



Percolation and de-confinement in relativistic nuclear collisions

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ICPAQGP-2023
Feb. 7-10, Puri, India

Annual Review of Nuclear and Particle Science

Heavy Ion Collisions: The Big Picture and the Big Questions

Wit Busza,¹ Krishna Rajagopal,^{1,2}
and Wilke van der Schee^{2,3}

Ann. Rev. Nucl. Part. Sci. 68 (2018) 339-376

Big Questions:

5. Can we obtain an experimental determination, even indirectly, of the temperature of the matter produced in a heavy ion collision at a time at which we can also determine its energy density? If so, we could obtain an experimental determination of the number of thermodynamic degrees of freedom, the quantity whose increase reflects the liberation of color above the crossover in the QCD phase diagram.

**De-Confinement and Clustering of Color Sources
In
Nuclear Collisions**

Color Strings

Multiparticle production at high energies is currently described in terms of color strings stretched between the projectile and target.

These strings decay into new ones by $q-\bar{q}$ production and subsequently hadronize to produce the observed hadrons. Particles are produced by the Schwinger 2D mechanism.

As the no. of strings grow with energy and or no. of participating nuclei they start to interact and overlap in transverse space as it happens for disks in the 2-D percolation theory

In the case of a nuclear collisions, the density of disks –elementary strings

$$\xi = \frac{N^s S_1}{S_N}$$

N^s = # of strings

S_1 = Single string area

S_N = total nuclear overlap area

Percolation : General

Percolation, statistical topography, and transport in random media

M. B. Isichenko

Reviews of Modern Physics, Vol. 64, No. 4, October 1992

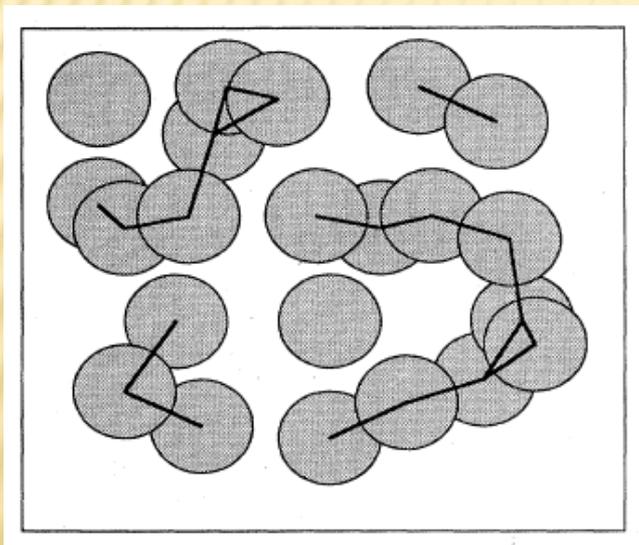
$$\xi = \pi n r^2$$

ξ is the percolation density parameter
 n is the density and r the radius of the disc
disc

ϕ is the fractional area covered by the cluster

$$\phi = 1 - e^{-\xi}$$

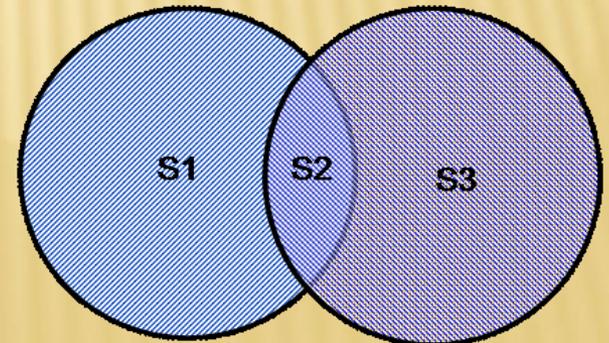
ξ_c is the critical value of the percolation density at which a communicating cluster appears



For example at $\xi_c = 1.2$, $\phi \sim 2/3$

It means $\sim 67\%$ of the whole area is covered by the cluster

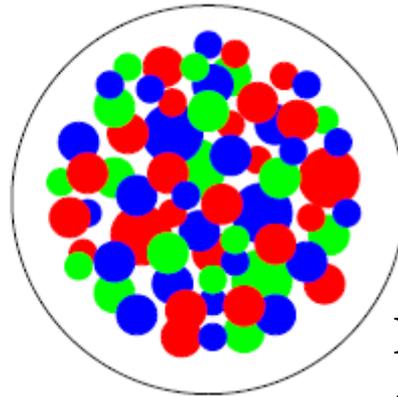
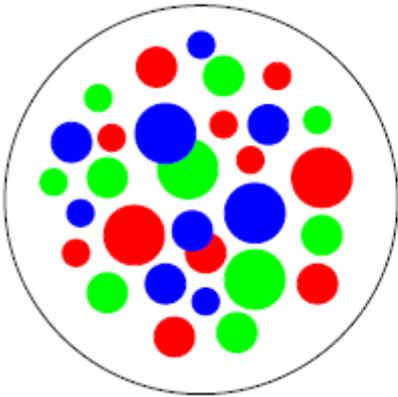
In the nuclear case it is the overlap area



Clustering of Color Sources

De-confinement is expected when the density of quarks and gluons becomes so high that it no longer makes sense to partition them into color-neutral hadrons, since these would overlap strongly.

We have clusters within which color is not confined : De-confinement is thus related to cluster formation very much similar to cluster formation in percolation theory and hence a connection between percolation and de-confinement seems very likely.



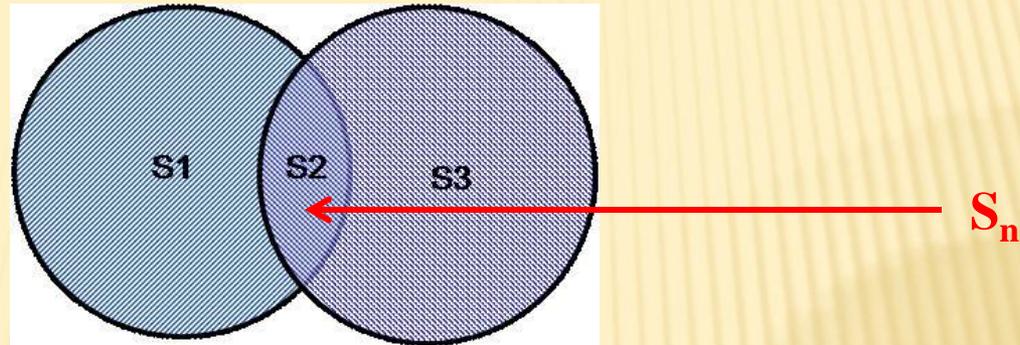
Parton distributions in the transverse plane of nucleus-nucleus collisions

In two dimensions, for uniform string density, the percolation threshold for overlapping discs is: $\xi_c = 1.18$

H. Satz, Rep. Prog. Phys. 63, 1511(2000).
H. Satz, hep-ph/0212046

Critical Percolation Density

Color Sources



The transverse space occupied by a cluster of overlapping strings split into a number of areas in which different no of strings overlap, including areas where no overlapping takes place.

A cluster of n strings that occupies an area S_n behaves as a single color source with a higher color field \vec{Q}_n corresponding to vectorial sum of color charges of each individual string \vec{Q}_1

$$\vec{Q}_n^2 = n\vec{Q}_1^2 \quad \text{If strings are fully overlap}$$

$$\vec{Q}_n^2 = n \frac{S_n}{S_1} \vec{Q}_1^2 \quad \text{Partially overlap}$$

Schwinger mechanism for the Fragmentation

Multiplicity and $\langle p_T^2 \rangle$ of particles produced by a cluster of n strings

Multiplicity (μ_n)

$$\mu_n = F(\xi) N^s \mu_1$$

Average Transverse Momentum

$$\langle p_T^2 \rangle_n = \langle p_T^2 \rangle_1 / F(\xi)$$

$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

= **Color suppression factor**
(due to overlapping of discs).

$$\xi = \frac{N^s S_1}{S_N}$$

N^s = # of strings
 S_1 = disc area
 S_N = total nuclear overlap area

ξ is the string density parameter

M. A. Braun and C. Pajares, Eur.Phys. J. C16,349 (2000)

M. A. Braun et al, Phys. Rev. C65, 024907 (2002)

Percolation and Color Glass Condensate

Both are based on parton coherence phenomena.

Percolation : Clustering of strings

CGC : Gluon saturation

- Many of the results obtained in the framework of percolation of strings are very similar to the one obtained in the CGC.
- In particular , very similar scaling laws are obtained for the product and the ratio of the multiplicities and transverse momentum.
- Both provide explanation for multiplicity suppression and $\langle p_t \rangle$ scaling with dN/dy .

Momentum Q_s establishes the scale in CGC with the corresponding one in percolation of strings

$$Q_s^2 = \frac{k \langle p_t^2 \rangle_1}{F(\xi)}$$

For large value of ξ

$$Q_s^2 \propto \sqrt{\xi}$$

The no. of color flux tubes in CGC and the effective no. of clusters of strings in percolation have the same dependence on the energy and centrality.

This has consequences in the Long range rapidity correlations and the ridge structure.

Color String Percolation Model for Nuclear Collisions

from
SPS-RHIC-LHC

Elementary partonic collisions

Formation of Color String

SU(3) random summation of charges

Reduction in color charge

Increase in the string tension

String breaking leads to formation of secondaries

Probability rate ->Schwinger

Fragmentation proceeds in an iterative way

- 1. Multiplicity**
- 2. pt distribution**
- 3. Particle ratios**
- 4. Elliptic flow**
- 5. Suppression of high pt particles R_{AA}**
- 6. J/ψ production**
- 7. Forward-Backward Multiplicity Correlations at RHIC**

Schwinger : p_t distribution of the produced quarks

$$\frac{dn}{d^2 p_{\perp}} \sim \exp\left(-\frac{\pi p_t^2}{k^2}\right)$$

Thermal Distribution

$$\frac{dn}{d^2 p_{\perp}} \sim \exp\left(-\frac{\pi p_t}{T}\right)$$

The Schwinger formula can be reconciled with the thermal distribution if the String tension undergoes fluctuations

$$P(k)dk = \sqrt{\frac{2}{\pi \langle k^2 \rangle}} \exp\left(-\frac{k^2}{2 \langle k^2 \rangle}\right) dk$$

which gives rise to thermal distribution

$$\frac{dn}{d^2 p_{\perp}} \sim \exp\left(-p_{\perp} \sqrt{\frac{2\pi}{\langle k^2 \rangle}}\right)$$

$$T = \sqrt{\frac{\langle k^2 \rangle}{2\pi}}$$

Initial temperature

$$\sqrt{\langle p_t^2 \rangle} = \sqrt{\frac{\langle k^2 \rangle}{\pi}} = \sqrt{\frac{\langle p_t^2 \rangle_1}{F(\xi)}}$$

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

Thermalization

- ❑ **The origin of the string fluctuation is related to the stochastic picture of the QCD vacuum . Since the average value of color field strength must vanish, it cannot be constant and must vanish from point to point. Such fluctuations lead to the Gaussian distribution of the string.**

H. G. Dosch, Phys. Lett. 190 (1987) 177

A. Bialas, Phys. Lett. B 466 (1999) 301

- ❑ **The fast thermalization in heavy ion collisions can occur through the existence of event horizon caused by rapid deceleration of the colliding nuclei. Hawking-Unruh effect encountered in black holes and for accelerated objects.**

D. Kharzeev, E. Levin , K. Tuchin, Phys. Rev. C75, 044903 (2007)

H.Satz, Eur. Phys. J. 155, (2008) 167

Data Analysis

Using the p_T spectrum to extract $F(\xi)$

The experimental p_T distribution from pp data is used

$$\frac{d^2 N}{dpt^2} = \frac{a}{(p_0 + pt)^n}$$

a , p_0 and n are parameters fit to the data.

This parameterization can be used for nucleus-nucleus collisions to account for the clustering :

$$\frac{d^2 N}{dpt^2} = \frac{b}{\left(p_0 \sqrt{\frac{F(\xi_{pp})}{F(\xi_{AuAu})}} + pt \right)^n}$$

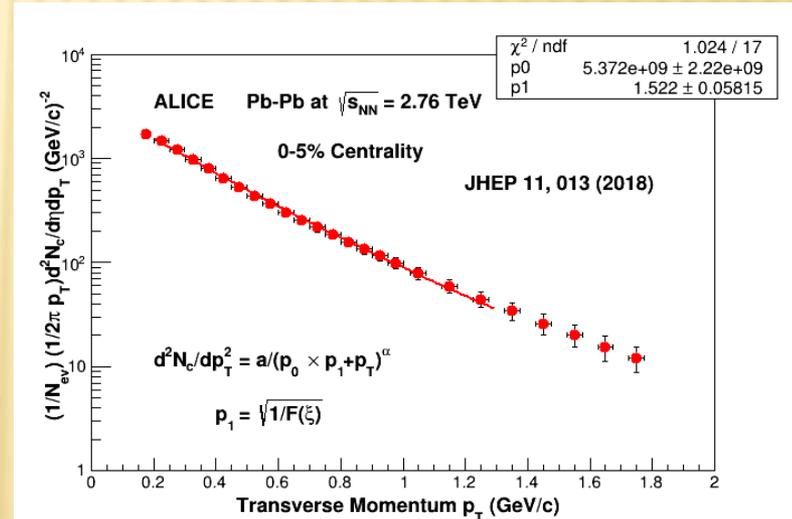
$$F(\xi)_{pp} = 1$$

Parameterization of STAR 200 GeV

$p_0 = 1.982$ and $n = 12.877$
 Phys. Rep. 599 (2015) 1-50

Parameterization of UA1 data
 from 200, 500 and 900 GeV
 ISR 53 and 23 GeV pp

$p_0 = 1.71$ and $n = 12.42$
 Nucl. Phys. A698, 331 (2002)



Determination of the Color Suppression Factor $F(\xi)$ from the ALICE data for pp and Pb-Pb collisions

Thermodynamics

- Temperature
- Energy Density
- Degrees of Freedom

Temperature

$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

For Au+Au@ 200 GeV
0-10% centrality $\xi = 2.88$
 $T = 193 \pm 3.5$ MeV

PHENIX:

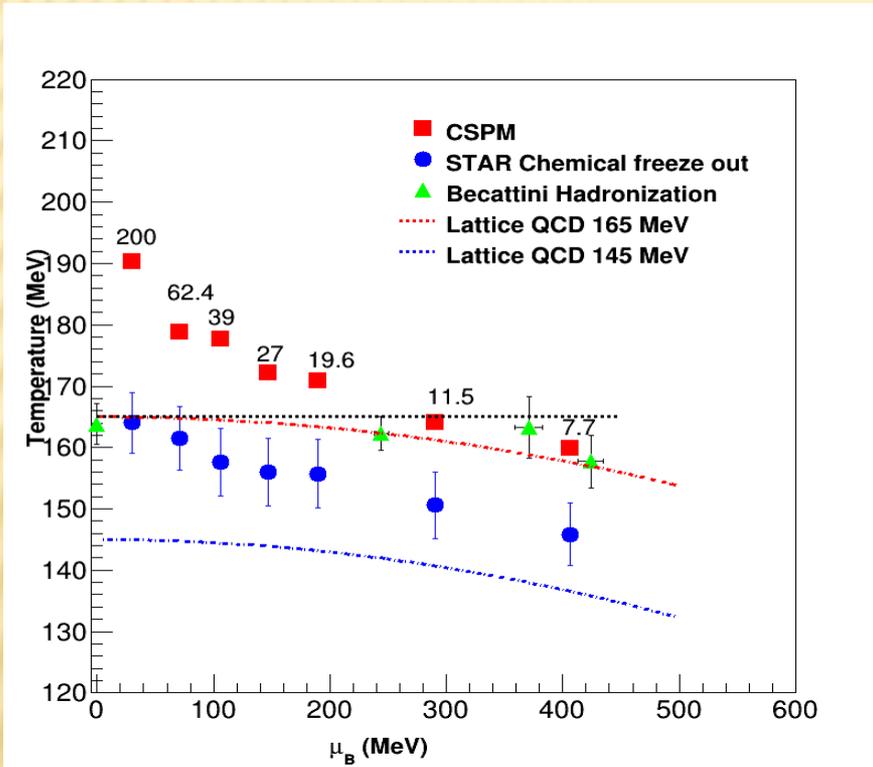
Temperature from direct photon
Exponential (consistent with thermal)
Inverse slope = 220 ± 20 MeV
PRL 104, 132301 (2010)

Braun, Dias de Deus, Hirsch, Pajares, Scharenberg, & Srivastava
Phys. Rep. 599 (2015) 1-50.

Mod. Phys. Lett A 34, 1950034 (2019)

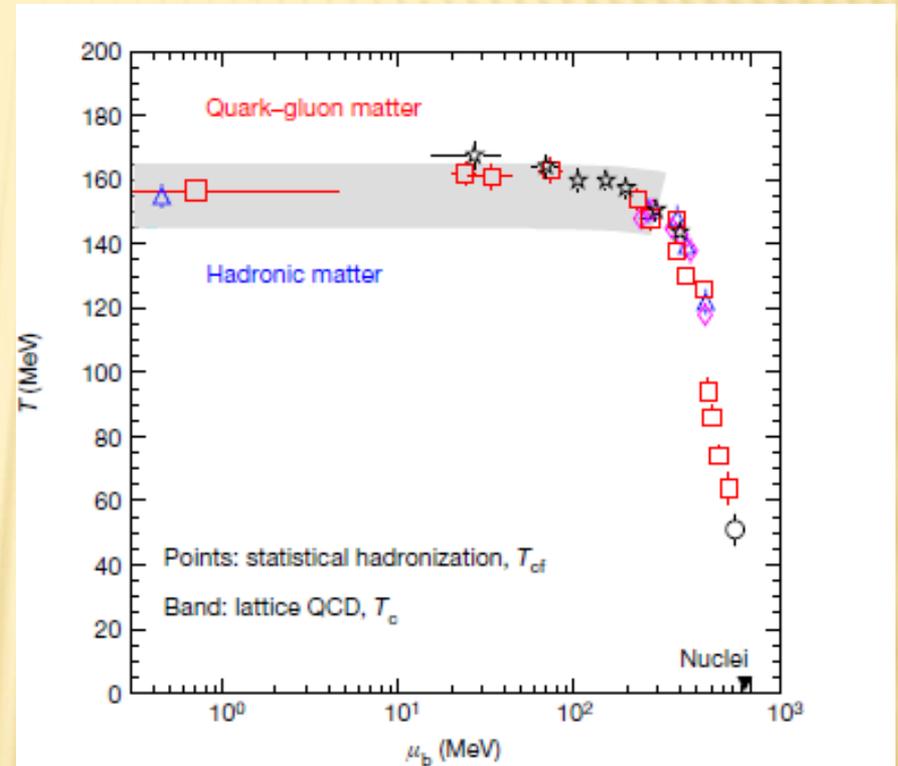
BES STAR

Pragati et al.



Naure 561, 322 (2018)

Decoding the phase structure
Andronic, Braun-Munzinger, Redlich ,
Stachel



Chemical freeze-out : 156.5 ± 1.5 MeV

LQCD (Bazavov et al.) : 154 ± 9

LQCD (Borsanyi et al.) : 156 ± 9

Summary I : Heavy Ion

- The Clustering of Color Sources leading to the Percolation Transition may be the way to achieve de-confinement in High Energy collisions.
- This picture provide us with a microscopic partonic structure which explains the early thermalization. The relevant quantity is transverse

string density $\xi = \frac{N^s S_1}{S_N}$

- A further definitive test of clustering phenomena can be made at LHC energies by comparing *h-h* and A-A collisions.

**Braun, Dias de Deus, Hirsch, Pajares, Scharenberg and Srivastava
Phys. Rep. 599 (2015) 1-50**

Application of Clustering Picture to Small System

pp at LHC energies 5.02 and 13 TeV

Determination of the Color Suppression Factor $F(\xi)$ using transverse momentum spectra of in high multiplicity events



Temperature

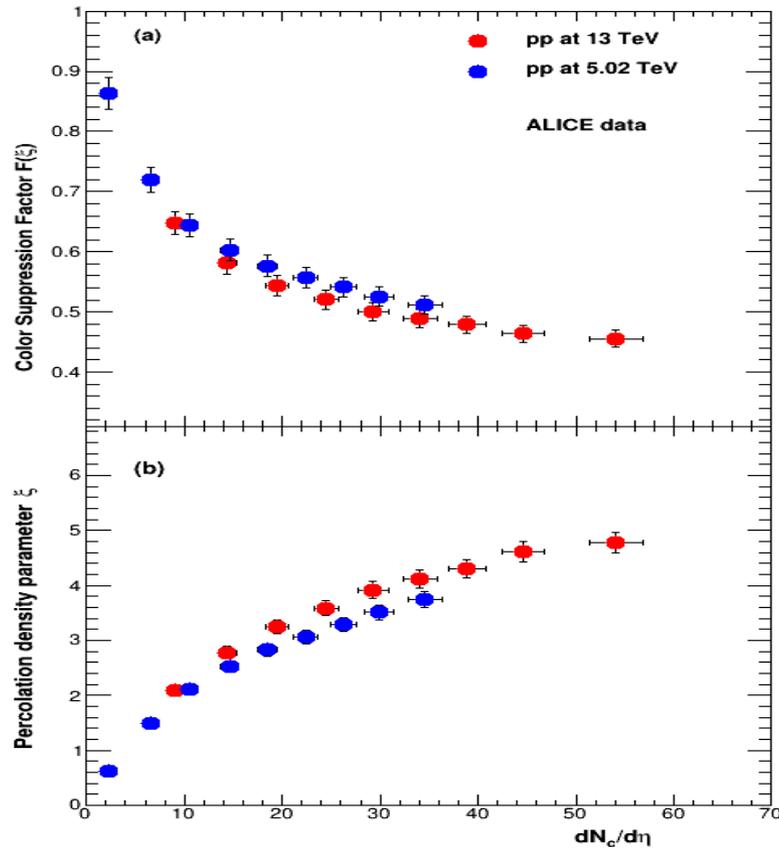


Comparison between AA and pp

Pb+Pb@2.76 and 5.02 TeV,
Xe+Xe@5.44 TeV

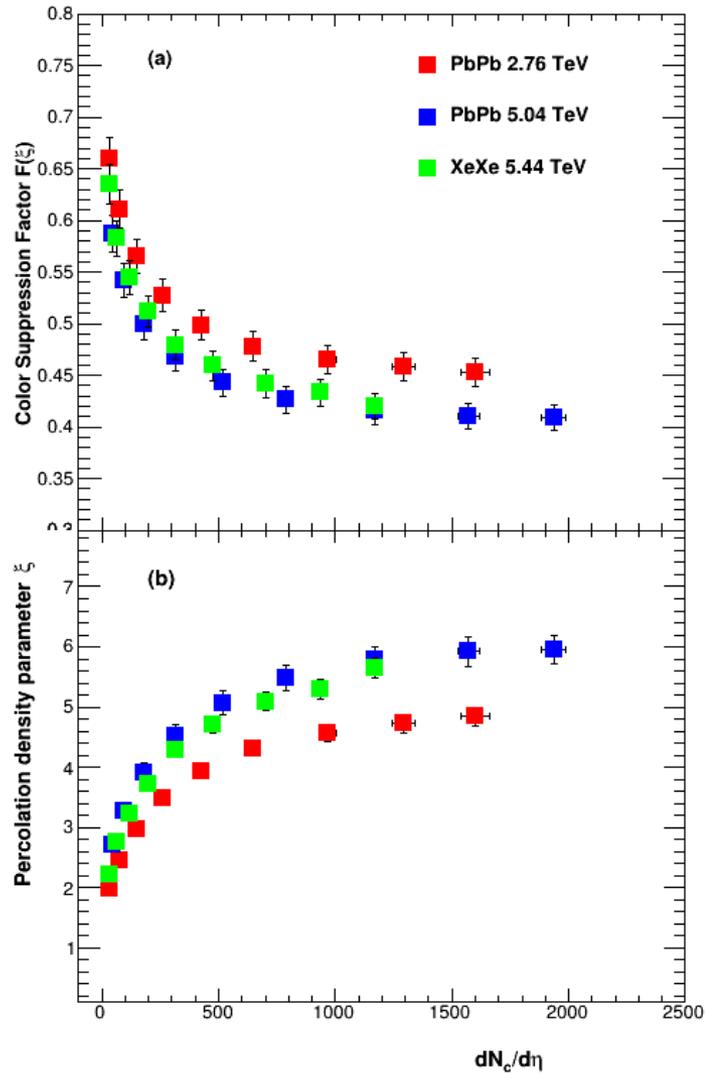
Results : pp data

Analysis of ALICE data to extract Color Suppression Factor $F(\xi)$ from the transverse momentum spectra at 5.02 and 13 TeV as a function of multiplicity



$$F(\xi) = \sqrt{\frac{1 - e^{-\xi}}{\xi}}$$

Results : Heavy Ions



$F(\xi)$ for Pb-Pb and Xe-Xe Collisions as a function of Charged particle multiplicity

Transverse momentum of protons, pions and kaons
in high multiplicity pp and pA collisions:
Evidence for the color glass condensate?

Larry McLerran ^{a,b,c}, Michal Praszalowicz ^d, Björn Schenke ^{a,*}

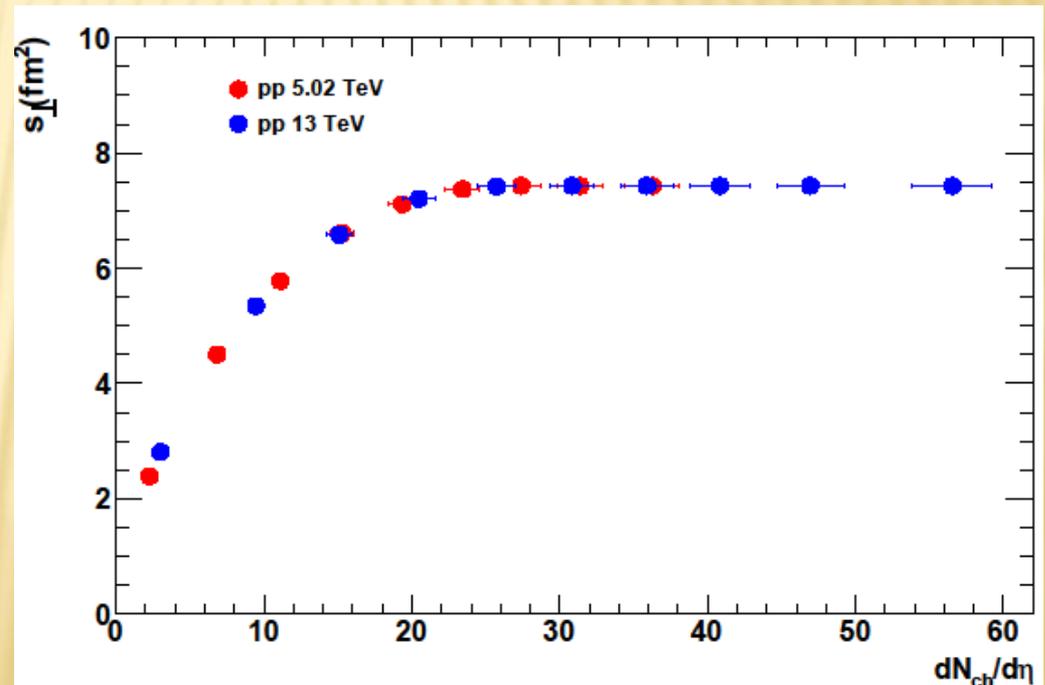
Interaction area is computed: IP-Glasma model

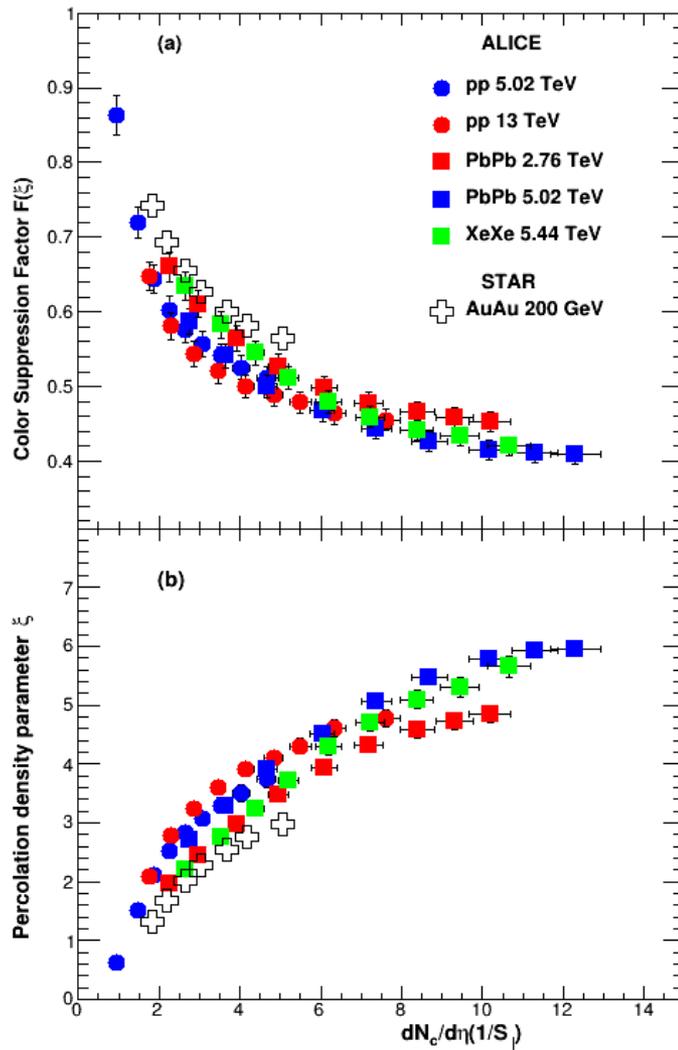
The gluon multiplicity can be
approx. related to the no of
tracks

$$\frac{dN_g}{dy} \approx K \frac{3}{2} \frac{1}{\Delta\eta} N_{track}$$

Transverse area : $S_{pp} = \pi R_{pp}^2$

$$R_{pp} = 1 \text{ fm} \times f_{pp} \left(\sqrt[3]{dN_g/dy} \right)$$

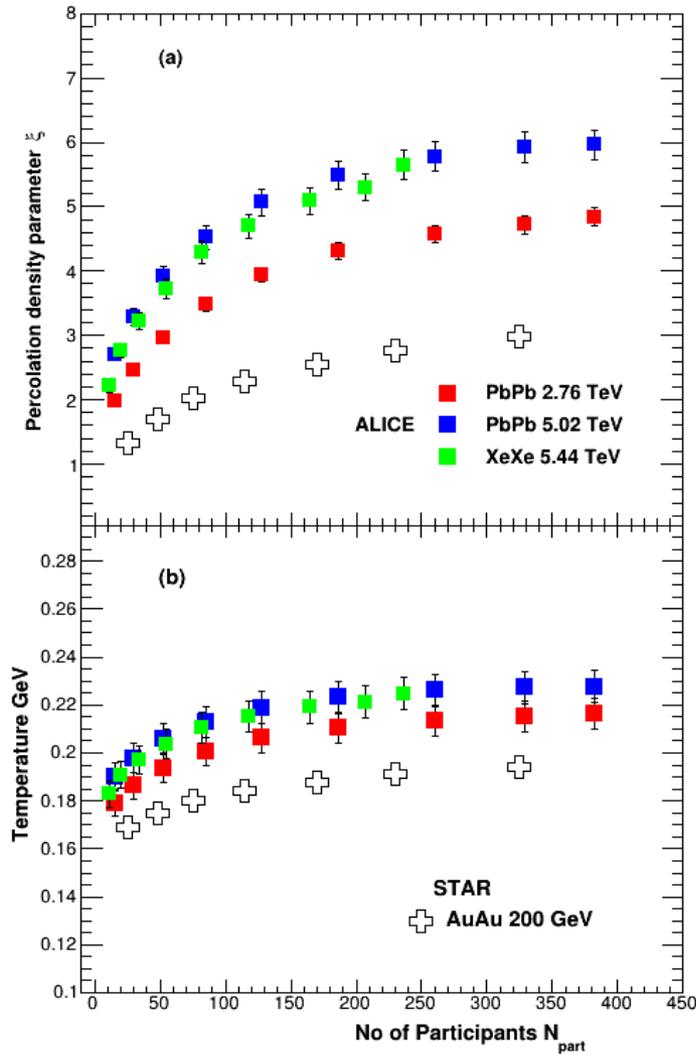




$F(\xi)$ in pp , Pb-Pb and Xe-Xe Collisions vs as a function of $dN_c/d\eta$ scaled by the transverse area S_\perp . For pp collisions S_\perp is multiplicity dependent obtained from IP Glasma model. In case of Pb-Pb and Xe-Xe collisions the nuclear overlap area was obtained using Glauber model.

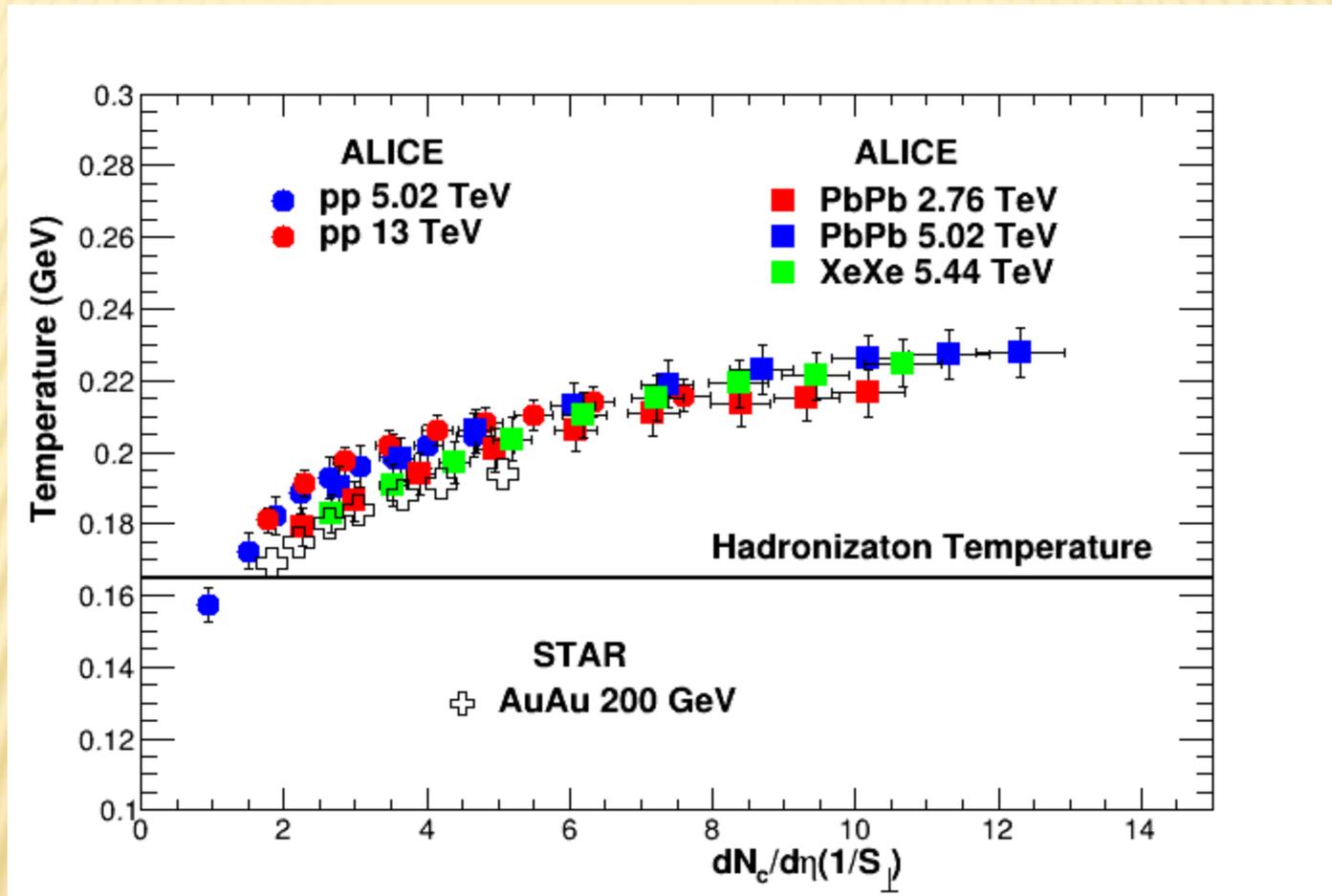
String density parameter ξ . Results are also shown for Au-Au collisions @200 GeV

Temperature and ξ



$$T = \sqrt{\frac{\langle p_t^2 \rangle_1}{2F(\xi)}}$$

Temperature



Becattini et al.
Eur. Phys. J C66, 377 (2010)

Energy Density

Bjorken Phys. Rev. D 27, 140 (1983)

$$\varepsilon = \frac{3}{2} \frac{dN_c}{dy} \frac{\langle m_t \rangle}{A} \frac{1}{\tau_{pro}} \text{GeV} / \text{fm}^3$$

Transverse overlap area

Proper Time

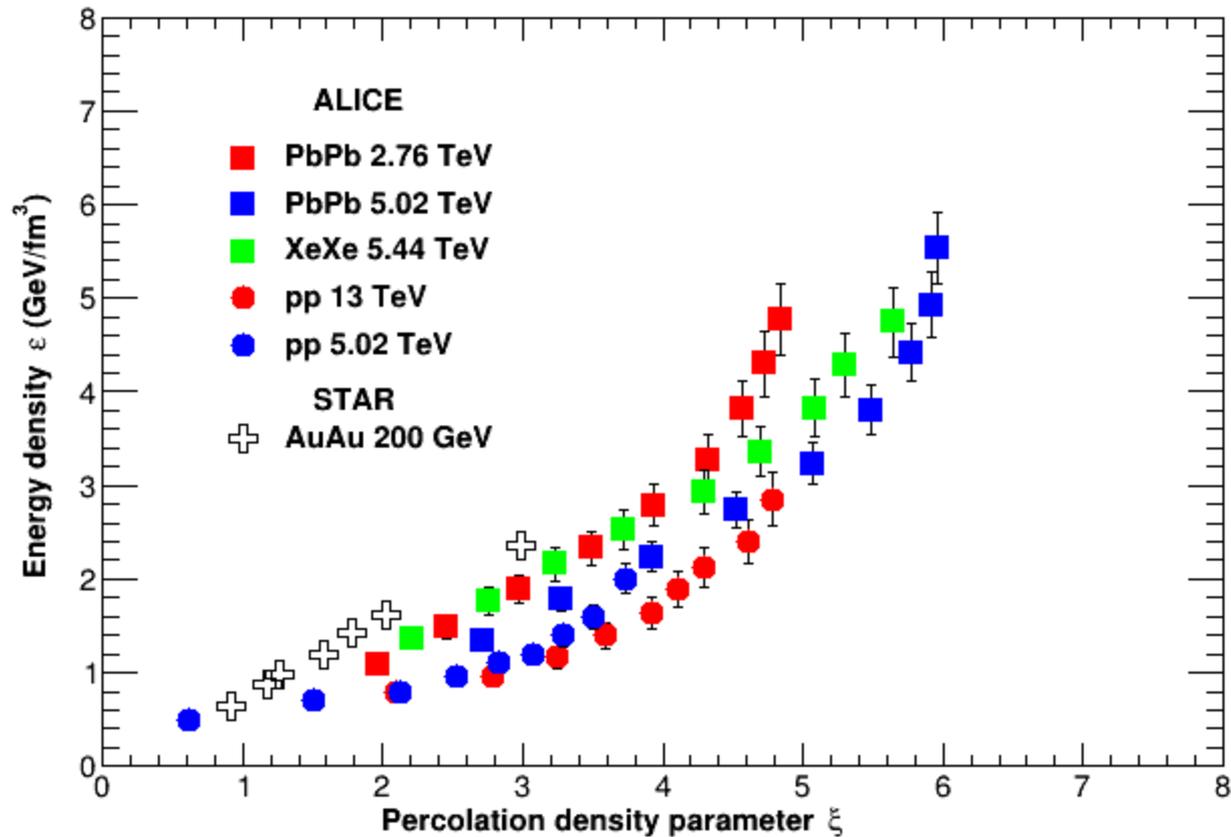
τ_{pro} is the QED production time for a boson which can be scaled from QED to QCD and is given by

$$\tau_{pro} = \frac{2.405\hbar}{\langle m_t \rangle}$$

Introduction to high energy
heavy ion collisions
C. Y. Wong

J. Dias de Deus, A. S. Hirsch, C. Pajares ,
R. P. Scharenberg , B. K. Srivastava
Eur. Phys. J. C 72, 2123 (2012)

Energy Density



Thermodynamic Relations

Equation of State

For an ideal gas following relations hold

$$\varepsilon = G(T) \frac{\pi^2}{30} T^4$$

$$Ts = (\varepsilon + p)$$

$$C_s^2 = \frac{dT}{d\varepsilon} s$$

$G(T)$ is the no. of degrees of freedom

For example for an ideal gas of massless pions $G(T) = 3$ for three charge states of pions.

For an ideal quark-gluon plasma

$$G(T) = [G_g + \frac{7}{8}(G_q + G_{\bar{q}})]$$

$G_g, G_q, G_{\bar{q}}$



Degeneracy of the gluons, the quarks and anti quarks

LATTICE QCD

Among the most important and fundamental problems in finite-temperature QCD are the calculation of the bulk properties of **HOT** QCD matter and the characterization of the nature of the QCD phase transition. QCD describes the interaction of quarks and gluons very similar to QED, which deals with the interaction of electrons and photons.

Understanding finite temperature QCD has direct application in interpreting the results **from heavy ion collision experiments**.

Reaching a detailed understanding of bulk thermodynamics of QCD , e.g. **the temperature dependence of pressure , energy density as well as the EOS $p(\epsilon)$ vs ϵ , is one of the central goal of studies of the QCD on the Lattice .**

Lattice Simulation results

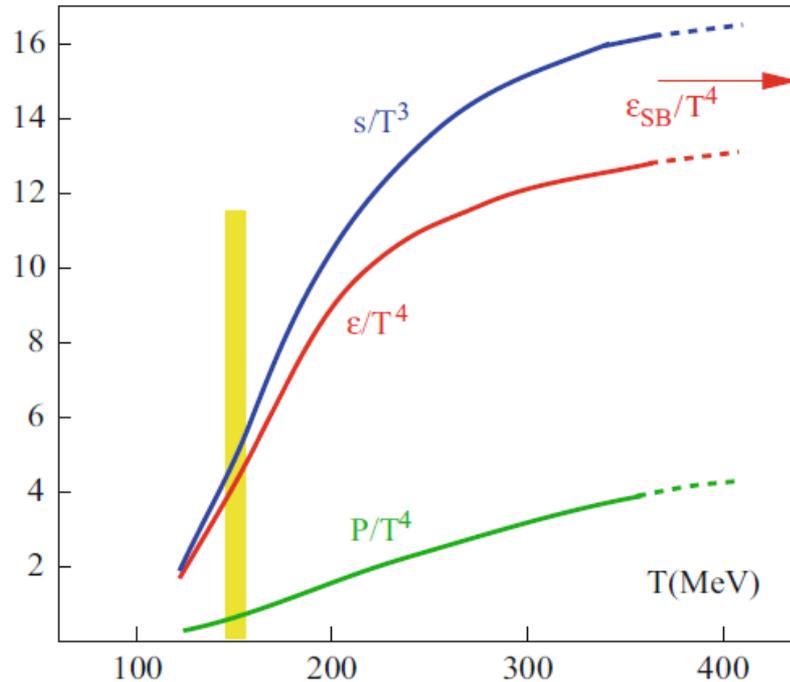


Fig. 5.6 Energy density, entropy density and pressure in full QCD with (2+1) physical quarks [45]. The ideal gas energy density is indicated by ϵ/T^4

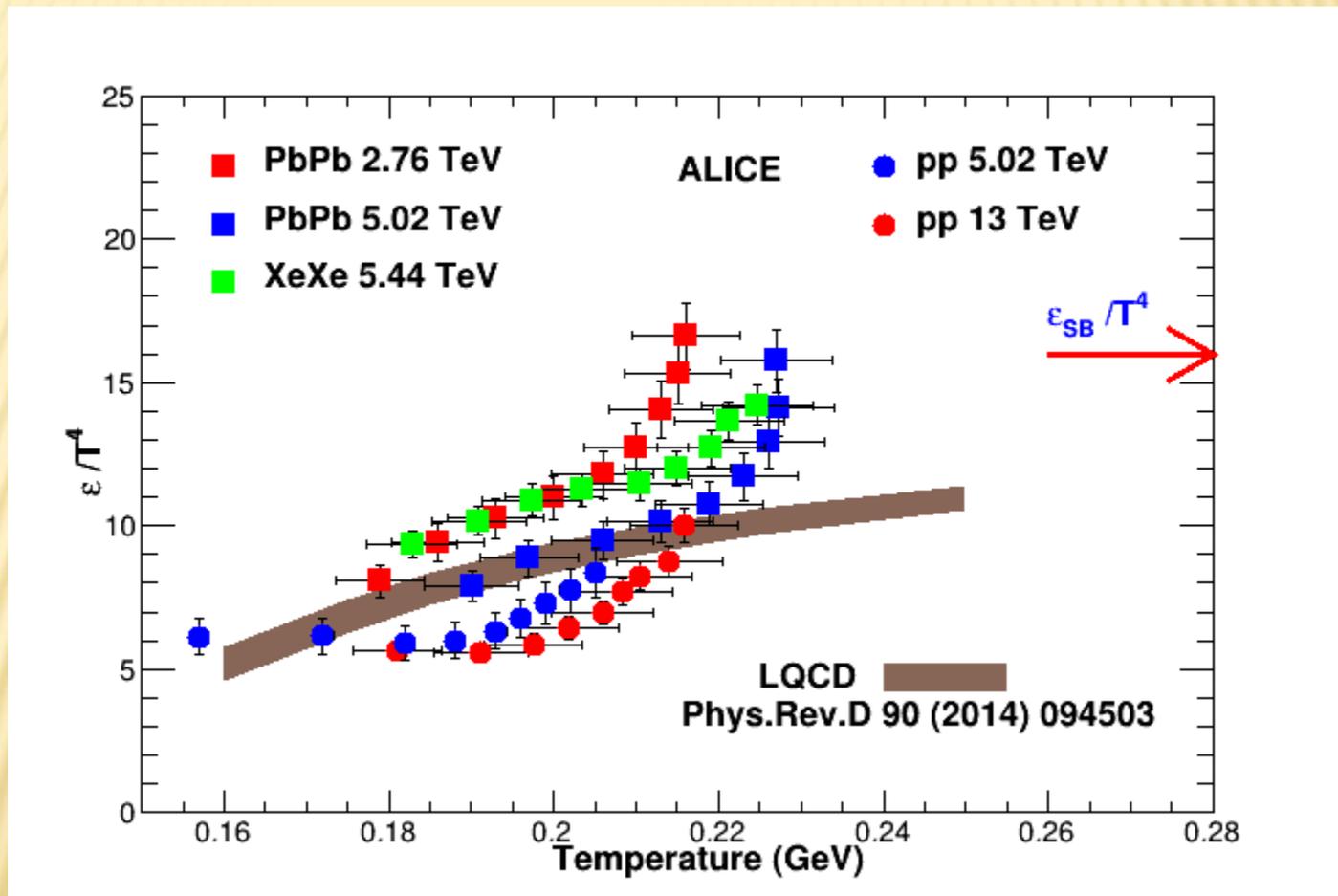
H. Satz

Extreme states of matter in strong interaction physics.

Lecture notes in physics 945, 2018

$$\epsilon/T^4 = \left\{ \begin{array}{l} \left(\frac{37}{30} \right) \pi^2 \approx 12 \text{ for } N_f=2 \\ \left(\frac{47.5}{30} \right) \pi^2 \approx 16 \text{ for } N_f=3 \end{array} \right\}$$

ε/T^4 as a function of temperature



Our results agree with LQCD results up to temperature of $T \sim 210$ MeV.

Above 210 MeV CSPM $\frac{\varepsilon}{T^4}$ rises much faster and reaches the ideal gas value of 16.

It has been argued that QCD could lead to three –state phase structure as a function of temperature. In such a scenario, color deconfinement would result in a plasma of massive “dressed quarks”. At still higher temperature this gluonic dressing of quarks would then “evaporate”, leading to a plasma of deconfined massless quarks and gluons : a “QGP”.

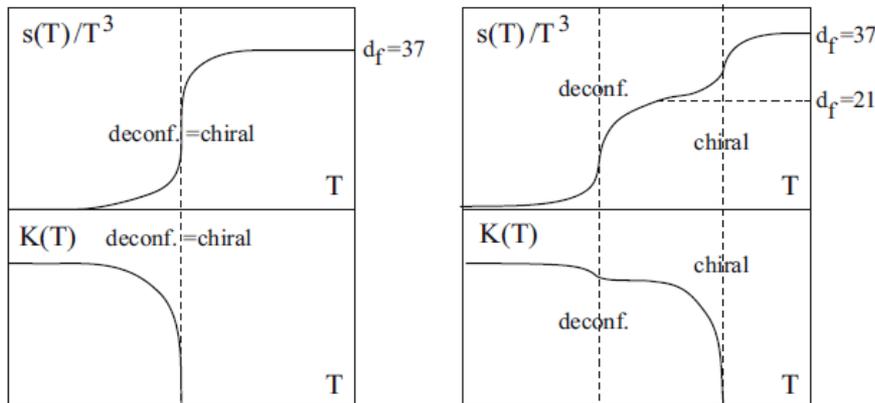


Fig. 6.6 Deconfinement and chiral symmetry restoration for one (left) and two (right) distinct transitions

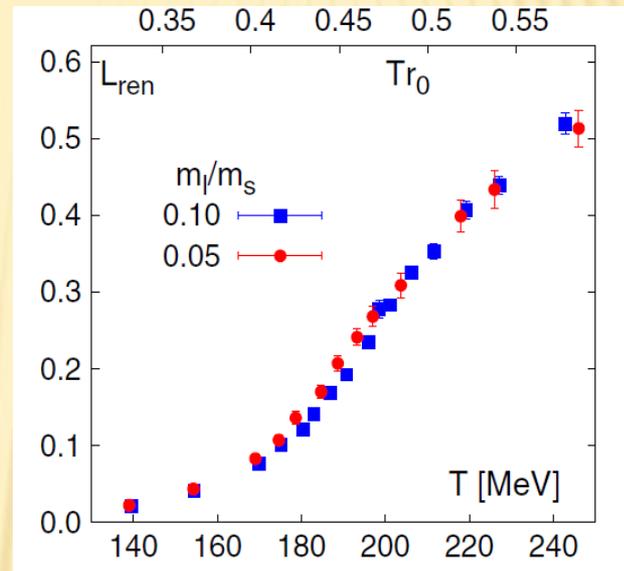
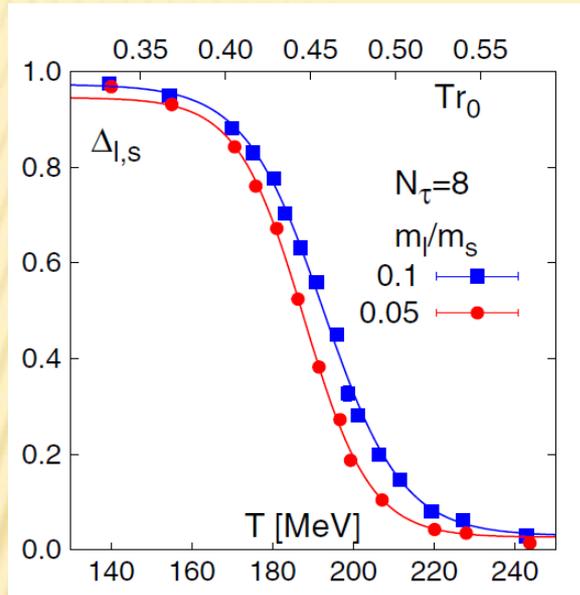
**Castorina, Gavai,
Satz, Eur. Phys. J
C 69 (2010) 69.**

Deconfinement is basically the transition from bound to unbound quark constituents, from a state of color-neutral hadrons to one of colored quarks. Chiral symmetry restoration is the transition from a state of massive “dressed” constituent quarks to one of massless current quarks. These two phenomena need not coincide, and there exist theories in which they do not, for quarks in the adjoint, rather than the fundamental color group representation [18, 29]. In this case, deconfinement occurs at a much lower temperature than chiral symmetry restoration, i.e., deconfinement leads to a state of colored massive “dressed” quarks. The basic indications of the two phenomena are therefore

- a sudden change in the number of degrees of freedom, from bound to unbound color, for deconfinement, and
- a sudden change in the effective quark mass, from finite to zero, for chiral symmetry restoration.

Subtracted chiral condensate

Normalized Polyakov loop



The normalized Polyakov loop rises in the temperature interval $T=170-200$ MeV. At the same time the subtracted chiral condensate rapidly drops in the transition region indicating the approx. restoration of chiral symmetry.

**Equation of state for physical quark masses
Phys. Rev. D 81, 054504 (2010), Cheng et al.**

Summary

- ❑ The Clustering of Color Sources produced by overlapping strings has been applied to both A-A and pp collisions.
- ❑ The most important quantity in this picture is the multiplicity dependent interaction area in the transverse plane S_{\perp}
- ❑ The temperature from AA and pp scales as $\frac{dN_c}{d\eta} \left(\frac{1}{S_{\perp}} \right)$
- ❑ Quantum tunneling through color confinement leads to thermal hadron production in the form of Hawking-Unruh radiation. In QCD we have string interaction instead of gravitation.
- ❑ We observe for the first time a two-step behavior in the increase of DOF. Results for Pb-Pb and Xe-Xe collisions show a sharp increase in ϵ/T^4 above $T \sim 210$ MeV and reaching the ideal gas of quarks and gluons at $T \sim 230$ MeV.



Thank You

Extra Slides

Chiral Symmetry Restoration

The essential features of hadron structure are color confinement and spontaneous chiral symmetry breaking. The former binds colored quarks interacting through colored gluons to color-neutral hadrons. The latter brings in pions as Goldstone bosons and gives the essentially massless quarks in the QCD Lagrangian a dynamically generated effective mass. We had already noted that both features will come to an end when hadronic matter is brought to sufficiently high temperatures and/or baryon densities. A priori, they need not end simultaneously; however, rather basic arguments suggest that chiral symmetry restoration occurs either together with or after color deconfinement [1].

In principle, QCD could thus lead to a three–state phase structure as a function of the temperature T and the baryochemical potential μ .

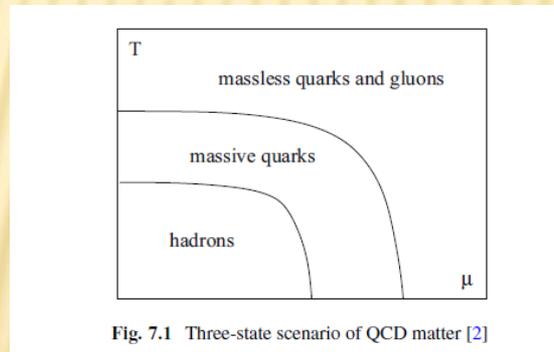
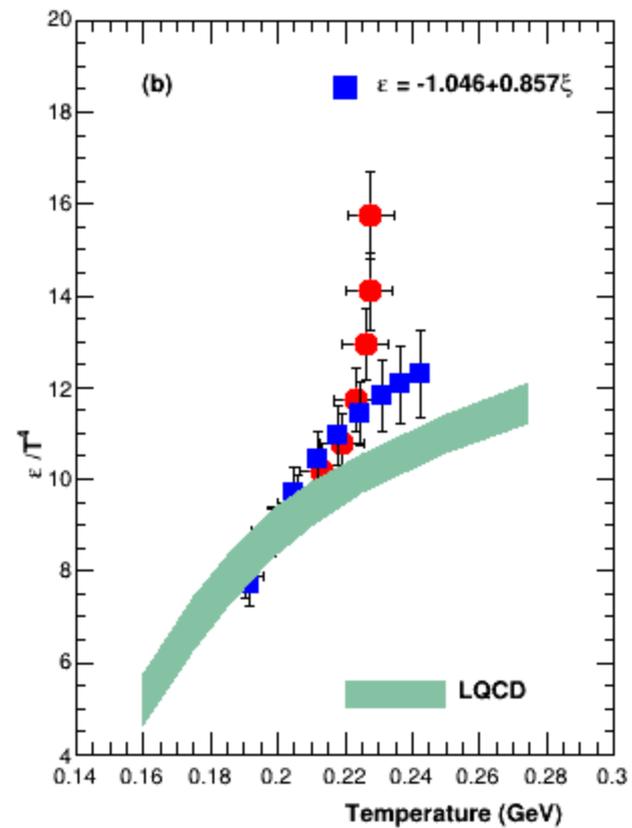
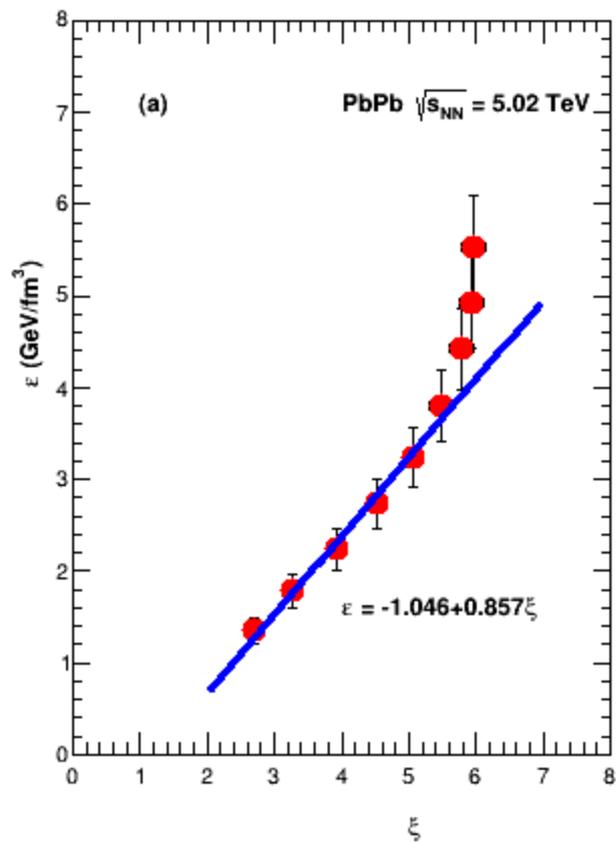


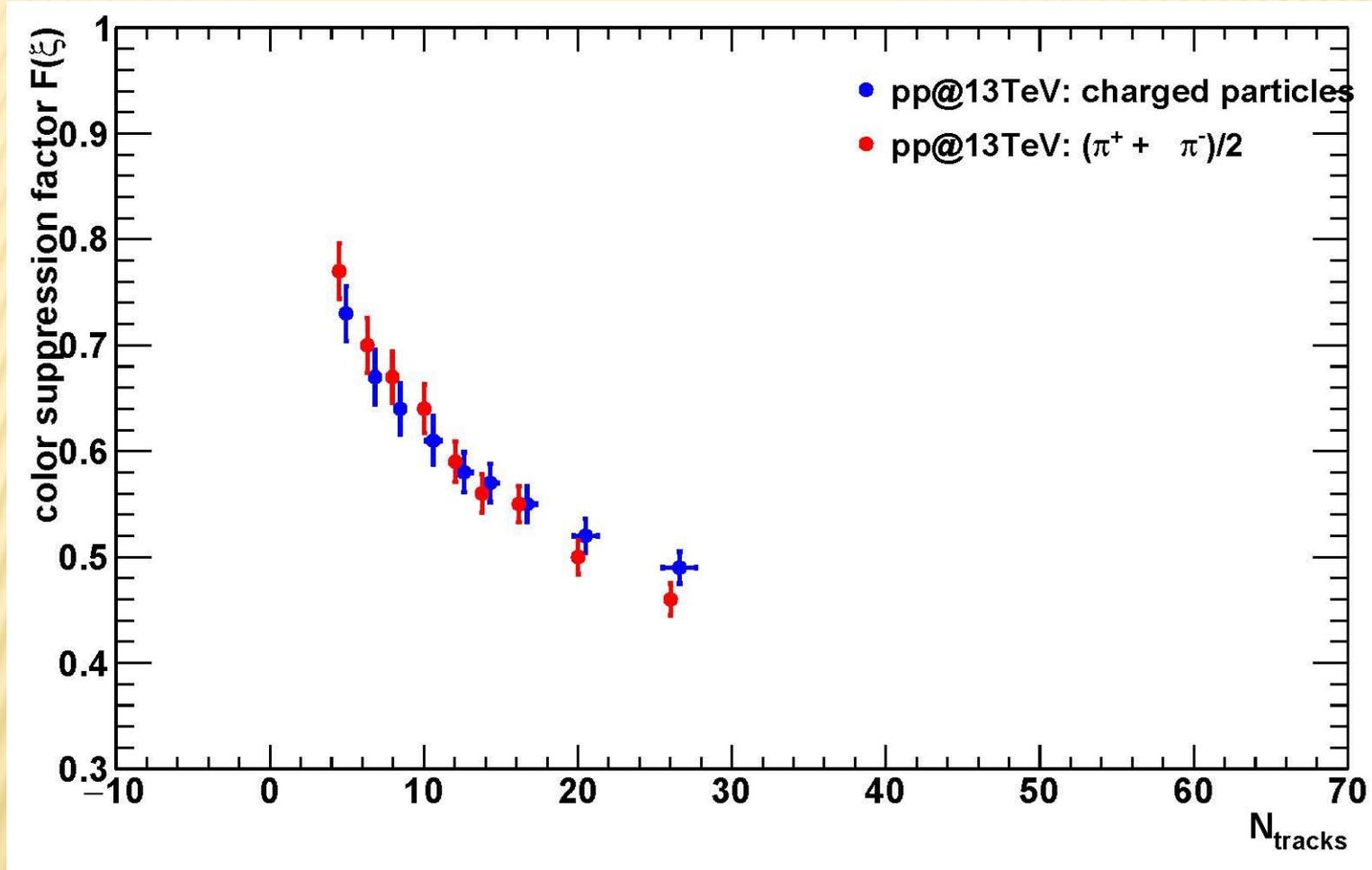
Fig. 7.1 Three-state scenario of QCD matter [2]

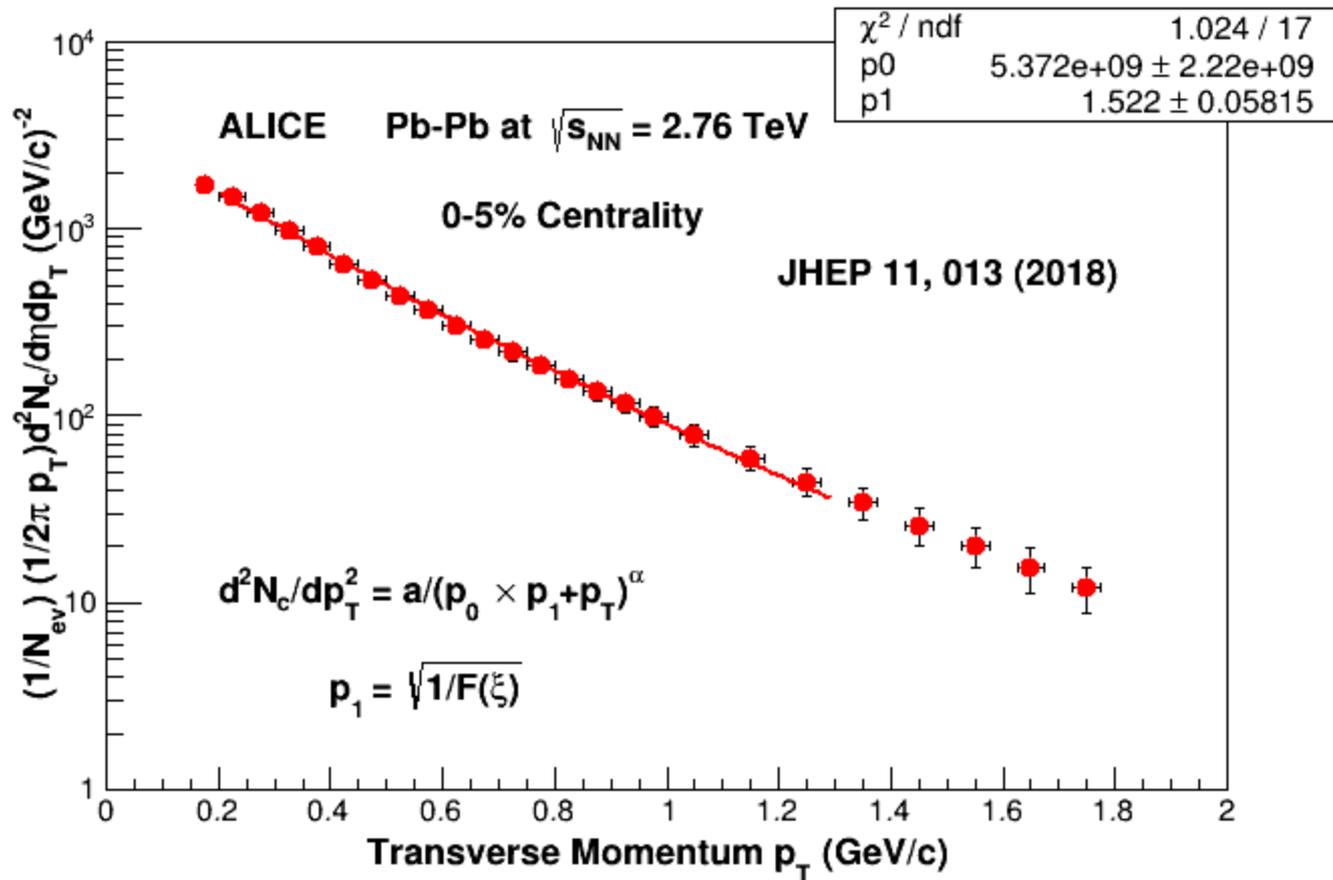
- In such a scenario, color de-confinement would result in a plasma of massive “dressed” quarks, the only role of gluon in this state would be to dynamically generate the effective quark mass, maintaining spontaneous chiral symmetry breaking.
- At still higher T and μ , this gluonic dressing of the quarks would then “evaporate” or “melt”, leading to a plasma of de-confined massless quarks and gluons with restored chiral symmetry.
- This aspect of QCD is studied in terms of renormalized Polyakov loop, subtracted chiral condensate, and strangeness susceptibility.

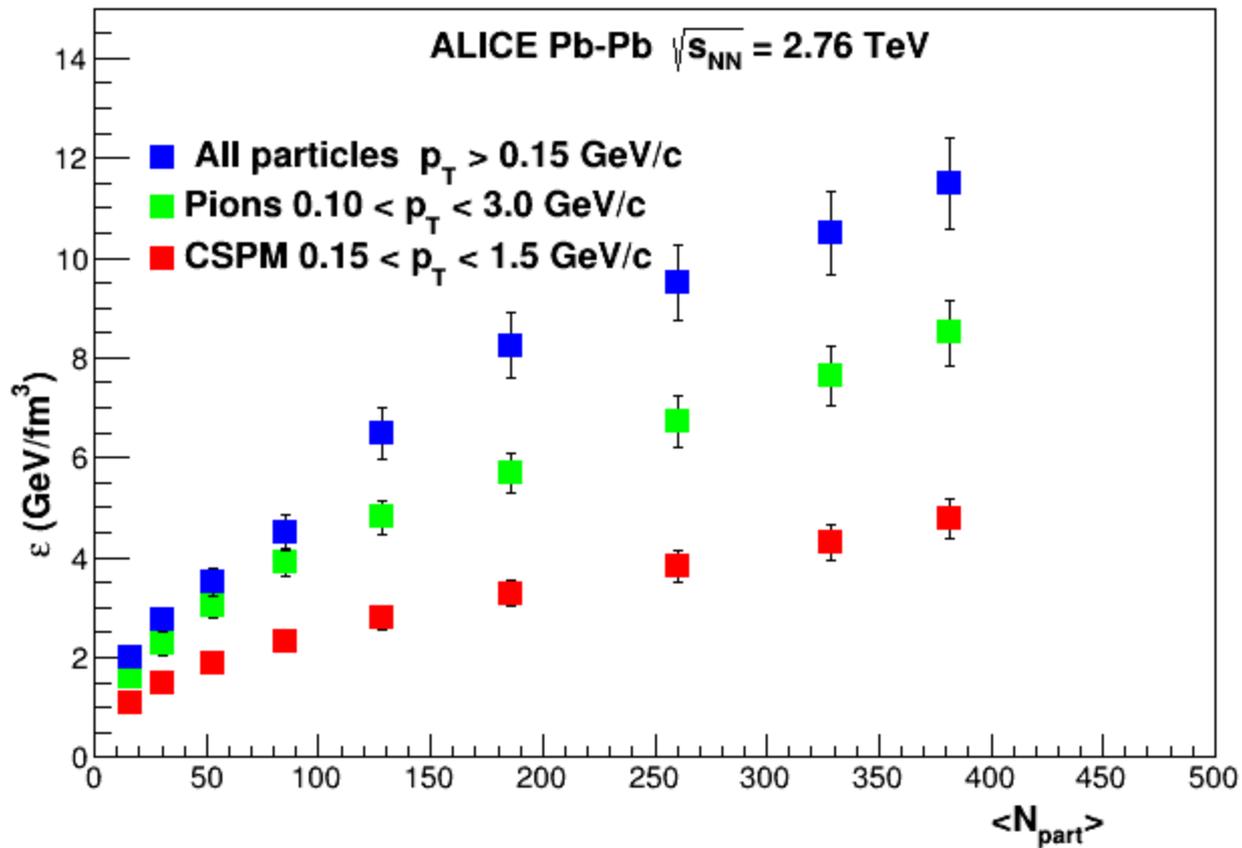
At zero quark mass, the chiral condensate needs to be renormalized only multiplicatively. At nonzero values of the quark mass, an additional renormalization is necessary to eliminate singularities that are proportional to m_q/a^2 . An appropriate observable that takes care of the additive renormalizations is obtained by subtracting a fraction, proportional to m_l/m_s , of the strange quark condensate from the light quark condensate. To remove the multiplicative renormalization factor we divide this difference at finite temperature by the corresponding zero temperature difference, calculated at the same value of the lattice cut-off, i.e.,

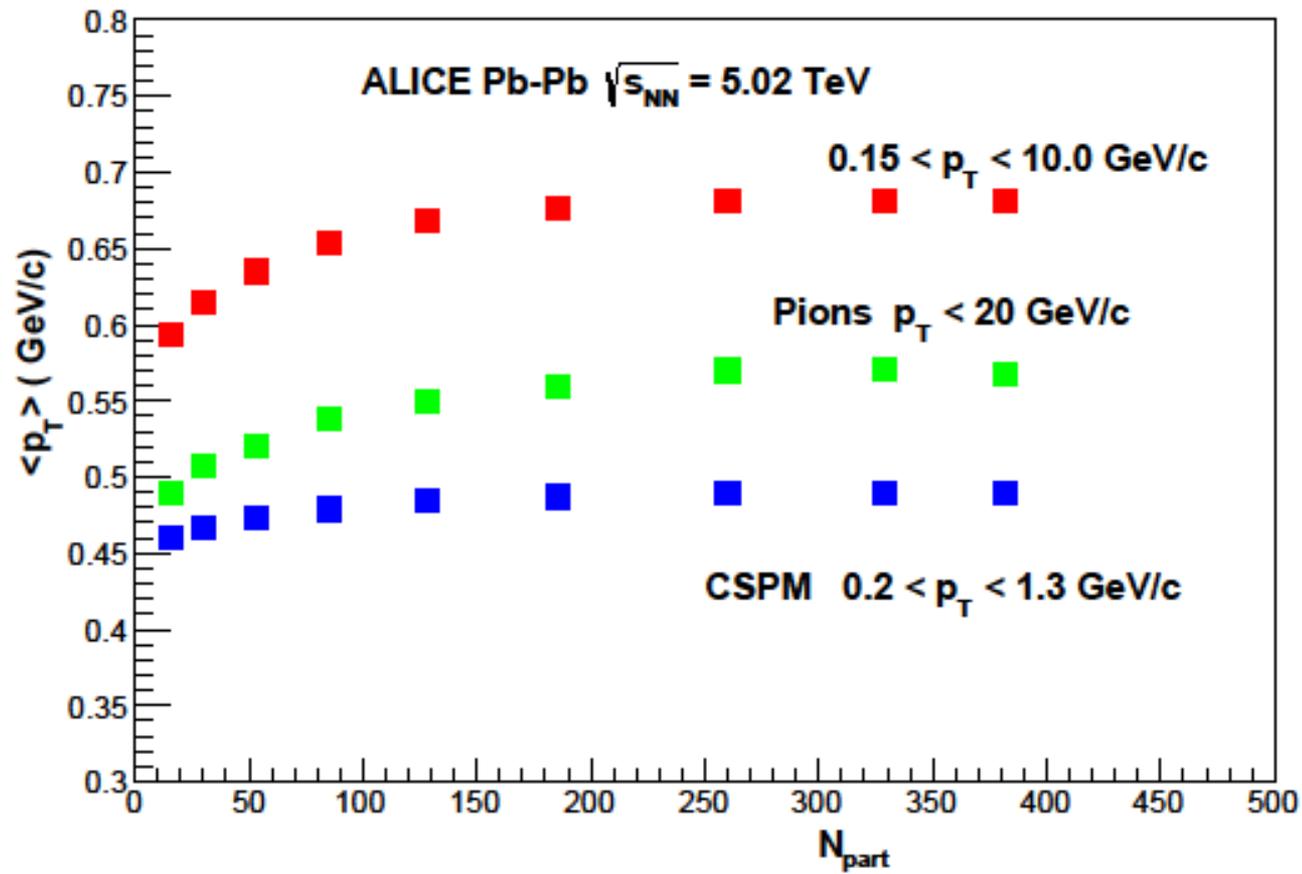
$$\Delta_{l,s}(T) = \frac{\langle \bar{\psi} \psi \rangle_{l,T} - \frac{m_l}{m_s} \langle \bar{\psi} \psi \rangle_{s,T}}{\langle \bar{\psi} \psi \rangle_{l,0} - \frac{m_l}{m_s} \langle \bar{\psi} \psi \rangle_{s,0}}. \quad (16)$$

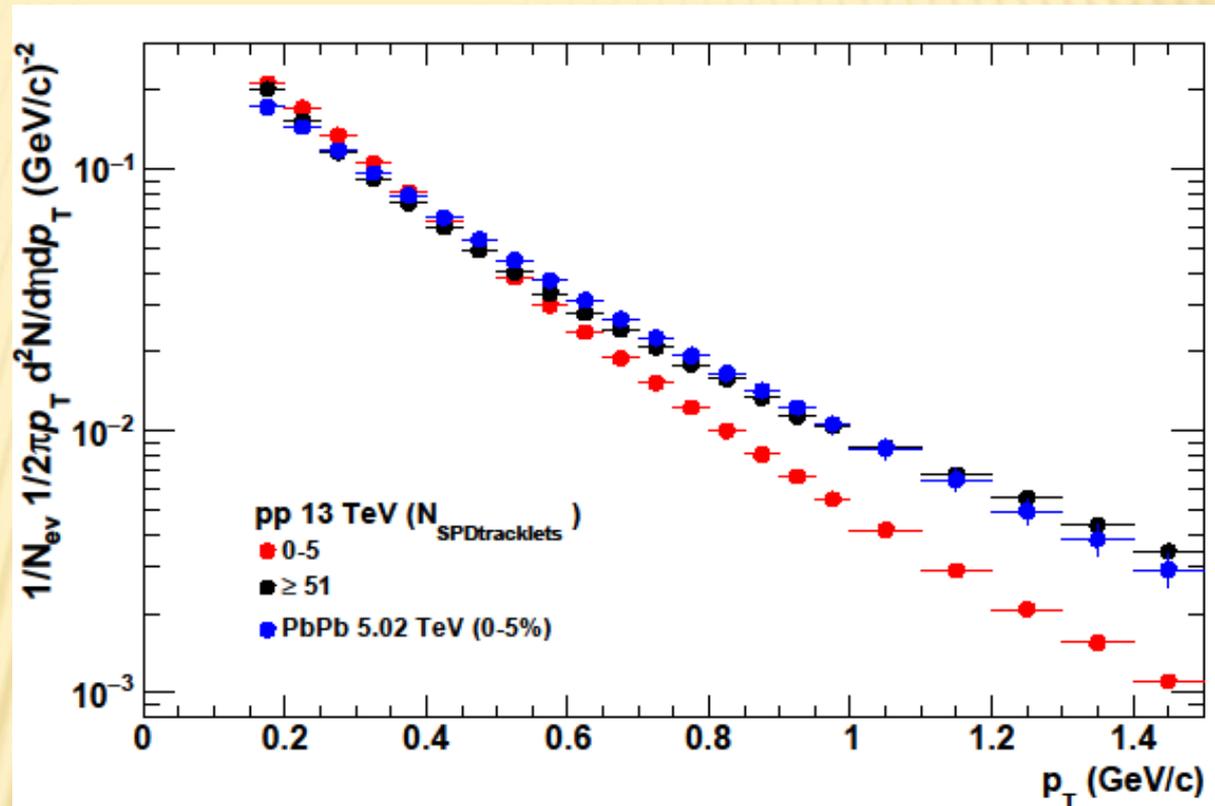




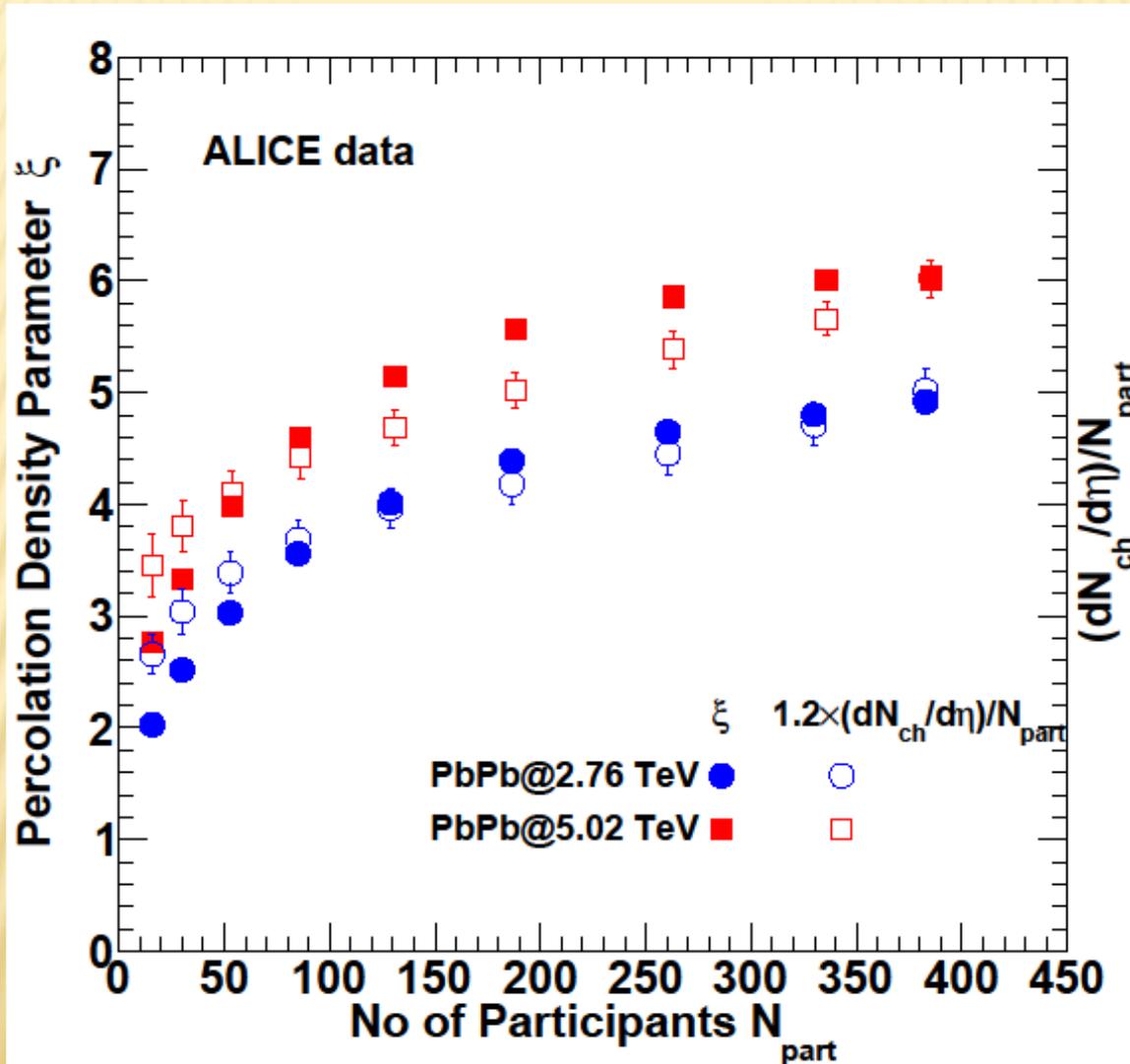








Percolation Density Parameter vs N_{part}



Pb+Pb @ 2.76TeV for 0-5%

$T = 230 \pm 7 \text{ MeV}$

ALICE : Direct Photon Measurement

$T = 297 \pm 12 \pm 41 \text{ MeV}$ Phys. Lett. B 754 , 235 (2016)