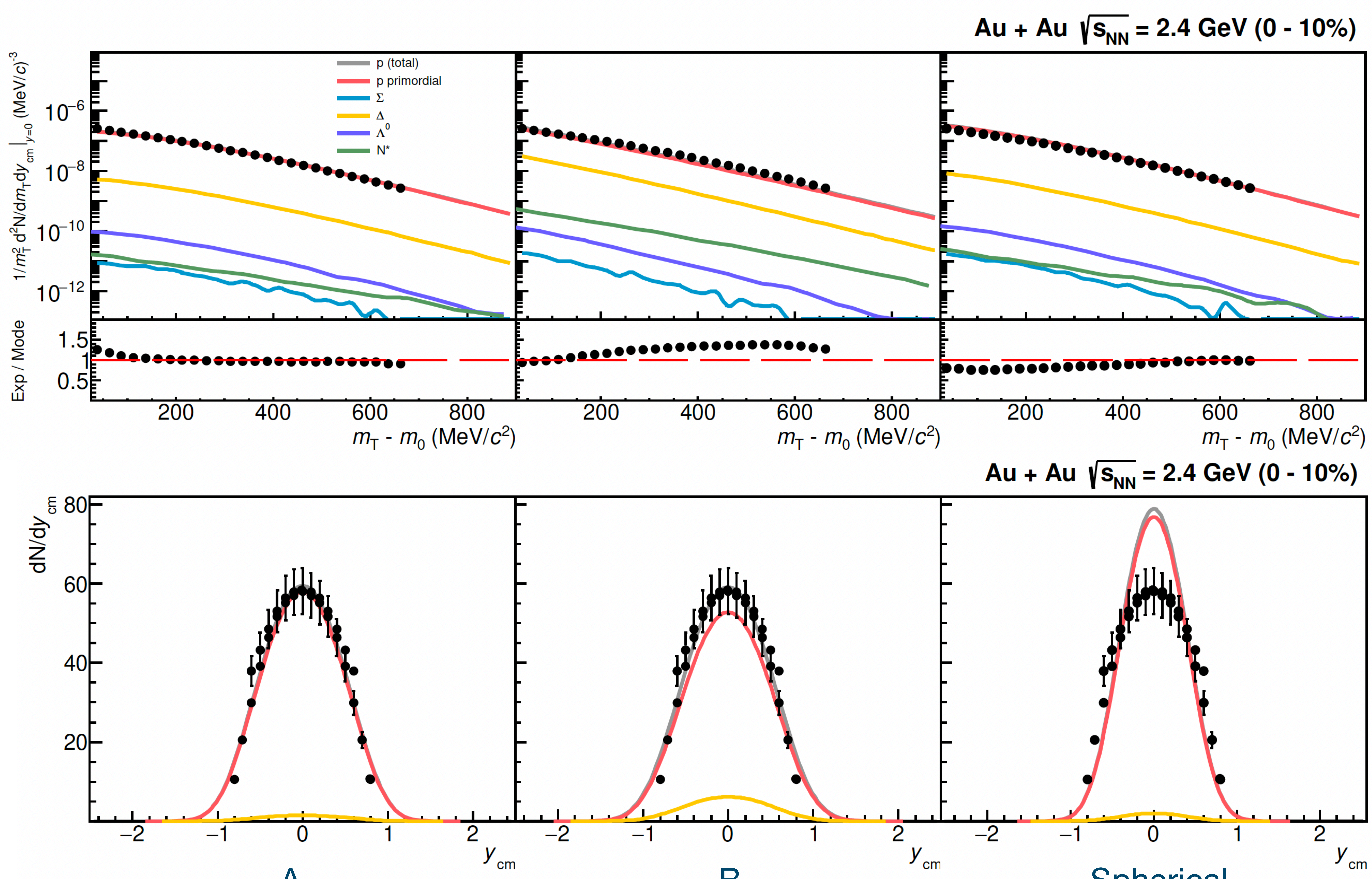
We take for comparison transverse mass distributions of protons, + and - pions in five center-of-mass rapidity intervals:
[0:45;0:35],
[0:25;0:15],
[0:05; 0:05],
[0:15; 0; 25],
[0:35; 0:45]

Parameter	Spherical geometry, Ref. [29]	Case A	Case B
T (MeV)	49.6	49.6	70.3
R (fm)	16.0	15.7	6.06
μ_B (MeV)	776	776	876
μ_S (MeV)	123.4	123.4	198.3
μ_{I_3} (MeV)	-14.1	-14.1	-21.5
γ_S	0.16	0.16	0.05
$\chi^2/N_{\rm df}$	$N_{\rm df}=0$	$N_{\rm df}=0$	$1.13/2$
H (GeV)	0.008	0.01	0.0225
δ	0	0.2	0.4
$\sqrt{Q^2}$	0.285	0.238	0.256

$S=0$ and $Q/B=0.4$

A. Motornenko, J. Steinheimer, V. Vovchenko, R. Stock, and H. Stoecker PLB 822, 136703 (221),

RESULTS

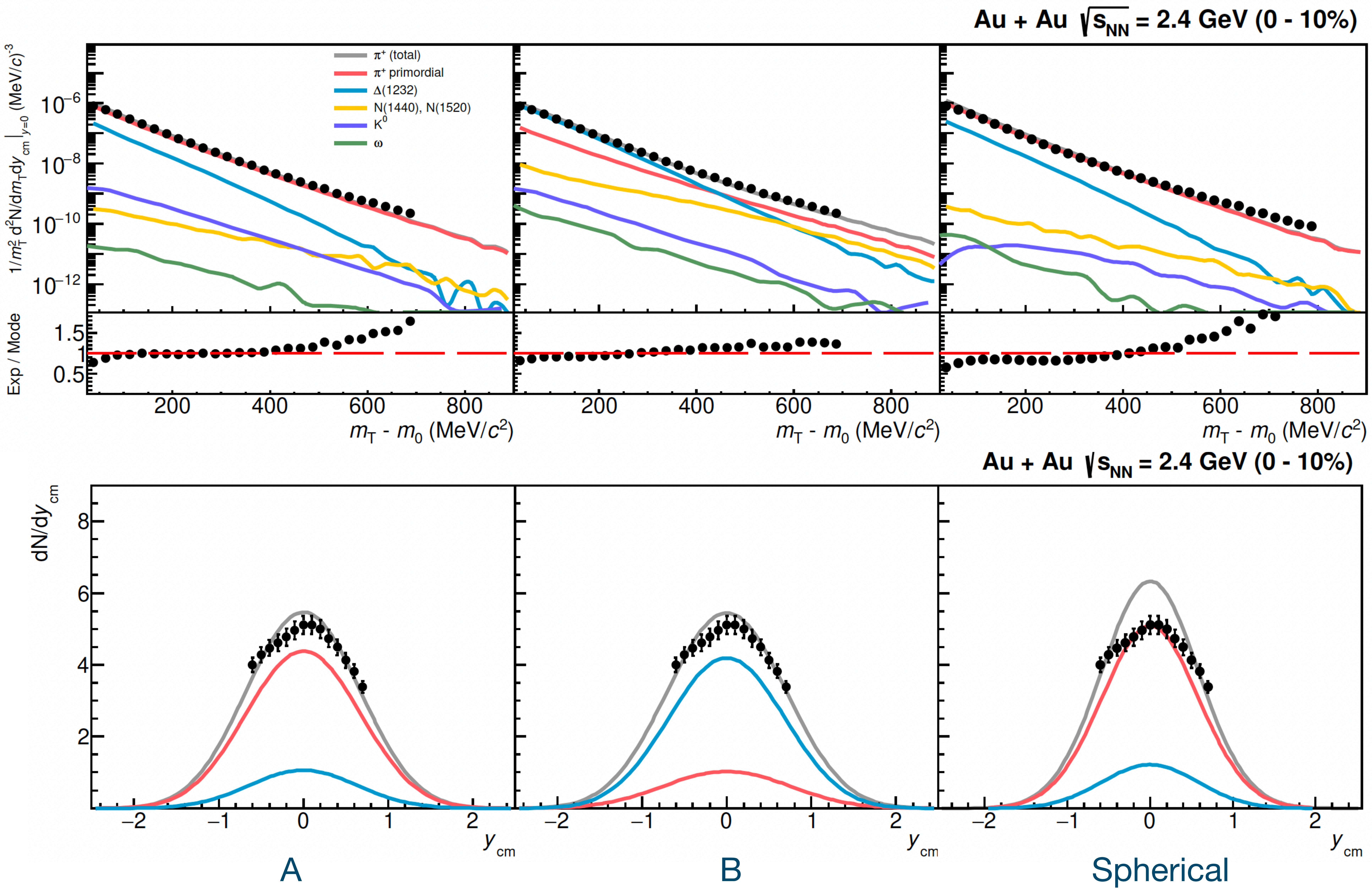


A

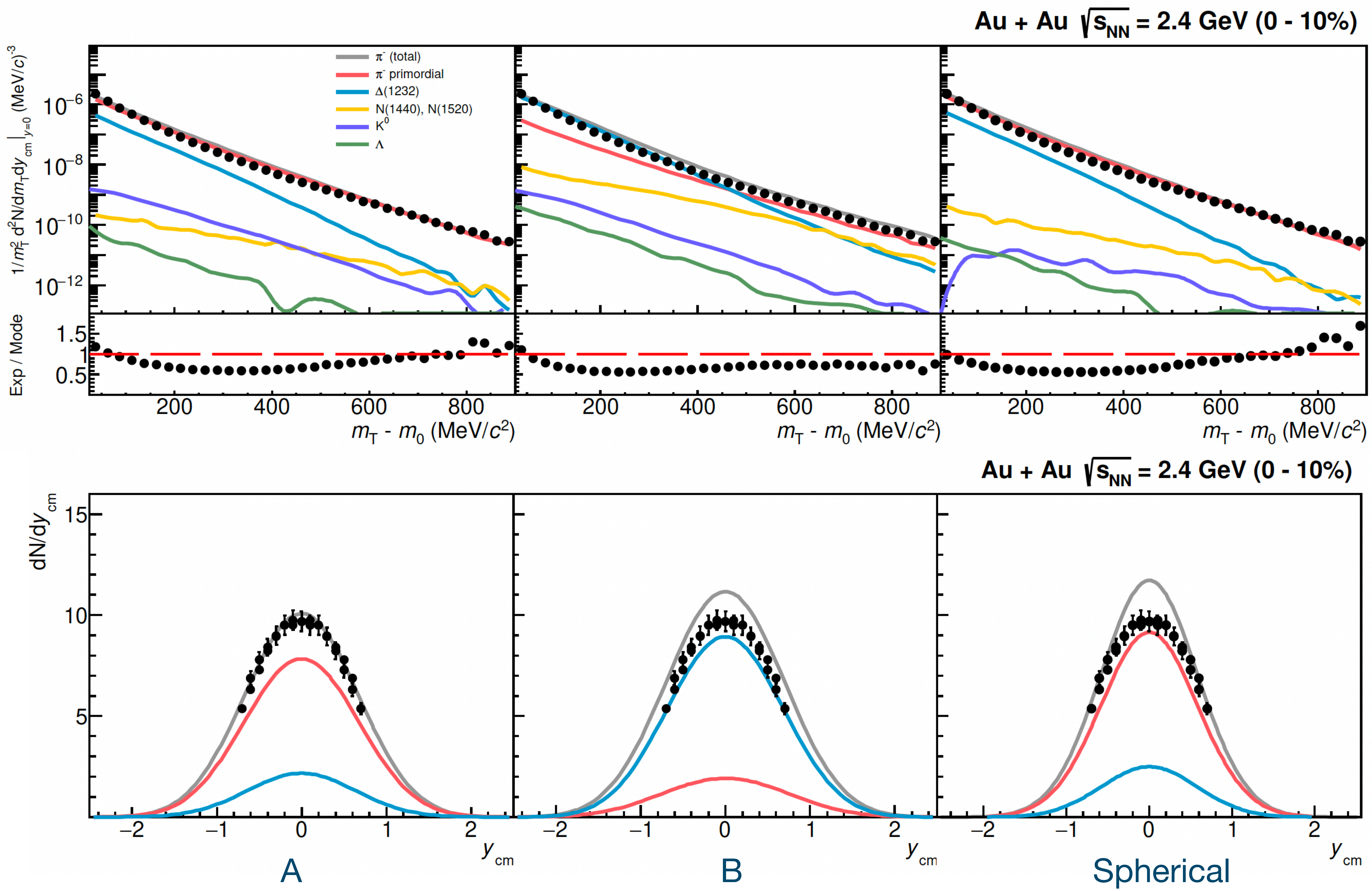
B

Spherical

RESULTS


Au + Au √sNN = 2.4 GeV (0 - 10%)
π+ (total)
π+ primordial
Δ(1232)
N(1440), N(1520)
K0
ω
1/mT2 d2N/dmTdycm|ycm=0 (MeV/c)-3
Exp / Mode
mT - m0 (MeV/c2)
Au + Au √sNN = 2.4 GeV (0 - 10%)
dN/dycm
ycm
A
B
Spherical

Results



Au + Au $\sqrt{s_{NN}}$ = 2.4 GeV (0 - 10%)
π⁻ (total)
π⁻ primordial
Δ(1232)
N(1440), N(1520)
K⁰
Λ
$1/m_T^2\, d^2N/dm_T dy_{cm}|_{y=0}$ (MeV/c)$^{-3}$
Exp / Mode
$m_T - m_0$ (MeV/c^2)
Au + Au $\sqrt{s_{NN}}$ = 2.4 GeV (0 - 10%)
dN/dy_{cm}
y_{cm}
A
B
Spherical

Conclusions

We have **studied the rapidity and transverse mass spectra of protons and pions** produced in Au-Au collisions at 2.4 GeV and measured by HADES.

We have found that they can be **well reproduced in a extended SR model** that assumes **single freeze-out of hadrons from a hypersurface spheroidal along beam direction**.

Our framework modifies and extends RS approach by incorporation of the Hubble-like expansion of matter, inclusion of the resonance decays, and spheroidal deformation of the source.

We have found that the presence of the **Delta resonance affects the spectra of pions**, while the contributions from other resonances can be neglected.

The **obtained thermodynamic parameters agree well with the universal freeze-out curve established by other groups**.

Our results bring **evidence for substantial thermalization of the matter produced in the few-GeV energy range and its nearly spherical expansion.**

THANK YOU FOR YOUR ATTENTION.