Transport coefficient calculations at finite magnetic field

Sabyasachi Ghosh (Physics Dept, IIT Bhilai)

Outline of talk

- **Time line of the topic (Slide-2)**
- **Non-Dissipative part (Slide-3-4)**
- **Dissipative part (Slide 5-7)**
- **Dissipation at finite magnetic field (Slide 8)**
- **References (Slide 9)**
- **CM to QM to QFT (Slide 10-11)**
- **Concluding points (Slide 12)**

Collaborators








+.......

Time line of Transport & Magnetic field in QGP topic

2004 — **SQGP**

2009 — **Strong B**

2012 — **IMC**

2017 — **Vorticity**

2022 — **CME test**

Estimate of the magnetic field strength in heavy-ion collisions

V. Skokov (Darmstadt, GSI and Frankfurt U., FIAS and Dubna, JINR), A.Yu. Illarionov (Trento U.), V. Toneev (Darmstadt, GSI and Dubna, JINR)

Jul, 2009

8 pages

Published in: *Int.J.Mod.Phys.A* 24 (2009) 5925-5932

QCD quark condensate in external magnetic fields

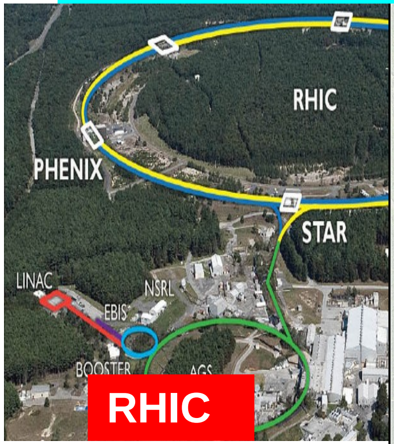
G.S. Bali (Regensburg U. and Tata Inst.), F. Bruckmann (Regensburg U.), G. Endrodi (Regensburg U.), Z. Fodor (Wuppertal U. and Eotvos U. and Julich, Forschungszentrum), S.D. Katz (Julich, Forschungszentrum) et al.

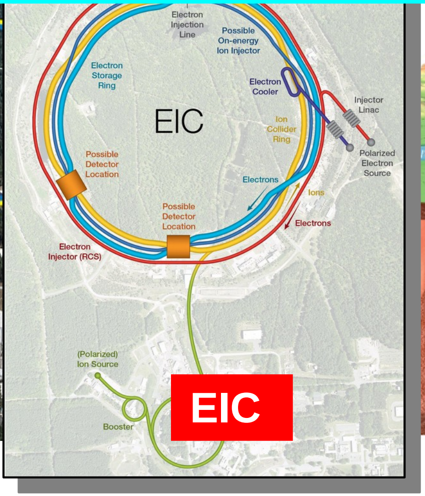
Phys.Rev.D 86 (2012) 071502 • e-Print: 1206.4205 • DOI: 10.1103/PhysRevD.86.071502

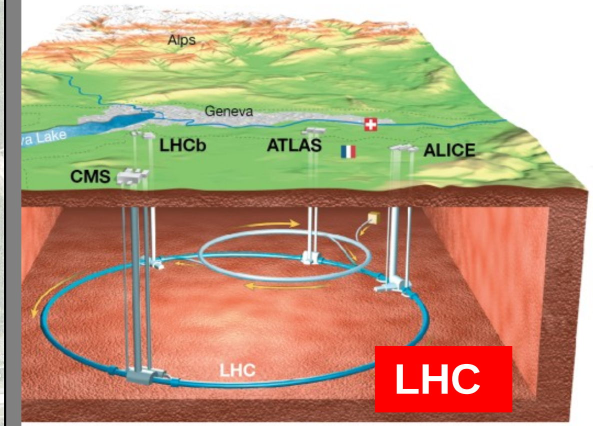
STAR Collaboration (Sep 7, 2022)

e-Print: 2209.03467 [nucl-ex]

Accelerator Facilities ...past to Present to Future.....

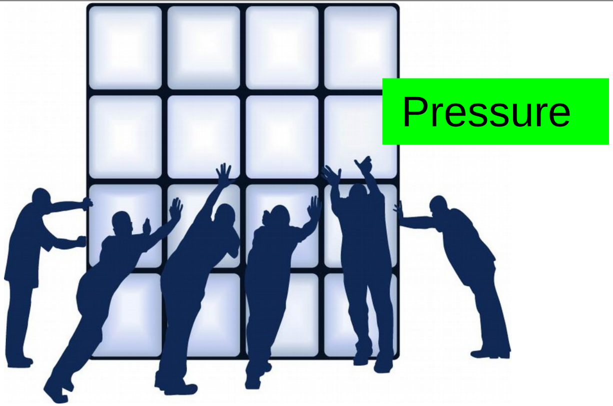






(Relativistic) Hydro/Fluid Dynamics

(Relativistic) Statistical Mechanics



Ideal Part

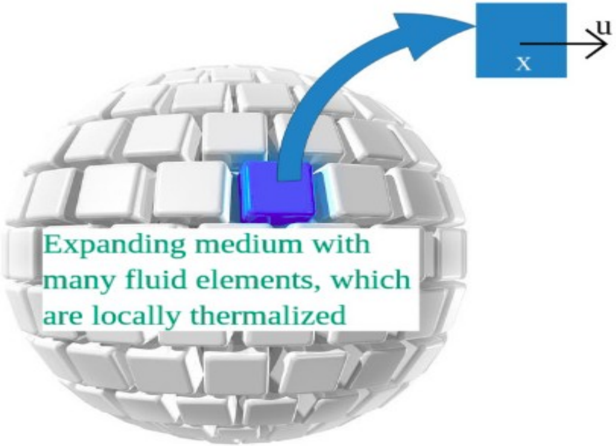
$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

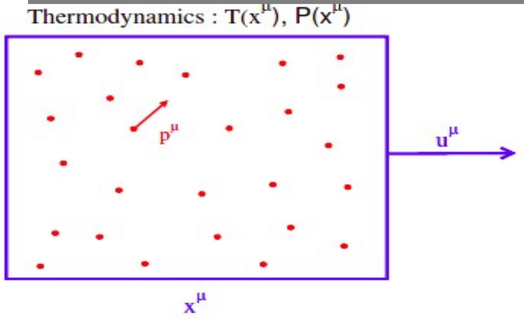
+

Dissipative Part

$$T_D^{\mu\nu} = W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}$$

Macro/many-body/Thermodynamics	SM	Micro/one-body/CM
ϵ	$\epsilon = \frac{1}{V}\sum_0 E$	$E = \sqrt{\vec{p}^2 + m^2}$
P	$P = \frac{1}{V}\sum_0 \frac{\vec{p}\cdot\vec{v}}{3}$	$\vec{v} = \frac{\vec{p}}{E}$
n	$n = \frac{1}{V}\sum_0 1$	1





$$\sum_0 \equiv g \int \frac{d^3x d^3p}{(2\pi)^3} f_0$$

$$T_0^{\mu\nu} = \epsilon u^\mu u^\nu - P(g^{\mu\nu} - u^\mu u^\nu)$$

Pressure

The general expression

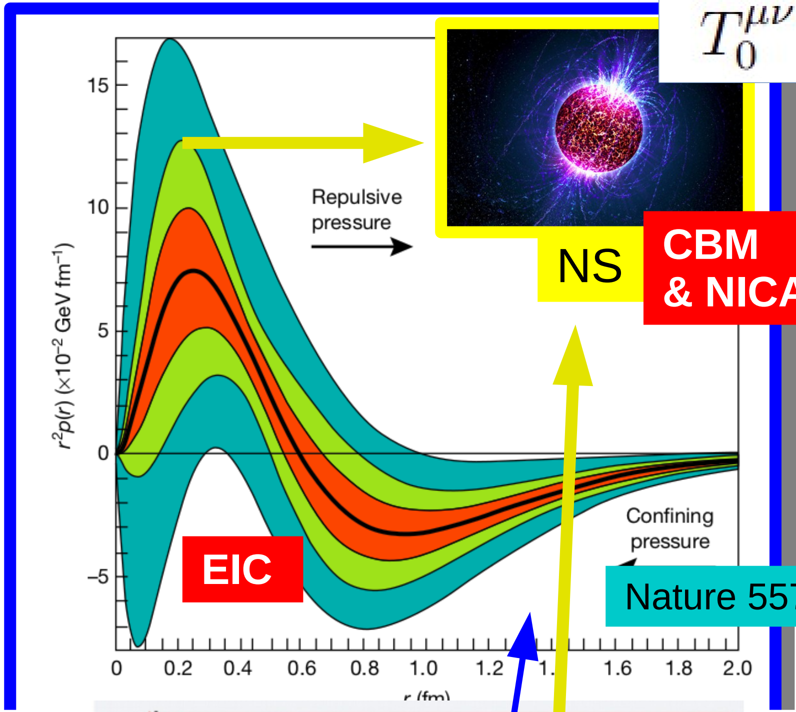
$$P_{UR} = \frac{2}{\pi^2} T^4 f_4(A) \tag{37}$$

- **Case:1** T=0K

$$P_{UR} = \frac{1}{12\pi^2}\mu^4 \tag{38}$$

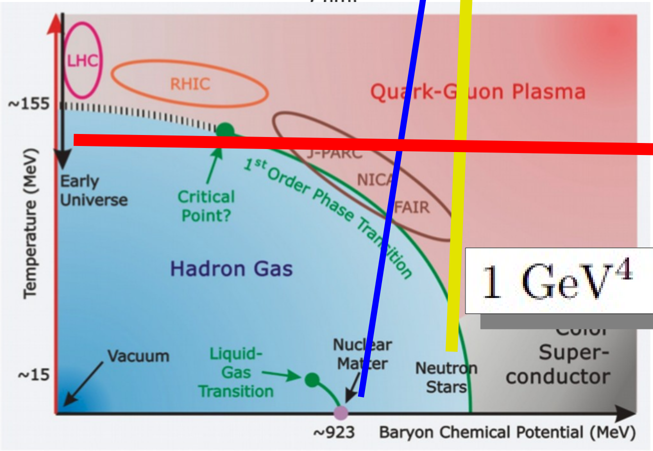
- **Case:2** $\mu = 0$

$$P_{UR} = \frac{1.89}{\pi^2} T^4 \tag{39}$$



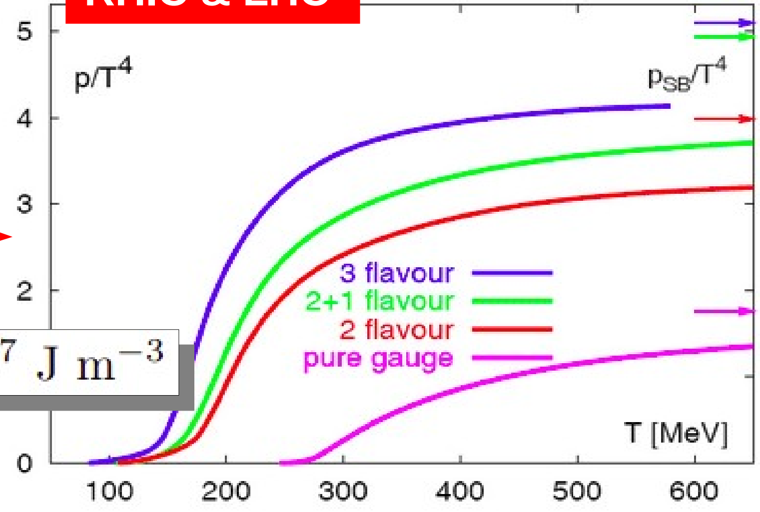
Nature 557, 396 (2018)

See the Poster of Thandar et al.



$$1\ \mathrm{GeV}^4 \rightarrow 2.0852 \times 10^{37}\ \mathrm{J\ m}^{-3}$$

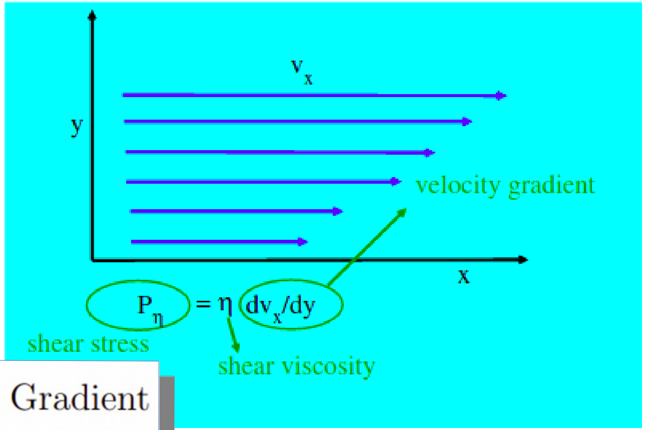
RHIC & LHC

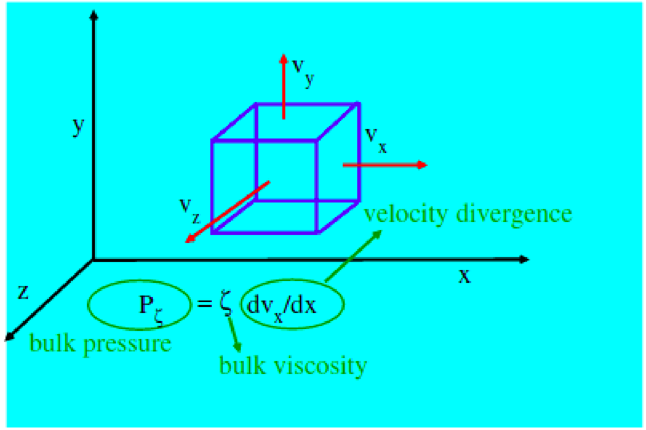


Dissipative Part

$$T_D^{\mu\nu} = W^\mu u^\nu + W^\nu u^\mu + \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}$$

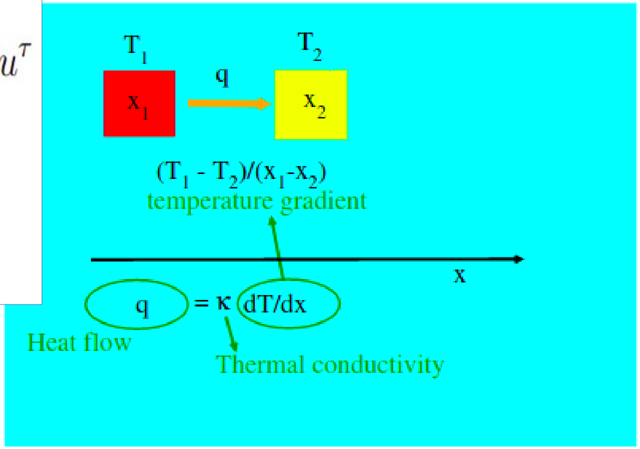
$$\begin{aligned} \pi^{\mu\nu} &= 2\eta\mathcal{U}_\eta^{\mu\nu} , \\ \Pi &= -\zeta\mathcal{U}_\zeta , \\ \text{and } q^\mu &= \kappa\mathcal{U}_\kappa^\mu , \end{aligned}$$

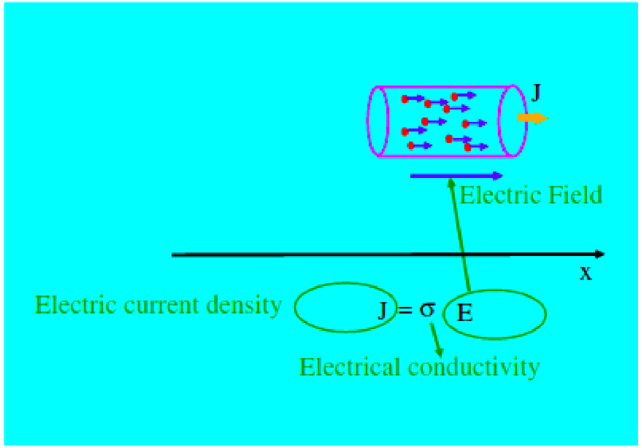




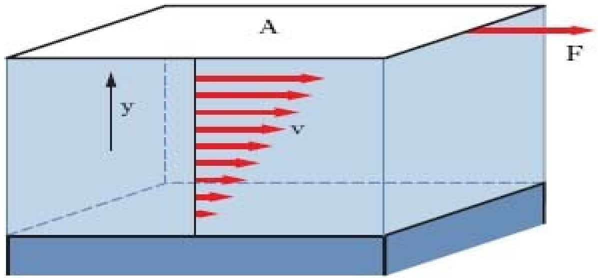
$$\text{Dissipating pressure} = (\text{transport coefficient}) \times \text{Gradient}$$

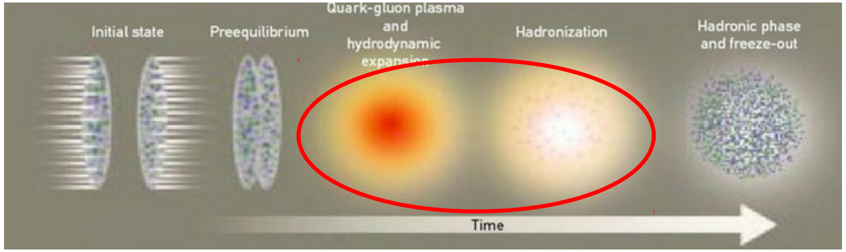
$$\begin{aligned} \mathcal{U}_\eta^{\mu\nu} &\equiv \left[\frac{1}{2}(\Delta_\sigma^\mu\Delta_\tau^\nu + \Delta_\sigma^\nu\Delta_\tau^\mu) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\sigma\tau}\right]\nabla^\sigma u^\tau \\ \text{and } \mathcal{U}_\zeta &\equiv \partial_\rho u^\rho , \\ \frac{\mathcal{U}_\kappa^\mu}{T} &\equiv -\frac{T^2}{h}\Delta^{\mu\rho}\partial_\rho\left(\frac{\mu}{T}\right) . \end{aligned}$$





Kinetic Theory (Relaxation Time Approximation) for B=0





$$\delta f = (A_{ij}U_\eta^{ij} + C_i E^i) f(1 \pm f) \ ,$$

Macro | **Micro**

$$\eta U_\eta^{ij} = T^{ij} = g \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} \delta f$$

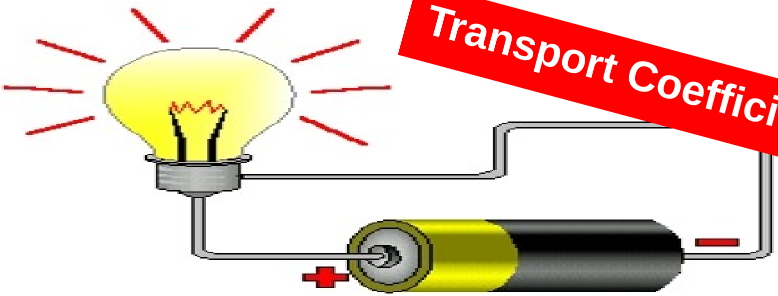
$$\sigma^{ij} E_j = J^i = g_e e \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{E} \delta f \ ,$$

$$\frac{p^\mu}{E}\partial_\mu f^\pm + F^\mu \frac{\partial f^\pm}{\partial p^\mu} = -\left(\frac{p^\mu u_\mu}{E}\right)\frac{\delta f^\pm}{\tau_c}$$

Relativistic Boltzmann Equation

$$\eta_{g,Q} = \frac{g_{g,Q}}{15T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}^2}{\omega}\right)^2 \tau f(1 \pm f)$$

$$\sigma_Q = \frac{e_Q^2 g_e}{3T} \int \frac{d^3\vec{p}}{(2\pi)^3} \left(\frac{\vec{p}}{\omega}\right)^2 \tau f(1 - f) \ .$$



Transport Coefficient = Relaxation time * Phase-space

Green-Kubo Relation of Transport coefficients

R. Kubo, J. Phys. Soc. Jpn. 12, 570 (1957).

Operators

Energy-momentum tensor & conserved current

$$\pi^{ij} \equiv T^{ij} - g^{ij}T^k_k/3\ ,$$
$$\mathcal{P} \equiv -T^k_k/3 - c_s^2 T^{00}\ \text{(for vanishing chemicalpotential } \mu = 0),$$
$$\mathcal{K}^i \equiv T^{0i} - hN^i\ ,$$

Transport Coefficients

Static Limit

Dynamical structure

Thermal Corelators

Shear Viscosity

$$\eta = \frac{1}{20}\lim_{q_0,\vec{q}\to 0}\frac{A_\eta(q_0,\vec{q})}{q_0}\ ,\ A_\eta = \int d^4x e^{iq\cdot x}\langle[\pi^{ij}(x),\pi_{ij}(0)]\rangle_\beta\ ;$$

Bulk Viscosity

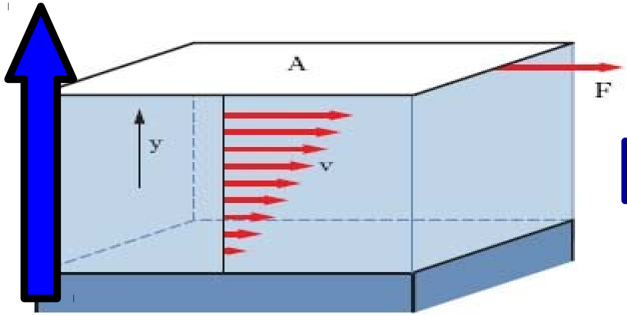
$$\zeta = \frac{1}{2}\lim_{q_0,\vec{q}\to 0}\frac{A_\zeta(q_0,\vec{q})}{q_0}\ ,\ A_\zeta = \int d^4x e^{iq\cdot x}\langle[\mathcal{P}(x),\mathcal{P}(0)]\rangle_\beta\ ;$$

Thermal Conductivity

$$\kappa = \frac{\beta}{6}\lim_{q_0,\vec{q}\to 0}\frac{A_\kappa(q_0,\vec{q})}{q_0}\ ,\ A_\kappa = \int d^4x e^{iq\cdot x}\langle[\mathcal{K}^i(x),\mathcal{K}_i(0)]\rangle_\beta$$

Heat Energy Transport

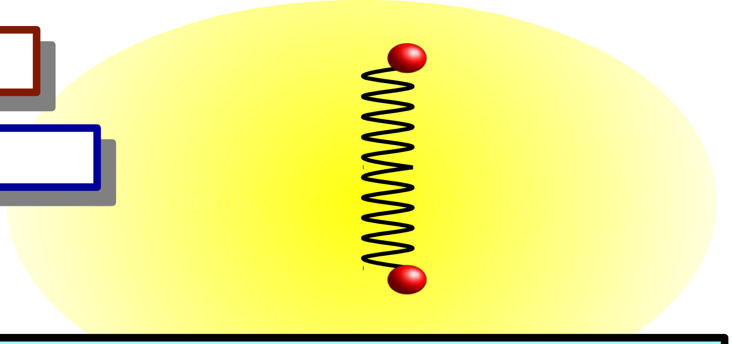
Momentum Transport



Classical Picture

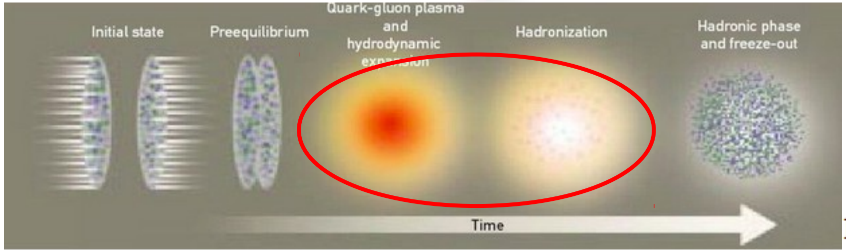
Velocity gradient

Temperature gradient



Picture of QFT at finte temperature

Kinetic Theory (Relaxation Time Approximation) for finite B



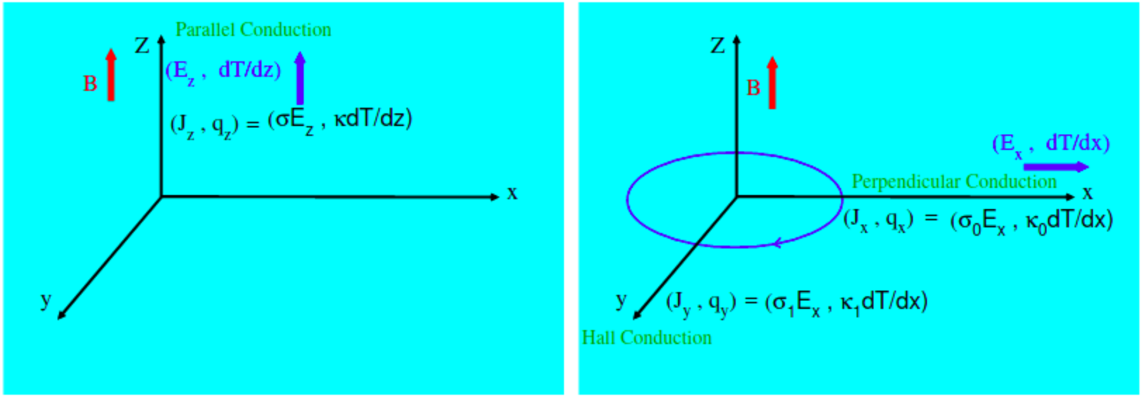


Fig. 5 Schematic diagram of parallel (Left), perpendicular and Hall (Right) components ... rical and thermal conductivity.

Macro | **Micro**

$$\sigma^{ij}E_j = J^i = g_e e\int \frac{d^3p}{(2\pi)^3}\frac{p^\mu}{E}\delta f\,,$$

$$\sigma_Q = \frac{e_Q^2 g_e}{3T}\int\frac{d^3\vec{p}}{(2\pi)^3}\left(\frac{\vec{p}}{\omega}\right)^2\tau f(1-f)\,.$$

τ_c

$\tau_c/[1+(\tau_c/\tau_B)^2]$

$\tau_c(\tau_c/\tau_B)/[1+(\tau_c/\tau_B)^2]$

Relativistic Boltzmann Equation

$$\frac{p^i}{E}\partial_i f_0^\pm + e_{Q,\bar{Q}}\mathcal{E}^i\frac{\partial f_0^\pm}{\partial p^i} + e_{Q,\bar{Q}}Bb^{ij}v_j\frac{\partial}{\partial p^i}\left(\delta f^\pm\right) = -\frac{\delta f^\pm}{\tau_c}$$

Transport Coefficient = Relaxation time * Phase-space

References on Microscopic calculation of transport coefficients at finite magnetic field

Shear Viscosity	• **K. Tuchin**, J. Phys. G 39, 025010 (**2012**) • **G.S. Denicol, X-G Huang, E. Molnar, G.M. Monterio, H.Neimi**, Phys.Rev.D 98 (**2018**) 7, 076009 • **S. Ghosh, B. Chatterjee, P. Mohanty, A. Mukharjee, H. Mishra**, Phys. Rev. D 100, 034024 (**2019**) • **P. Mohanty, A. Dash, and V. Roy**, Eur. Phys. J. A 55, 35 (**2019**) • **A. Das, H. Mishra, and R. K. Mohapatra**, Phys. Rev. D 100, 114004 (**2019**) • **M Kurian, S. Mitra, S. Ghosh,V. Chandra**, Eur. Phys. J. C 79 (**2019**) 2, 134 • **A. Dash, S. Samanta, J. Dey, U. Gangopadhyaya, S. Ghosh, V. Roy**, Phys. Rev. D 102, 016016 (**2020**) • **S. Rath, B.K. Patra**, Phys.Rev.D 102 (**2020**) 3, 036011 • **J. Dey, S. Satapathy, P. Murmu, and S. Ghosh**, Pramana 95, 125 (**2021**) • **J. Dey, S. Satapathy, A. Mishra, S. Paul, and S. Ghosh**, Int. J. Mod. Phys. E 30, 2150044 (**2021**); • **S. Ghosh, S. Ghosh**, Phys. Rev. D 103 (**2021**) 096015 • **R. Ghosh, N. Haque**, Phys.Rev.D 105 (**2022**) 11, 11 • **U. Gangopadhyaya, V. Roy**, JHEP 09 (**2022**) 114 • + ...
Electrical Conductivity	• **K. Hattori, D. Satow**, Phys. Rev. D 94, 114032 (**2016**) • **A. Harutyunyan and A. Sedrakian**, Phys. Rev. C 94, 025805 (**2016**) • **M. Kurian, V. Chandra**, Phys.Rev.D 96 (**2017**) 11, 114026 • **K. Fukushima and Y. Hidaka**, Phys. Rev. Lett. 120, 162301 (**2018**) • **A. Das, H. Mishra, R. K. Mohapatra**, Phys. Rev. D 101, 034027 (**2020**) • **A. Bandyopadhyay, S. Ghosh, R. L. S. Farias, J. Dey, G. a. Krein**, Phys. Rev. D 102, 114015 (**2020**) • **P. Kalikotay, S. Ghosh, N. Choudhory, P. Roy, S. Sarkar**, Phys.Rev.D 102 (**2020**) 7, 076007 • **S. Rath, B.K. Patra**, Eur.Phys.J.C 80 (**2020**) 8, 747 • **B. Chatterjee, R. Rath, G. Sarwar, R. Sahoo**, Eur.Phys.J.A 57 (**2021**) 2, 45 • **K.K. Gowthama, M Kurian, V. Chandra**, Phys.Rev.D 104 (**2021**) 9, 094037 • **S. Sarkar, S.P. Adhya**, Phys.Scripta 97 (**2022**) 10, 104003 • **J. Dey, S. Samanta, S. Ghosh, S. Satapathy**, Phys.Rev.C 106 (**2022**) 4, 044914 • **K. Singh, J. Dey, R Sahoo, S. Ghosh** in poster presentation • + ...

Last 5 Years

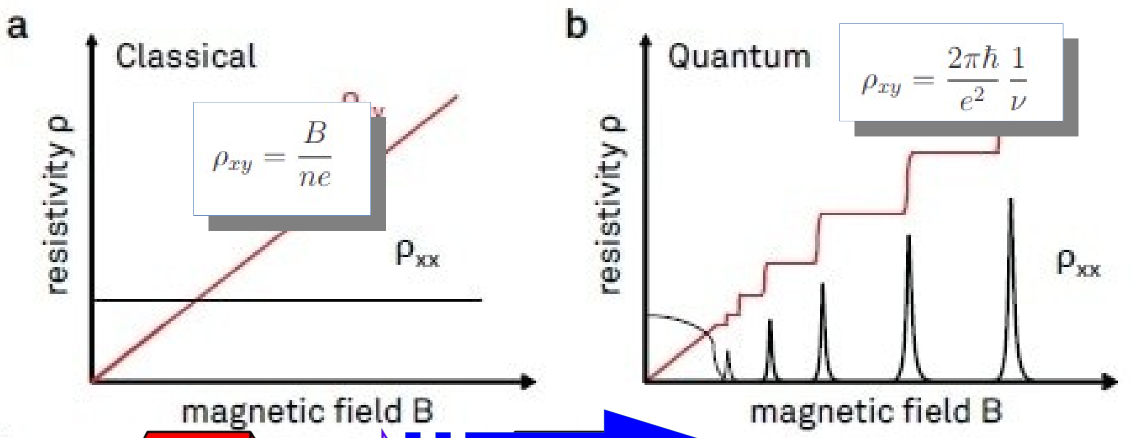
Resistivity Matrix:

The *resistivity* is defined as the inverse of the conduc
both are matrices,

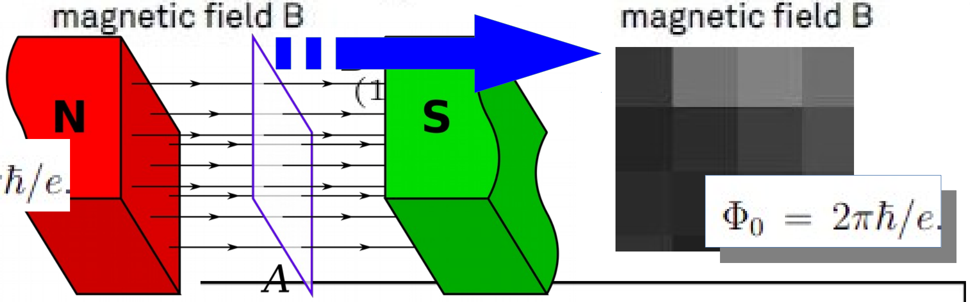
$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$$

From the Drude model, we have

$$\rho = \frac{1}{\sigma_{DC}} \begin{pmatrix} 1 & \omega_B \tau \\ -\omega_B \tau & 1 \end{pmatrix}$$

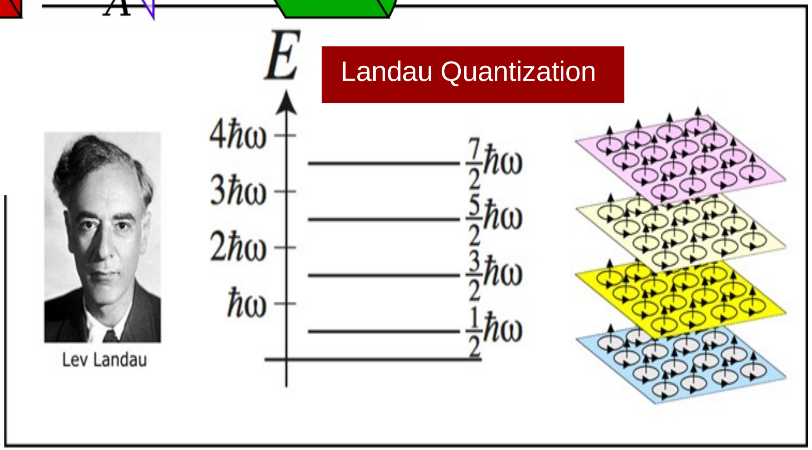


$$\Phi_0 = 2\pi\hbar/e.$$



Electrical Conductivity Matrix:

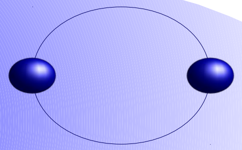
$$\sigma = \frac{\sigma_{DC}}{1+\omega_B^2\tau^2} \begin{pmatrix} 1 & -\omega_B \tau \\ \omega_B \tau & 1 \end{pmatrix} \quad \text{with} \quad \sigma_{DC} = \frac{ne^2\tau}{m}$$



Kubo Formulism: QFT ->TFT -> TFT for finite B

S. Satapathy, S. Ghosh, S. Ghosh, Phys.Rev.D 106 (2022) 3, 036006

Fermion Self-energy in Medium



$$\langle \mathcal{T}_C J^\mu(x) J^\nu(0)\rangle_{11}^B$$

QFT

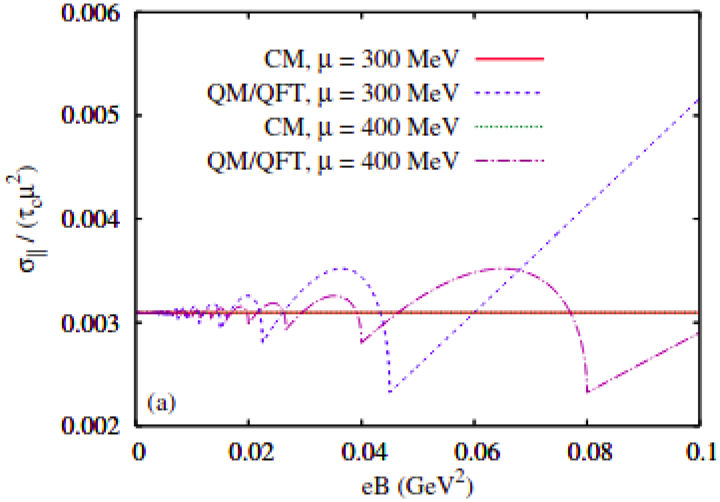
$$\sigma_\parallel = e^2\left(\frac{eB}{2\pi^2}\right)\frac{1}{\Gamma\mu}\sum_{l=0}^{l_{\max}}(2-\delta_l^0)\sqrt{\mu^2-m_l^2}\,\Theta(\mu-m_l),$$

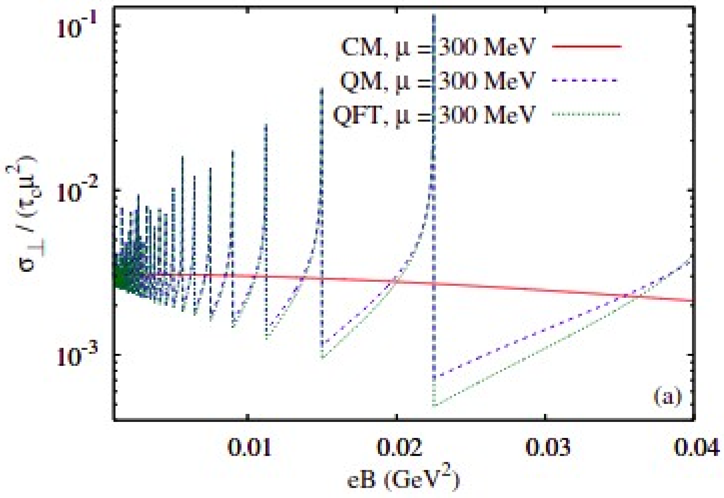
$$\sigma_\perp = e^2\left(\frac{eB}{2\pi^2}\right)\frac{\Gamma}{\Gamma^2+(\mu-\sqrt{\mu^2-2eB})^2}\frac{1}{\sqrt{\mu^2-2eB}}\sum_{l=1}^{l_{\max}}\frac{(2l-1)eB}{\sqrt{\mu^2-m_l^2}}\Theta(\mu-m_l),$$

$$l_{\max} = \left\lfloor \frac{\mu^2-m^2}{2eB} \right\rfloor$$

CM

$$\sigma^{\rm RTA}_{\parallel,\perp} = 2e^2\int\frac{d^3k}{(2\pi)^3}\tau_c^{\parallel,\perp}\frac{\vec{k}^2}{3\omega_k^2}\delta(\omega_k-\mu) = \frac{e^2}{3\pi^2}\frac{(\mu^2-m^2)^{3/2}}{\mu}\tau_c^{\parallel,\perp}$$





Concluding Bullet Points:

- **Due to magnetic field multi component transport coefficients structure is expected (Parallel, Perpendicular & Hall)**
- **Two time scales (relaxation time and cyclotron time) will create an effective relaxation time in different direction/component**
- **Landau quantization effect will modify the transport coefficients values from its Classical values**
- **Apart from Landau quantization effect, QFT calculation can introduce a QFT version of cyclotron time**

$$\tilde{\tau}_c^{\perp} = \frac{\Gamma}{\Gamma^2 + \left(\mu - \sqrt{\mu^2 - 2eB}\right)^2} = \tau_c \Big/ \left(1 + \frac{\tau_c^2}{\tilde{\tau}_B^2}\right),$$

where,

$$\frac{1}{\tilde{\tau}_B} = \left(\mu - \sqrt{\mu^2 - 2eB}\right) = \frac{eB}{\mu}\left\{1 + \frac{eB}{2\mu^2} + \frac{(eB)^2}{2\mu^4} + \frac{5(eB)^3}{8\mu^6} + \cdots\right\} = \frac{1}{\tau_B}\left\{1 + \frac{1}{2\mu\tau_B} + \frac{1}{2(\mu\tau_B)^2} + \frac{5}{8(\mu\tau_B)^3} + \cdots\right\}$$

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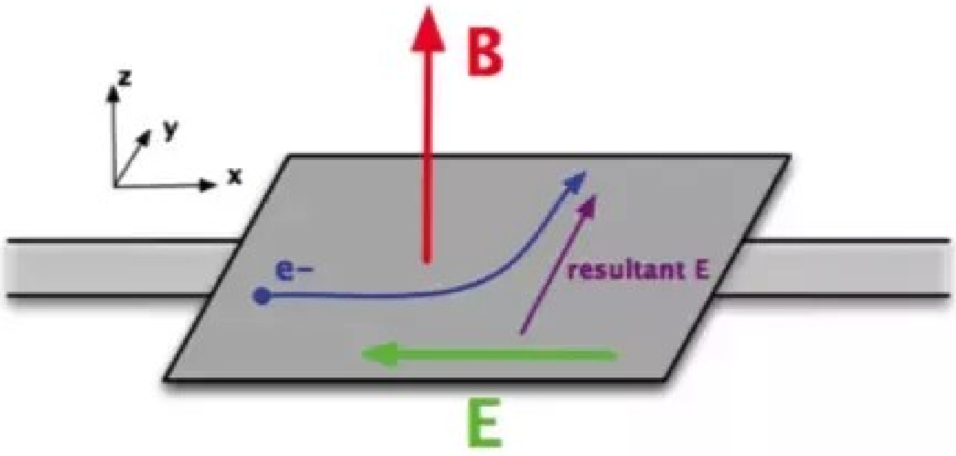

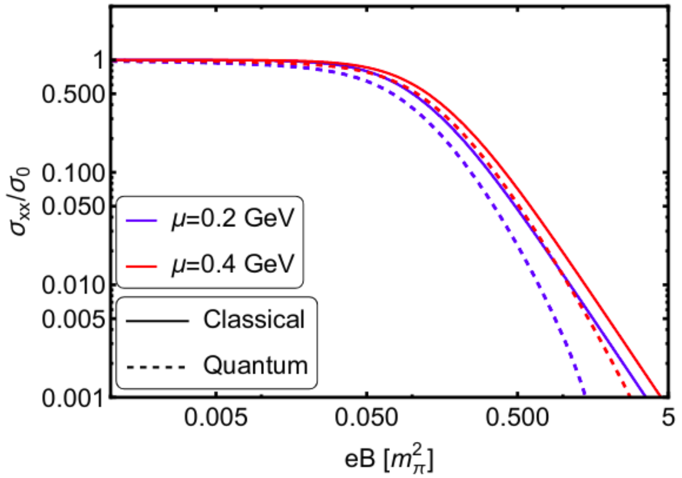

ArXiv 2103.15364 [hep-ph]