Physics of heavy quarks and their bound states in a quark-gluon plasma

Student day

ICPAQGP

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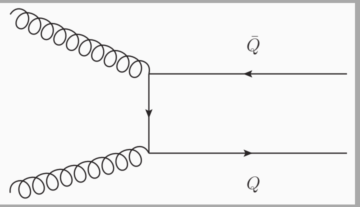
Introductory remarks

Heavy quarks and quarkonia as 'hard probes' or 'test particles'

Heavy quarks are produced in pairs in the early stages of URHIC. Their number remains constant.



Formation time of a $Q\bar{Q}$ pair is small

$$\Delta t \sim \frac{1}{2M_Q}$$

$$J/\Psi \qquad M_c \simeq 1.5 \text{ Gev} \qquad \Delta t \simeq 0.07 \text{ fm/c}$$

$$\Upsilon \qquad M_b \simeq 4.5 \text{ Gev} \qquad \Delta t \simeq 0.02 \text{ fm/c}$$

Dynamics of heavy quarks is non-relativistic

$$\Upsilon \quad v_0^2 \sim 0.1$$

$$J/\Psi \quad v_0^2 \sim 0.3$$

$$H = \frac{P^2}{M_Q} + V(r) \qquad \left(V(r) = \frac{\alpha_s}{r} + \sigma r \right)$$

The potential can be obtained using effective theory (pNRQCD)

[see N. Brambilla, A.Pineda, J. Soto, A. Vairo, NPB566 (2000) 275]

Heavy quark interaction at finite T

Mass is large compared to the typical temperature

$$M_Q \gg T$$

Initial suggestion (Matsui-Satz 86): screening of the potential

$$H = \frac{P^2}{M_Q} + V(r)$$

$$V(r) = -\frac{\alpha}{r} \mathrm{e}^{-r\, m_D(T)} + \sigma(T) r$$

This picture predicts a "suppression" of bound states at high temperature, the most "fragile" ones (bigger, less bound) disappearing first as the temperature increases ("sequential suppression").

Hence the idea of using quarkonia to diagnose the formation of quark-gluon plasma in URHI

A nice idea....

A considerable experimental effort...

But a very difficult many-body problem !

A plethora of theoretical approaches

-potential models
-spectral functions
-Euclidean correlators (lattice), maximum entropy techniques
-coupled channels
-in-medium T matrix
-path integrals
-open quantum systems
-influence functional
-Lindblad equation
-effective field theories
-strong coupling techniques
-effect of magnetic field
-etc

In most of these approaches only one specific aspect of the problem is addressed

What do we need ?

A robust picture that encompasses in a coherent framework
all the main features of the dynamics

Why is it a difficult problem ?

Complex, multifaceted, multi scale dynamics

- The formation of bound state is not instantaneous. Nor is the establishment of a screening cloud.

  Typical formation time:

$$\tau_0 \sim \frac{2}{M\alpha^2}$$

$$\tau_\Upsilon \sim 1 \text{ fm/c}$$

$$\tau_{J/\Psi} \sim 2.5 \text{ fm/c}$$

- The effect of the medium does not reduce to an instantaneous modification of the potential.

- As the bound state "forms" interactions with the medium take place.

- The formation and fate of a bound state is affected both by screening of the potential and by collisions with the plasma constituents

Typical scales in Coulomb bound states

Bohr radius $$a_0 \sim \frac{2}{M\alpha}$$

Momentum $$p_0 \sim \frac{1}{a_0} \sim \frac{M}{2}\mathrm{v}_0 \qquad \mathrm{v}_0 \sim \alpha$$

$\Upsilon \quad \mathrm{v}_0^2 \sim 0.1$

$J/\Psi \quad \mathrm{v}_0^2 \sim 0.3$

Energy $$M\mathrm{v}_0^2 \sim M\alpha^2$$

(p)NRQCD hierarchy of scales:

$$\Lambda_{QCD} \lesssim M\alpha^2 \ll \alpha M \ll M \qquad\qquad M \gg T$$

NB. Hierarchy of scale is convenient for theoretical analysis.
But in practice, scales are not well separated.

Typical time scale for HQ motion in a bound state

$$\tau_0 \sim \frac{2}{M\alpha^2}$$

When does "melting" occur ?

various approximate criteria

(a) Thermal velocity equals velocity in bound state

$$\mathrm{v}_Q \sim \sqrt{\frac{T}{M}} \sim \mathrm{v}_0 \sim \alpha \qquad\qquad T \sim M\alpha^2$$

(b) momentum of thermal particles matches HQ bd state momentum

$$T \sim M\alpha$$

(c) The bound state typical time scales match that of plasma response

$$\frac{\tau_{\rm pl}}{\tau_0} \sim \frac{M}{m_D}\frac{\alpha^2}{2} \sim 1$$

Binding energy is of the order of the Debye mass

Why is it interesting ?

- New data of high quality and precision, for two distinct systems, charmonium and bottomonium
- New phenomena observed ("regeneration")
- New theoretical developments: lattice, spectral functions, effective field theories, NRQCD, pNRQCD, **open quantum system approaches**, etc...
- More broadly, connections with other interesting issues: modifications of bound state properties in medium, production and survival of fragile states in a hot environment (in particular, multi quark sates), etc.

Several conceptual issues remain to be clarified before we can get a robust picture that can be confronted to phenomenology and quarkonia can be used as "probes"

An 'open quantum system'

In the rest of this talk, I focus mostly on a simple problem

Put a number of $Q\bar{Q}$ pairs into a quark-gluon plasma at temperature T, and study their evolution.

Typical questions that one would like to answer

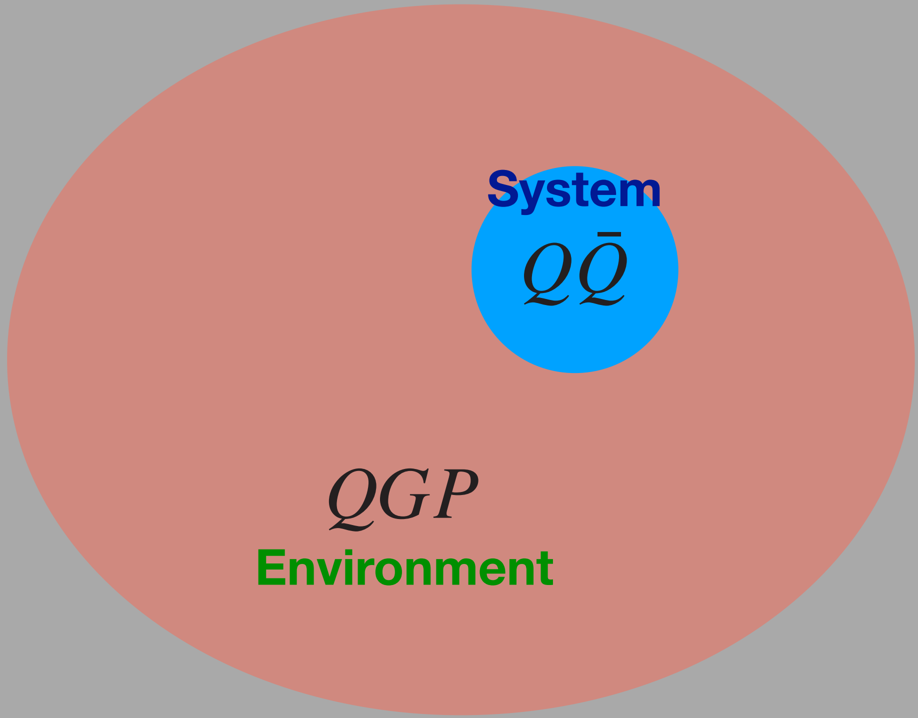
How does a $Q\bar{Q}$ pair (or collection of pairs) evolve towards "equilibrium"? Can we find robust features of the dynamics? For instance, can we identify various regimes as a function of external parameters (T, M, ...). Etc.

Towards an effective theory for heavy quarks in QGP

$$H = H_Q + H_{\rm pl} + H_{\rm int} \qquad\qquad H_{\rm int} \sim J_Q \cdot A_{\rm pl}$$

Prototptype of an "open quantum system"

Open quantum system



Density matrix of total system

$$\mathcal{D}(t)$$

with equation of motion $i\dfrac{\mathrm{d}\mathcal{D}}{\mathrm{d}t} = [H, \mathcal{D}]$.

Reduced density matrix of heavy quarks: $\mathcal{D}_Q(t) = \mathrm{Tr}_{\mathrm{pl}}\mathcal{D}(t)$

Equation of motion for $\mathcal{D}_Q(t)$?

Typical form of the hamiltonian

$$H = H_Q + H_{\mathrm{pl}} + H_{\mathrm{int}} \qquad H_{\mathrm{int}} \sim J_Q \cdot A_{\mathrm{pl}}$$

Various strategies:

- Feynman-Vernon Influence functional
- Lindblad equation,
- Schwinger-Keldysh diagrammatic techniques,
- Etc

For a recent review in the heavy quark context, see Y. Akamatsu, 2009.10559

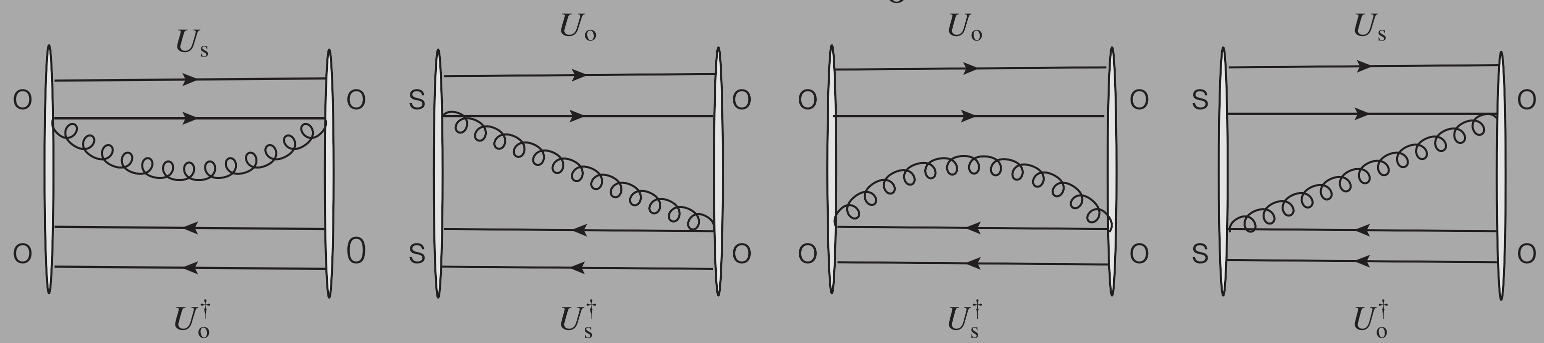
- Our goal: construct an effective theory for the HQ, by eliminating the plasma dof's.

- Tool of choice: reduced density matrix for heavy quarks

$$\mathcal{D}_Q(t) = \mathrm{Tr}_{\rm pl}\,\mathcal{D}(t)$$

- $\mathcal{D}(t)$ obeys equation of motion of the form

$$\frac{\mathrm{d}}{\mathrm{d}t}\mathcal{D}_Q(t) = -i[H_Q, \mathcal{D}_Q(t)] + \int_0^{t-t_0} \mathrm{d}\tau\, \mathcal{L}(\tau)\mathcal{D}_Q(t-\tau)$$



- This does not reduce to changing the HQ hamiltonian: $H_Q \to H_Q^{\rm eff}$

- Non hamiltonian dynamics (dissipation, transport, etc)

[For details, see JPB, M. Escobedo-Espinosa, 1711.10812,1803.07996]

Typical approximations

(i) Weak coupling between HQ and the plasma

$$H_1 = -g \int_{\boldsymbol{r}} A_0^a(\boldsymbol{r}) n^a(\boldsymbol{r}),$$

gauge potential of plasma

HQ density

$$n^a(\boldsymbol{x}) = \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, t^a \otimes \mathbb{I} - \mathbb{I} \otimes \delta(\boldsymbol{x} - \hat{\boldsymbol{r}})\, \tilde{t}^a.$$

The presence of the heavy quarks does not modify significantly the equilibrium state of the plasma.

The influence of the plasma on the heavy quark dynamics is characterized by simple response functions (correlators)

$$\Delta(t_1, t_2) \equiv \langle A_{\mathrm{pl}}(t_1) A_{\mathrm{pl}}(t_2) \rangle_T = \mathrm{Tr}\left[A_{\mathrm{pl}}(t_1) A_{\mathrm{pl}}(t_2) \mathcal{D}_{\mathrm{pl}} \right]$$

No weak or strong coupling assumption needs to be made concerning the plasma. The correlators can, in some cases, be obtained from lattice calculations.

(ii) The response of the plasma is "fast"

plasma response characterized by a single energy scale, the Debye mass

$$m_D = CT \qquad (C \simeq 2)$$ In strict weak coupling $C = g$

$$m_D \ll M$$

collisions with plasma constituents involve small energy transfer

soft gluon exchanges

$$q \lesssim m_D \ll M$$

small energy transfer

$$\frac{q^2}{M} \sim \frac{m_D^2}{M} \ll m_D$$

also, during time $t \sim m_D^{-1}$ the heavy quark moves a distance that is small compare to the size of the screening cloud ($v_{\rm th} \simeq \sqrt{T/M} \ll 1$)

the relevant correlator is then generically of the form

$$\Delta(\omega = 0, \mathbf{r}) = \Delta^R(\omega = 0, \mathbf{r}) + i\Delta^<(\omega = 0, \mathbf{r})$$

$$V(\boldsymbol{r}) = -\Delta^R(\omega = 0, \boldsymbol{r}), \qquad W(\boldsymbol{r}) = -\Delta^<(\omega = 0, \boldsymbol{r})$$

Imaginary potential

Key to obtain a Markovian approximation

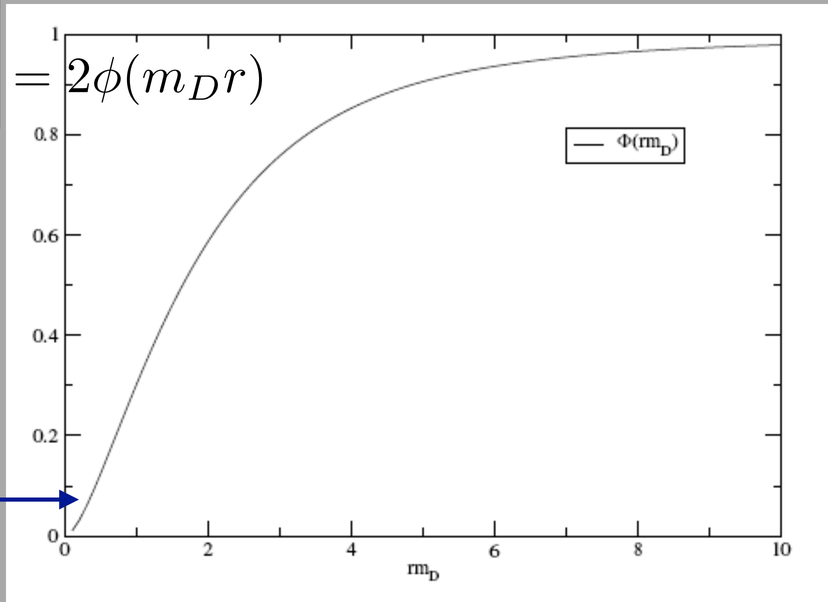
Static response and "Optical potential"

(*first obtained by M. Laine et al hep-ph/ 0611300)

$$\mathcal{V}(r) = V(r) + iW(r)$$

$$\Delta^R(\omega = 0, r) = -V(r) \qquad \Delta^<(\omega = 0, r) = -W(r)$$

$$\boxed{\Gamma(\boldsymbol{r}) = W(\boldsymbol{r}) - W(0)} = 2\phi(m_D r)$$

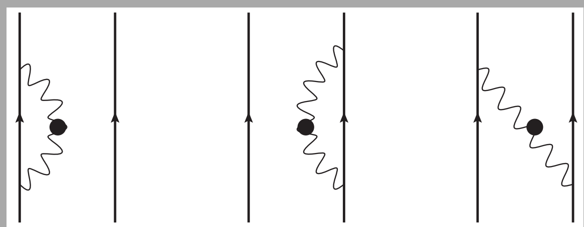


At large distance the imaginary part is twice the damping rate of the heavy quark

$$\Phi(x) = \frac{x^2}{3}\left(-\ln x + \frac{4}{3} - \gamma_E\right)$$

makes density matrix nearly diagonal in coordinate space

At short distance, interference produces cancellation: a small dipole does not "see" the electric field fluctuations.



For one heavy quark

$$\partial_t \langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle = \cdots - \Gamma(\boldsymbol{r}-\boldsymbol{r}')\langle \boldsymbol{r}|\mathcal{D}_Q|\boldsymbol{r}'\rangle$$

Semi-classical approaches for abelian plasmas

(iii) semi-classical approximation

$$M \gg T$$

HQ thermal wavelength $\lambda_{\rm th} \sim \dfrac{1}{\sqrt{MT}} \ll \dfrac{1}{T}$

Density matrix becomes nearly diagonal

$\langle \mathbf{r} | \mathcal{D}_Q | \mathbf{r}' \rangle \simeq 0$ when $|\mathbf{r} - \mathbf{r}'| \gtrsim \lambda_{\rm th}$

Expansion in $|\mathbf{r} - \mathbf{r}'|$ $\longrightarrow$ Fokker-Planck and Langevin equations

Semi-classical expansion for heavy quark motion

- Equation for the density matrix $\longrightarrow$ Langevin equation

- Langevin equation for the relative motion

$$\frac{M}{2}\ddot{\boldsymbol{r}}^i = -\gamma_{ij}\boldsymbol{v}^j - \boldsymbol{\nabla}^i V(\boldsymbol{r}) + \xi^i(\boldsymbol{r}, t)$$

$$\gamma_{ij}(\boldsymbol{r}) = \frac{1}{2T}\eta_{ij}(\boldsymbol{r}) \qquad \langle \xi^i(\mathbf{r}, t)\xi^j(\mathbf{r}, t')\rangle = \eta_{ij}(\mathbf{r})\delta(t - t')$$

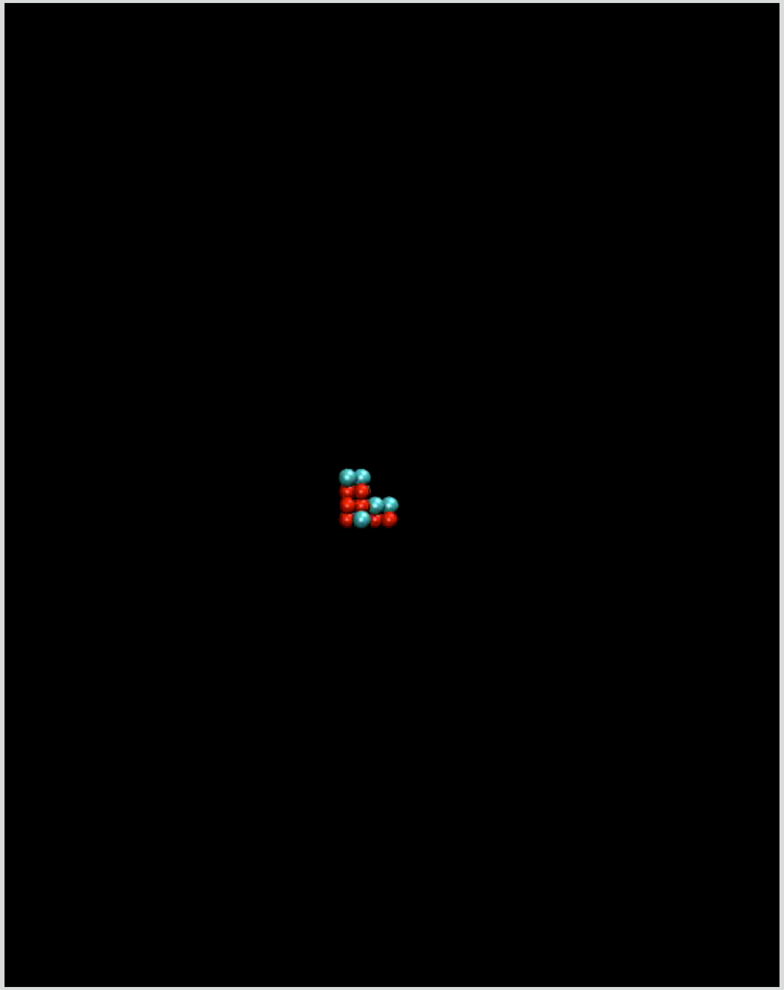
Non trivial noise

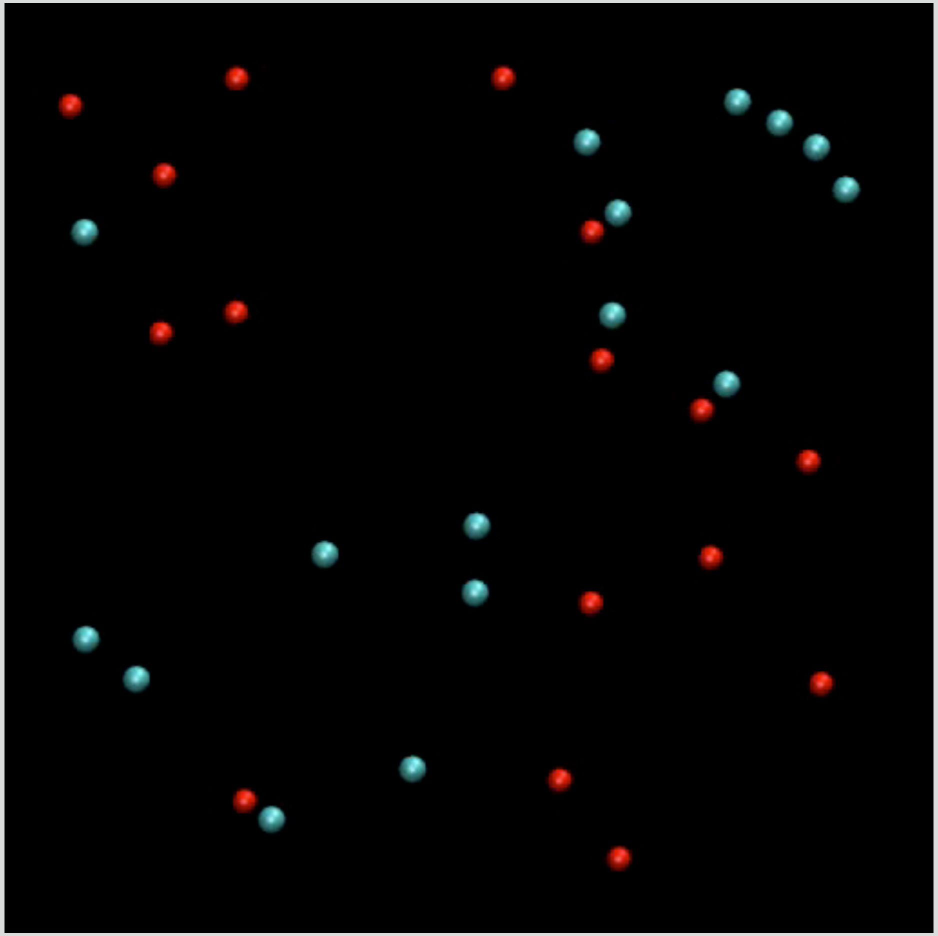
- For an isotropic plasma

$$\eta_{ij}(\boldsymbol{r}) = \delta_{ij}\eta(\boldsymbol{r}) \qquad \eta(\boldsymbol{r}) = \frac{1}{6}\left(\nabla^2 W(0) + \nabla^2 W(\boldsymbol{r})\right)$$

- All ingredients of the dynamics are calculated from the plasma correlation functions

The semi-classical approximation allows for very detailed simulations (abelian plasmas)

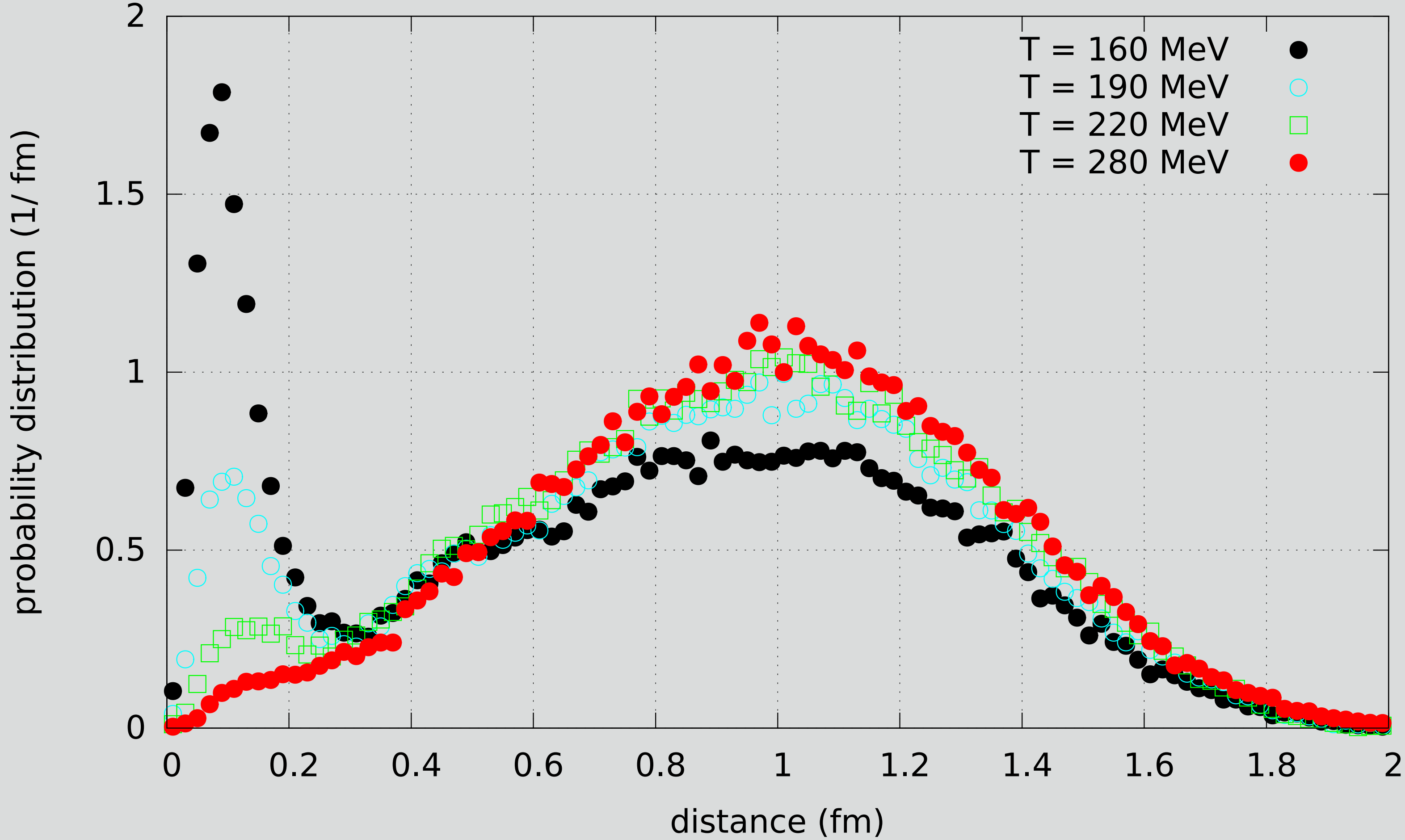




J-P.B, D. de Boni, P. Faccioli and G. Garberoglio, NPA (2016)

Probability distribution of distance to nearest neighbour

[10 cc pairs in a 4 fm cubic box]



$$P_{q\bar{q}}^{\text{ideal}}(r) = \frac{3}{a}\left(\frac{r}{a}\right)^2 \left(1 - \left(\frac{r}{a}\right)^3 \frac{1}{N}\right)^{N-1} \overset{N \gg 1}{\simeq} \frac{3}{a}\left(\frac{r}{a}\right)^2 e^{-(r/a)^{\frac{1}{3}}} \qquad a = \left(\frac{3}{4\pi\rho}\right)^{1/3}$$

From QED to QCD

Much of the previous discussion goes through...

...with essential differences

Force between HQ depends on color

$$V_{\rm s}(\mathbf{r}) = -C_f \frac{\alpha_s}{r} \qquad\qquad V_{\rm o}(\mathbf{r}) = 2N_c \frac{\alpha_s}{r}$$

New random force, dependent on color

Subtle interplay between color and coordinate space dynamics -> complicates the semi-classical description

In particular, the treatment of multiple pairs is difficult

[For details on various attempts, see JPB, M. Escobedo-Espinosa, 1711.10812]

Dipole approximation

Simplify the interaction

$$A(\mathbf{R} + \mathbf{r}) \simeq A(\mathbf{R}) + \mathbf{r} \cdot \nabla A(\mathbf{R})$$

Yields dipolar interaction

Correlator determined by two constants

$$\kappa = \frac{g^2}{6N_c} \int_0^\infty \left\langle \left\{ E_i^a(s, \vec{0}), E_i^a(0, \vec{0}) \right\} \right\rangle$$

$$\gamma = -i \frac{g^2}{6N_c} \int_0^\infty \left\langle \left[E_i^a(s, \vec{0}), E_i^a(0, \vec{0}) \right] \right\rangle$$

Linblad equation is then simplified and can be solved

Emphasizes singlet to octet dipolar transitions

[For details, see N. Brambilla et al, 2205.10289]

The imaginary potential
is energy dependent

Emergence of two regimes

Another criterion (d) for the disappearance of bound states

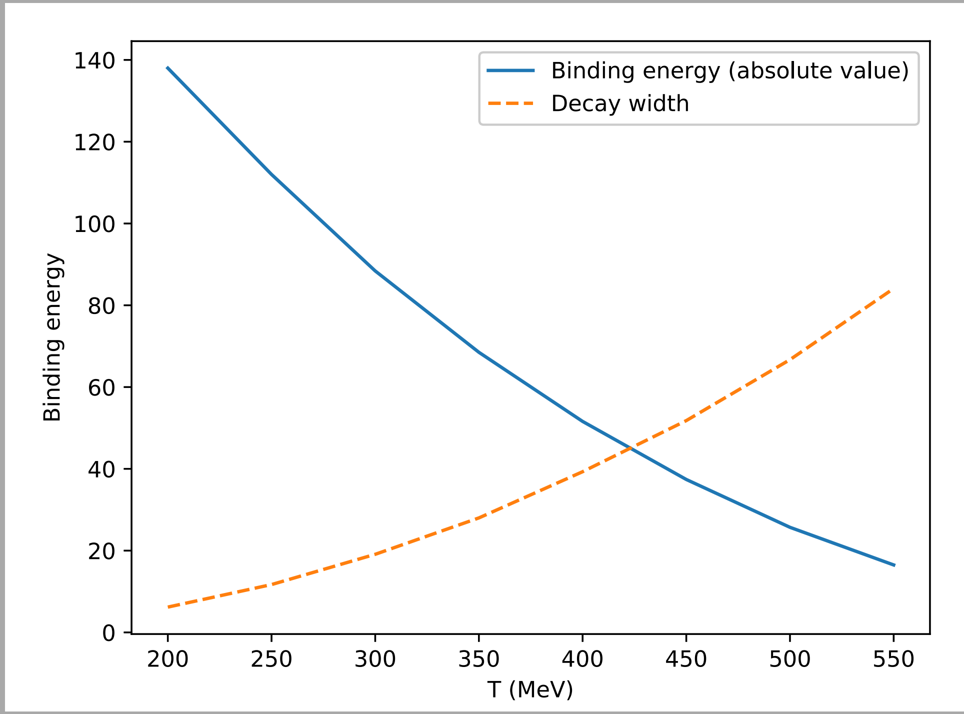
Competition between binding (and screening) and collisional effects (decay width)

$$V \sim \frac{\alpha}{r} \sim \alpha^2 M \qquad\qquad \Gamma \sim \alpha T \Phi(m_D r) = \alpha T \Phi(m_D/\alpha M)$$

Low T

Bound states

Rate equations



High T

Collision dominated

Langevin dynamics

$$\Phi \sim 1 \qquad T \sim \alpha M$$

$$\Phi \sim m_D^2/\alpha^2 M^2 \qquad m_D \sim CT \qquad T \sim \alpha \frac{M}{C^{2/3}}$$

Imaginary "potential" is energy dependent

Simple two-level model

(two massive quarks separated by r)

$N_c^2 - 1$

V_o ——— octet

V_s ——— singlet

$$Z = e^{-\frac{V_s}{T}} + (N_c^2 - 1)e^{-\frac{V_o}{T}}$$

$$\Delta V(r) = V_o - V_s$$

Low T $$F \approx V_s - T(N_c^2 - 1)e^{-\frac{\Delta V}{T}}$$ "Binding" dominates

High T $$F = -T\ln N_c^2 + \frac{V_s + (N_c^2 - 1)V_o}{N_c^2}$$ Entropy dominates

Transition occurs when

$$T \sim \Delta V$$

Approach to equilibrium

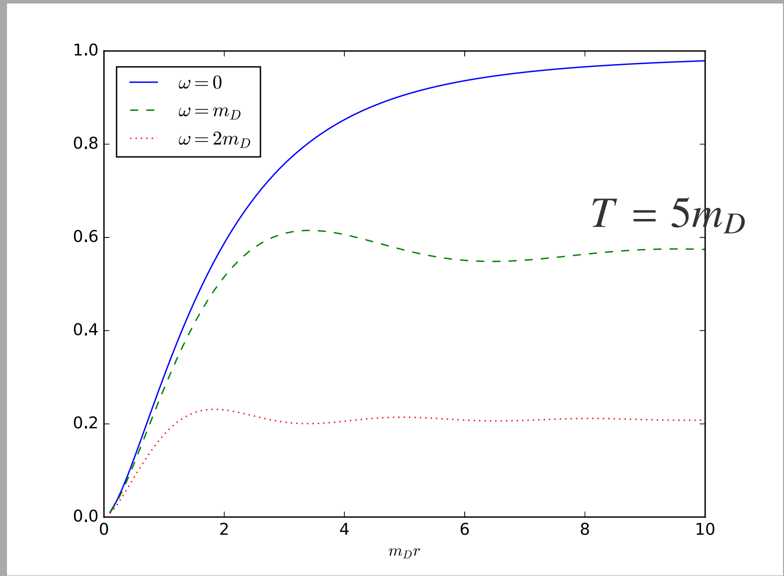
$$\frac{\mathrm{d}p_s}{\mathrm{d}t} = (N_c^2 - 1)p_o\Gamma_{o\to s} - p_s\Gamma_{s\to o}$$

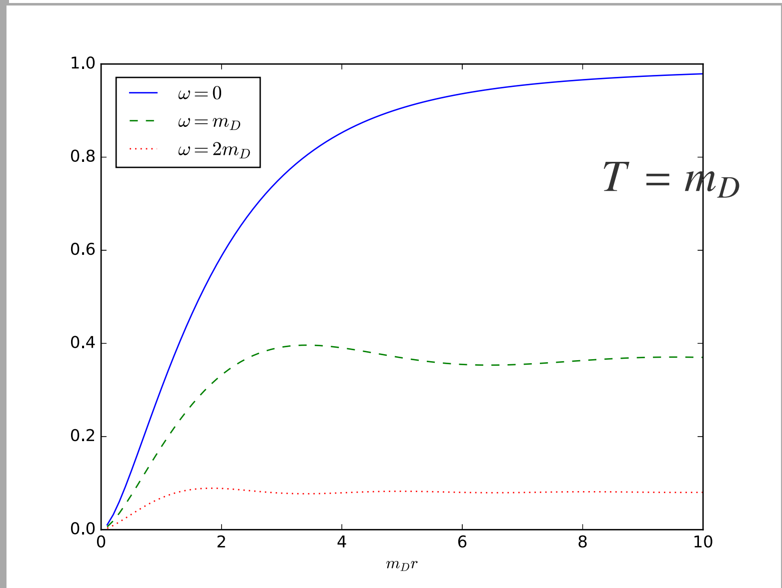
$$\Gamma_{s\to o} = g^2 C_F e^{-\frac{\Delta V}{T}} \int_q \Delta^>(\Delta V, q)|\mathcal{S}_{q\cdot\hat{r}}|^2$$

static approximation breaks down

The imaginary part of the potential is energy dependent

$$(4\pi/Tg^2)(W(\omega,r) - W(\omega,0))$$





[J-P.B, M. Escobedo Phys. Rev. D104 (2021) 5, 054034]

Final remarks

- A consistent framework is emerging, allowing to treat on par both screening and collisional effects. The theory of open quantum systems offers interesting perspectives, allowing us to derive most approaches from a common starting point.
- Two regimes: low and high temperature (fuzzy transition, multiple criteria)

$$T \sim M\alpha \qquad M\alpha^2 \sim m_D \qquad T \sim \alpha \frac{M}{C^{2/3}}$$

- Low T: bound states are weakly affected by the plasma, appropriate rate equations could be a good starting point
- High T: binding effects not essential, Langevin dynamics, equilibration, etc
- Intermediate region difficult: bound states are present, combination of Langevin and rate equations seems needed