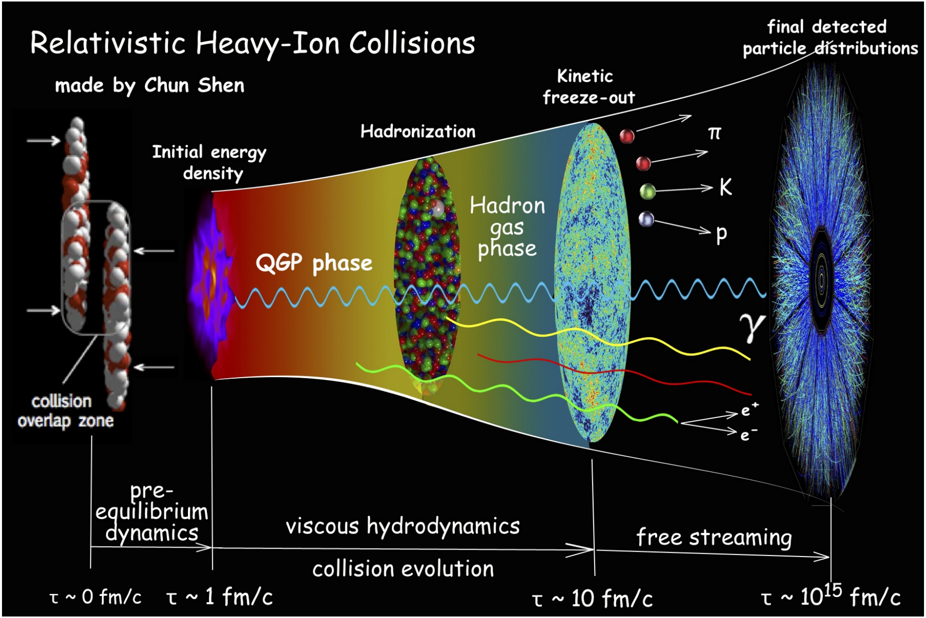
Relativistic hydrodynamics in heavy-ion collisions

Amaresh Jaiswal

School of Physical Sciences
NISER Bhubaneswar

ICPAQGP Student Day

February 6, 2023


Relativistic Heavy-Ion Collisions
made by Chun Shen
final detected particle distributions
Kinetic freeze-out
Hadronization
Initial energy density
Hadron gas phase
QGP phase
π
K
p
γ
e+
e-
collision overlap zone
pre-equilibrium dynamics
viscous hydrodynamics
free streaming
collision evolution
τ ~ 0 fm/c
τ ~ 1 fm/c
τ ~ 10 fm/c
τ ~ 10^15 fm/c

Hydrodynamics in heavy-ion collisions

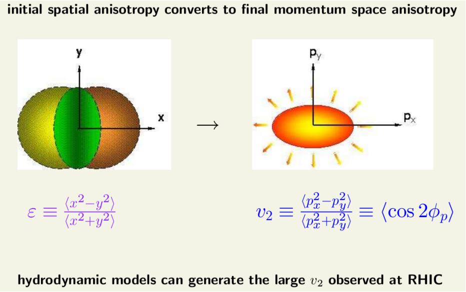
- Hydrodynamics/Fluid dynamics: Dynamics of a system exhibiting collective behaviour.
- An effective theory describing the long-wavelength, low-frequency limit of the microscopic dynamics of a system.
- Little bangs in relativistic heavy-ion collision create de-confined QCD matter.
- This hot and dense matter in vacuum undergoes violent expansion.
- Relativistic hydrodynamics applied successfully to explain the space-time evolution.
- Several heavy-ion collision observables at RHIC and LHC are explained quite accurately using hydrodynamic models.

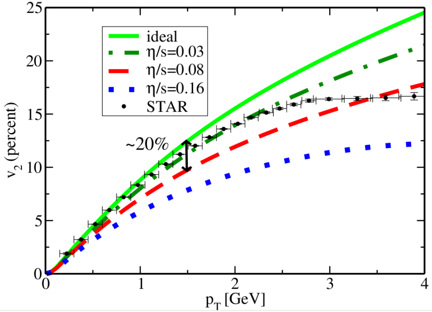
Anisotropic flow and viscosity

- Role of Hydrodynamics:

Initial state spatial deformation $\xrightarrow{Hydro}$ Final state momentum anisotropy

- Viscosity degrades conversion efficiency; necessary to explain data





Relativistic fluid dynamics

- We know that $\rho(t,\vec{x})$, $\vec{v}(t,\vec{x})$ and P are dynamical variables for non-relativistic hydrodynamics.
- For relativistic systems, the mass density $\rho(t,\vec{x})$ is not a good degree of freedom.
- For large kinetic energy, replace $\rho(t,\vec{x})$ by energy density $\epsilon(t,\vec{x})$.
- Similarly, $\vec{v}(t,\vec{x})$ should be replaced by

$$u^\mu = \frac{dx^\mu}{d\tau} = \frac{dt}{d\tau}\frac{dx^\mu}{dt} = \frac{1}{\sqrt{1-\vec{v}^2}}\begin{pmatrix} 1 \\ \vec{v} \end{pmatrix} = \gamma(\vec{v})\begin{pmatrix} 1 \\ \vec{v} \end{pmatrix}$$

- The fluid four-velocity u^μ is timelike: $u^2 \equiv u^\mu g_{\mu\nu} u^\nu = 1$.
- Hydrodynamic equations are essentially conservation equations:
  - Energy-momentum conservation: $\partial_\mu T^{\mu\nu} = 0$.
  - Current conservation: $\partial_\mu N^\mu = 0$.
- $T^{\mu\nu}$: Energy-momentum tensor, N^μ: Charge current.

Relativistic ideal fluids

- The energy-momentum tensor of an ideal fluid can be written in terms of the available tensor degrees of freedom:

$$T^{\mu\nu}_{(0)} = c_1 u^\mu u^\nu + c_2 g^{\mu\nu}$$

- Local rest frame (LRF), $u^\mu_{LRF} = (1, 0, 0, 0)$, implies

$$T^{00}_{(0)LRF} = \epsilon_0, \quad T^{i0}_{(0)LRF} = T^{0i}_{(0)LRF} = 0, \quad T^{ij}_{(0)LRF} = P_0 \delta^{ij}$$

- One can extract the scalars c_1 and c_2 in LRF

$$T^{\mu\nu}_{(0)LRF} = \mathrm{diag}(\epsilon_0,\, P_0,\, P_0,\, P_0) \Rightarrow c_1 = \epsilon_0 + P_0,\, c_2 = -P_0.$$

- Energy-momentum tensor for the ideal fluid, $T^{\mu\nu}_{(0)}$ is

$$T^{\mu\nu}_{(0)} = \epsilon_0 u^\mu u^\nu - P_0 \Delta^{\mu\nu}$$

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, \quad u_\mu \Delta^{\mu\nu} = 0$$

- Similarly, $N^\mu_{(0)} = n_0\, u^\mu$.

Non-relativistic vs relativistic hydrodynamics

- Consider incompressible non-relativistic fluid;
  trivia: absolute incompressibility not allowed by relativity.
- Assume that no conserved charges present in relativistic case.

Non-relativistic	**Relativistic**
Continuity equation: $\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$	$D\epsilon + (\epsilon + P)\partial_\mu u^\mu = 0$
Euler equation: $\rho\left[\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v}\right] + \vec{\nabla}P = 0$	$(\epsilon + P)Du^\mu - \nabla^\mu P = 0$
ρ measures inertia of the non-rel. fluid	$D = u^\mu\partial_\mu,\ \nabla^\mu = \Delta^{\mu\alpha}\partial_\alpha$

- The EoM for relativistic fluid is given by $\partial_\mu T^{\mu\nu} = 0$.
- Inertia of the relativistic fluid is determined by $(\epsilon + P)$.
- Remember, the entropy density is given by: $s = (\epsilon + P)/T$.

Relativistic kinetic theory: let's build intuition

- Kinetic theory: calculation of macroscopic quantities by means of statistical description in terms of distribution function.
- Let us consider a system of relativistic particles of rest mass m with momenta $\mathbf{p}$ and energy p^0
$$p^0 = \sqrt{\mathbf{p}^2 + m^2}$$
- For large no. of particles, introduce a function $f(x, p)$ which gives a distribution of the four-momenta $p = p^\mu = (p^0, \mathbf{p})$ at each space-time point.
- $f(x, p)\Delta^3 x \, \Delta^3 p$ gives average no. of particles at any given time in the volume element $\Delta^3 x$ at point x with momenta in the range $(\mathbf{p}, \mathbf{p} + \Delta\mathbf{p})$.
- Statistical assumptions:
  - No. of particles contained in $\Delta^3 x$ is large $(N \gg 1)$.
  - $\Delta^3 x$ is small compared to macroscopic volume $(\Delta^3 x / V \ll 1)$.

Relativistic kinetic theory: Particle four-flow

- To describe a non-uniform system, $n(x)$ is introduced: $n(x)\Delta^3 x$ is avg. no. of particles in volume $\Delta^3 x$ at x.
- Similarly particle flow $\mathbf{j}(x)$ is defined as the particle current along (x,y,z) directions.
- These two local quantities, particle density and particle flow constitute a four-vector field: $N^\mu = (n, \mathbf{j})$
- With the help of distribution function, the particle density and particle flow is given by:

$$n(x) = \frac{g}{(2\pi)^3}\int d^3p \; f(x,p); \quad \mathbf{j}(x) = \frac{g}{(2\pi)^3}\int d^3p \; \mathbf{v} \, f(x,p)$$

where $\mathbf{v} = \mathbf{p}/p^0$ is the velocity.

- Particle four-flow can be written in a unified way

$$N^\mu(x) = \frac{g}{(2\pi)^3}\int \frac{d^3p}{p^0} p^\mu \, f(x,p)$$

Relativistic kinetic theory: Energy-momentum tensor

- Energy per particle is p^0, the average can be written as

$$T^{00}(x) = \frac{g}{(2\pi)^3}\int d^3p\, p^0\, f(x,p) = \frac{g}{(2\pi)^3}\int \frac{d^3p}{p^0}\, p^0\, p^0\, f(x,p)$$

- Similarly energy flow and momentum density are defined as

$$T^{0i}(x) = \frac{g}{(2\pi)^3}\int d^3p\, p^0\, v^i\, f(x,p); \quad T^{i0}(x) = \frac{g}{(2\pi)^3}\int d^3p\, p^i\, f(x,p)$$

- For momentum flow (flow in direction j of momentum in direction i), we have

$$T^{ij}(x) = \frac{g}{(2\pi)^3}\int d^3p\, p^i\, v^j\, f(x,p); \qquad \left[v^j = \frac{p^j}{p^0}\right]$$

- Combining all this in compact covariant form:

$$T^{\mu\nu}(x) = \frac{g}{(2\pi)^3}\int \frac{d^3p}{p^0} p^\mu\, p^\nu\, f(x,p)$$

Relativistic dissipative fluids

- Fluids are in general dissipative; dissipation has to be included.
- Define an equilibrium state $\epsilon = \epsilon_0,\ n = n_0$ (matching conditions).

$$u_\mu u_\nu \delta T^{\mu\nu} = 0, \quad u_\mu \delta N^\mu = 0$$

$$\delta T^{\mu\nu} \equiv T^{\mu\nu} - T^{\mu\nu}_{(0)}, \quad \delta N^\mu = N^\mu - N^\mu_{(0)}$$

- Most general form with dissipation:

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + 2u^{(\mu} h^{\nu)} + \pi^{\mu\nu}, \qquad N^\mu = n u^\mu + n^\mu$$

- Orthogonality: $u_\mu h^\mu = u_\mu n^\mu = u_\mu \pi^{\mu\nu} = u_\nu \pi^{\mu\nu} = \Delta_{\mu\nu}\pi^{\mu\nu} = 0.$
- Freedom for LRF definition:

$$T^{\mu\nu} : 10, \quad N^\mu : 4 \quad \Rightarrow \quad \text{Total} : 14$$

$$\underbrace{\epsilon : 1,\ P : 1,\ n : 1}_{\text{EoS:1, 2}-\text{independent}},\ u^\mu : 3,\ \Pi : 1,\ h^\mu : 3,\ n^\mu : 3,\ \pi^{\mu\nu} : 5 \ \Rightarrow\ \text{Total} : 17$$

- Definition of velocity field to be specified.

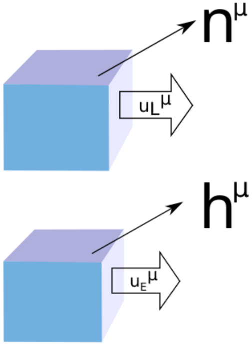
Local fluid rest frame definitions

- Two natural definitions for fluid field.
- Landau frame definition: $u_\nu T^{\mu\nu} = \epsilon u^\mu$

$$u^\mu = \frac{u_\nu T^{\mu\nu}}{u_\alpha u_\beta T^{\alpha\beta}} \quad \Rightarrow \quad h^\mu = 0$$

- Eckart frame definition: $N^\mu = n u^\mu$

$$u^\mu = \frac{N^\mu}{\sqrt{N_\alpha N^\alpha}} \quad \Rightarrow \quad n^\mu = 0$$



- Landau frame condition more convenient.
- Landau frame uniquely defined for multiple conserved charges.

Relativistic fluid dynamics

- Conservation equations for energy-momentum and charge current.

Ideal	Dissipative
$T^{\mu\nu} = \epsilon u^\mu u^\nu - P\Delta^{\mu\nu}$	$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}$
$N^\mu = n u^\mu$	$N^\mu = n u^\mu + n^\mu$
Unknowns: $\underbrace{\epsilon,\ P,\ n,\ u^\mu}_{1+1+1+3} = 6$	$\underbrace{\epsilon,\ P,\ n,\ u^\mu,\ \Pi,\ \pi^{\mu\nu},\ n^\mu}_{1+1+1+3+1+5+3} = 15$
Equations: $\underbrace{\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu N^\mu = 0, \quad EOS}_{4 \quad + \quad 1 \quad + \quad 1} = 6$	
Closed set of equations	9 more equations required

- Here $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$ and Landau frame chosen: $T^{\mu\nu} u_\nu = \epsilon u^\mu$.
- Equations required for dissipative currents Π, $\pi^{\mu\nu}$ and n^μ.

Relativistic Navier-Stokes [Landau and Lifshitz, *Fluid Mechanics*, 1987]

- Second law in covariant form: $\partial_\mu S^\mu \geq 0$, where

$$S^\mu = s\, u^\mu \quad ; \quad s = \frac{\epsilon + P - \mu n}{T}.$$

- Demanding second-law from this entropy current,

$$\pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu}, \quad \Pi = -\zeta\,\theta, \quad n^\mu = \kappa\,\nabla^\mu a$$

$$\sigma^{\mu\nu} \equiv \partial^{\langle\mu}u^{\nu\rangle} = \Delta^{\mu\nu}_{\alpha\beta}\,\partial^\alpha u^\beta, \quad \theta \equiv \partial_\mu u^\mu, \quad \nabla^\mu \equiv \partial^{\langle\mu\rangle} = \Delta^{\mu\alpha}\,\partial_\alpha, \quad a \equiv \mu/T$$

$$\Delta^{\mu\nu}_{\alpha\beta} \equiv \frac{1}{2}(\Delta^\mu_\alpha \Delta^\nu_\beta + \Delta^\mu_\beta \Delta^\nu_\alpha) - \frac{1}{3}\Delta^{\mu\nu}\Delta_{\alpha\beta}$$

$$\eta = \beta_\pi\,\tau_R, \quad \zeta = \beta_\Pi\,\tau_R, \quad \kappa = \beta_n\,\tau_R$$

- η: co-efficient of shear viscosity.
- ζ: co-efficient of bulk viscosity.
- κ: charge conductivity.

Dissipation effects

- In simple terms:

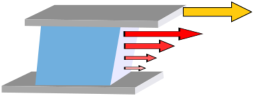
$$\pi^{yx} = 2\,\eta\,\partial^{\langle y}u^{x\rangle}$$

$$\Pi = -\zeta\,\partial\cdot u$$

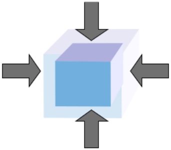
$$n^{x} = \kappa\,\partial^{\langle x\rangle}a$$

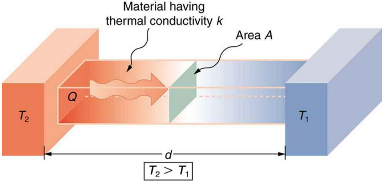
- Shear viscosity: resistance to shape change.
- Bulk viscosity: resistance to volume change.
- Charge/heat conductivity: fluid's resistance to flow of charge/heat.

▸ Shear viscosity: fluid's resistance to shear forces



▸ Bulk viscosity: fluid's resistance to compression





QGP shear viscosity

John Mainstone (Wikipedia)

Pitch

Start 1927

1^{st} drop 1938

8^{th} drop 2000

$\eta \sim 2\,10^8$ Pa s

$\sim 10^{11} \eta_{H_2O}$

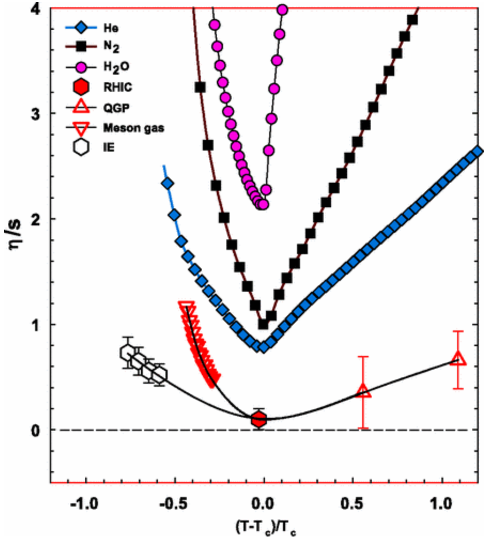
Wikipedia: The ninth drop touched the eighth drop on 17 April 2014. However, it was still attached to the funnel. On 24 April 2014, Prof. White decided to replace the beaker holding the previous eight drops before the ninth drop fused to them. While the bell jar was being lifted, the wooden base wobbled and the ninth drop snapped away from the funnel.

$$\eta_{\rm qgp} > \eta_{\rm pitch}$$

$$(\eta/s)_{\rm qgp} < \frac{3}{4\pi}$$

Specific shear viscosity is the relevant quantity for dynamics.

η/s of various fluids



[Lacey et. al., Phys. Rev. Lett. 98, 092301 (2007)]

Acausality problem [P. Romatschke, Int. J. Mod. Phys. E 19, 1 (2010)]

- Consider small perturbations of energy density and fluid velocity,

$$\epsilon = \epsilon_0 + \delta\epsilon(t,x), \quad u^\mu = (1,\, \mathbf{0}) + \delta u^\mu(t,x).$$

- For a particular direction y, we get a diffusion-type equation

$$\partial_t \delta u^y - \frac{\eta_0}{\epsilon_0 + P_0}\partial_x^2 \delta u^y = \mathcal{O}(\delta^2).$$

- Use mixed Laplace-Fourier wave ansatz to study the individual modes

$$\delta u^y(t,x) = \exp(-\omega t + ikx) f_{\omega,k}.$$

- We obtain the "dispersion-relation" for the diffusion equation

$$\omega = \frac{\eta_0}{\epsilon_0 + P_0} k^2.$$

- The speed of diffusion of a mode with wavenumber k

$$v_T(k) = \frac{d\omega}{dk} = 2\frac{\eta_0}{\epsilon_0 + P_0} k.$$

- Increases $\propto k$ without bound: acausal behavior.

Intuitive solution

- One possible way out is the "Maxwell-Cattaneo" law,

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}.$$

- The diffusion equation becomes a relaxation-type equation.
- A new transport coefficient: the relaxation time τ_π.
- The effect of this modification on the dispersion relation for the perturbation δu^y becomes,

$$\omega = \frac{\eta_0}{\epsilon_0 + P_0}\frac{k^2}{1 - \omega\tau_\pi}.$$

- The above equation describes propagating waves with a maximum propagation speed

$$v_T^{\max} \equiv \lim_{k\to\infty}\frac{d|\omega|}{dk} = \sqrt{\frac{\eta_0}{(\epsilon_0 + P_0)\tau_\pi}}.$$

- Interestingly, for all known fluids the limiting value of $v_T^{\max} < 1$.

Muller-Israel-Stewart theory from entropy current

- While Maxwell-Cattaneo law is successful in solving the acausality problem, it does not follow from a first-principles framework.
- Desirable to derive some variant of Maxwell-Cattaneo law which preserves causality: Muller-Israel-Stewart (MIS) theory.
- Assuming entropy current to be algebraic in the hydrodynamic degrees of freedom,

$$S^\mu = su^\mu - \frac{\beta_2}{2T}\pi_{\alpha\beta}\pi^{\alpha\beta}u^\mu + \mathcal{O}(\pi^3).$$

- Demanding second law of thermodynamics, $\partial_\mu S^\mu \geq 0$,

$$\tau_\pi\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle} - \frac{4}{3}\tau_\pi\theta\pi^{\mu\nu}.$$

- The relaxation time can be related as: $\tau_\pi = 2\eta\beta_2$.
- One more transport coefficient: β_2 or equivalently τ_π.

Other aspects and references

- Several other aspects pertaining to regime of applicability of hydrodynamics:
  - Re-summation of the asymptotic gradient expansion.
  - Emergent dynamical attractors and resurgence.
  - Power law behavior of solutions at early times.
- An extensive review on "New theories of relativistic hydrodynamics in the LHC era": [W. Florkowski, M. Heller, M. Spalinski, Rept.Prog.Phys. 81 (2018) 4, 046001, arXiv:1707.02282]
- For basic theoretical framework of relativistic hydrodynamics: [**AJ** and V. Roy, Adv.High Energy Phys. 2016 (2016) 9623034, arXiv:1605.08694]
- Also recommended is a recent book by Romatschke: "Relativistic Fluid Dynamics In and Out of Equilibrium" [P. Romatschke and U. Romatschke, arXiv:1712.05815]

First-order causal hydrodynamics [P. Kovtun BIRS workshop seminar]

- In general, out of equilibrium, the notions of "local rest frame", "local densities" etc are ambiguous, and are a matter of pure convention/taste.
- Identify the low-energy variables: $T,\ \mu,\ u^\mu$.
- Write down all possible terms in the constitutive relations consistent with the symmetry.
- Do this up to a given order (say, first order) in the derivative expansion.
- Do not use identities between different derivatives from lower-order equation of motion: exact first-order theory.
- Constrain the coefficients so that the physics is sensible, e.g. demand stability of equilibrium, stability of perturbation and causality.

[Kovtun, JHEP 10 (2019) 034; Bemfica, Disconzi, Noronha, Phys.Rev.D 100 (2019) 104020]

Minimal stable and causal uncharged hydro

[Kovtun, BIRS workshop]

$$T^{\mu\nu} = \mathscr{E} u^\mu u^\nu + \mathscr{P} \Delta^{\mu\nu} + \mathscr{Q}^\mu u^\nu + \mathscr{Q}^\nu u^\mu + \mathscr{T}^{\mu\nu}$$

$$\mathscr{E} = \epsilon + \varepsilon_1 \dot{T}/T + O(\partial^2)$$

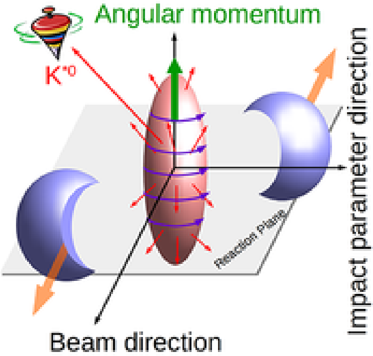
$$\mathscr{P} = p + \pi_1 \dot{T}/T + \left(-\zeta + v_s^2(\pi_1 - v_s^2 \varepsilon_1)\right) \partial\cdot u + O(\partial^2)$$

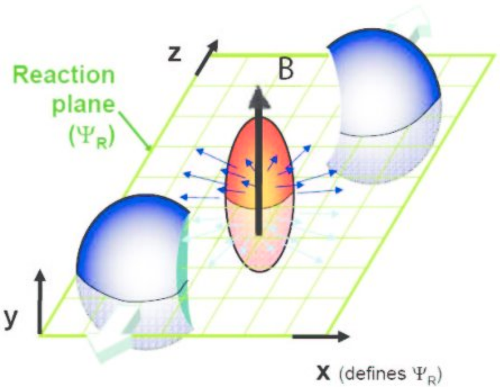
$$\mathscr{Q}^\mu = \theta \left(\dot{u}^\mu + \frac{1}{T} \Delta^{\mu\lambda} \partial_\lambda T \right) + O(\partial^2)$$

$$\mathscr{T}^{\mu\nu} = -\eta \sigma^{\mu\nu} + O(\partial^2)$$

Three parameters $\varepsilon_1(T), \pi_1(T), \theta(T)$ besides $\eta(T), \zeta(T)$

Angular momentum and Magnetic field





- Angular momentum and magnetic field generated in the fireball.
- Magneto-hydro formulation with angular momentum conservation
- Current state-of-the-art: both effects are treated separately.
- Possible unified formulation in future.

Angular momentum conservation: particles

- Orbital angular momentum of a particle with momentum $\vec{p}$:

$$\vec{L} = \vec{x} \times \vec{p} \quad \Rightarrow \quad L_i = \varepsilon_{ijk}\, x_i\, p_j$$

- One can obtain the dual tensor:

$$L_{ij} \equiv \varepsilon_{ijk}\, L_k \quad \Rightarrow \quad L_{ij} = x_i\, p_j - x_j\, p_i$$

- We know that both definitions are equivalent.
- In absence of external torque, $\dfrac{d\vec{L}}{dt} = 0$ and $\partial_i L_{ij} = 0$.
- This treatment valid for non-relativistic point particles.
- For fluids, particle momenta → "generalized fluid momenta"

The energy-momentum tensor

Angular momentum conservation: fluid

- The orbital angular momentum for relativistic fluids is defined as

$$L^{\lambda,\mu\nu} = x^\mu T^{\lambda\nu} - x^\nu T^{\lambda\mu}$$

- Keeping in mind the energy-momentum conservation, $\partial_\mu T^{\mu\nu} = 0$:

$$\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$$

- Obviously, for symmetric $T^{\mu\nu}$, orbital angular momentum is automatically conserved. Classically $T^{\mu\nu}$ always symmetric.

- For medium constituent with intrinsic spin, different story

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- Ensure total angular momentum conservation: $\partial_\lambda J^{\lambda,\mu\nu} = 0$.

- In absence of coupling terms, $\partial_\lambda S^{\lambda,\mu\nu} = 0$ (Spin Hydrodynamics).

Pseudo-gauge transformations

- Recall that the total angular momentum is given by,

$$J^{\lambda,\mu\nu} = L^{\lambda,\mu\nu} + S^{\lambda,\mu\nu}$$

- With $\partial_\mu T^{\mu\nu} = 0$, and $\partial_\lambda L^{\lambda,\mu\nu} = T^{\mu\nu} - T^{\nu\mu}$,

$$\partial_\lambda J^{\lambda,\mu\nu} = 0 \quad \Longrightarrow \quad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Hence the final hydrodynamic equations can be written as

$$\partial_\mu T^{\mu\nu} = 0, \qquad \partial_\lambda S^{\lambda,\mu\nu} = T^{\nu\mu} - T^{\mu\nu}$$

- Also holds with the following redefinition

$$\tilde{T}^{\mu\nu} = T^{\mu\nu} + \frac{1}{2}\partial_\lambda \left(\Phi^{\lambda,\mu\nu} - \Phi^{\mu,\lambda\nu} - \Phi^{\nu,\lambda\mu} \right)$$

$$\tilde{S}^{\lambda,\mu\nu} = S^{\lambda,\mu\nu} - \Phi^{\lambda,\mu\nu}$$

- Freedom due to space-time symmetry; including torsion fixes this.

[Gallegos et. al., SciPost Phys. 11, 041 (2021); Hongo et. al., JHEP 11 (2021) 150]

Theoretical development in spin-hydrodynamics

- Within kinetic theory [Florkowski et. al., PRC 97, 041901 (2018); PRD 97, 116017 (2018); Bhadury et. al. PLB 814, 136096 (2021); PRD 103, 014030 (2021)].
- Other parallel approaches from Wigner function [N. Weickgenannt, X.-l. Sheng, E. Speranza, Q. Wang and D. Rischke, PRD 100 (2019) 056018].
- Appraoch based on chiral kinetic theory [S. Shi, C. Gale and S. Jeon, PRC 103 (2021) 044906].
- Appraoch based on Lagrangian method [D. Montenegro and G. Torrieri, PRD 100 (2019) 056011].
- Formulation with torsion in metric [A. D. Gallegos, U. Gürsoy and A. Yarom, SciPost Phys. 11, 041 (2021); M. Hongo, X.-G. Huang, M. Kaminski, M. Stephanov, H.-U. Yee, JHEP 11 (2021) 150].
- Useful reviews on spin hydro: [W. Florkowski, R. Ryblewski and A. Kumar, Prog.Part.Nucl.Phys. 108 (2019) 103709; S. Bhadury, J. Bhatt, A. Jaiswal and A. Kumar, Eur.Phys.J.ST 230 (2021) 3, 655-672].
- Much work needed in this direction.

Electromagnetic field and fluid

- Let's consider non-polarizable, non-magnetizable fluid.
- The equations of motion of magento-hydrodynamics in absence of external charge current:

$$\partial_\mu T^{\mu\nu} = 0, \quad \partial_\mu J^\mu = 0$$

- Total energy momentum tensor and electric charge current

$$T^{\mu\nu} = T_{\rm f}^{\mu\nu} + T_{\rm em}^{\mu\nu}, \quad J^\mu = qN_{\rm f}^\mu$$

- The energy momentum of fluid not separately conserved

$$\partial_\mu T_{\rm f}^{\mu\nu} = F^{\nu\lambda} J_\lambda \quad \Rightarrow \quad \partial_\mu T_{\rm em}^{\mu\nu} = -\partial_\mu T_{\rm f}^{\mu\nu} = -F^{\nu\lambda} J_\lambda$$

- Energy momentum of field changes because field performs work on charged particles within the fluid.

Magnetohydrodynamics: non-resistive & non-dissipative

- The charge four-current induced by magnetic field is:

$$J^{\mu}_{\rm ind} = \sigma_E \, E^{\mu}$$

- For non-resistive, electrical conductivity $\sigma_E \to \infty$.
- For induced current to remain finite, $E^{\mu} \to 0$.
- In this limit, Faraday tensor and its dual becomes

$$F^{\mu\nu} \to B^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} u_\alpha B_\beta, \quad \tilde{F}^{\mu\nu} \to \tilde{B}^{\mu\nu} = B^{\mu} u^{\nu} - B^{\nu} u^{\mu}$$

- Maxwell's equations reduce to:

$$\epsilon^{\mu\nu\alpha\beta}(u_\alpha \, \partial_\mu B_\beta + B_\beta \, \partial_\mu u_\alpha) = J^{\mu}, \quad \dot{B}^{\mu} + B^{\mu}\theta = u^{\mu} \, \partial_\nu B^{\nu} + B^{\nu} \nabla_\nu u^{\mu}$$

- Field energy-momentum tensor becomes

$$T^{\mu\nu}_{\rm em} \to T^{\mu\nu}_{\rm B} = \frac{B^2}{2}(u^{\mu} u^{\nu} - \Delta^{\mu\nu} - 2 \, b^{\mu} \, b^{\nu})$$

Where $B^2 \equiv -B^{\mu} B_{\mu}$ and $b^{\mu} \equiv B^{\mu}/B$.

Magnetohydrodynamics in heavy-ion collisions

- Total energy momentum tensor of non-resistive non-dissipative fluids [V. Roy, S. Pu, L. Rezzola and D. Rischke, PLB 750 (2015) 45-52]:

$$T^{\mu\nu} = \left(\epsilon + \frac{B^2}{2}\right) u^\mu u^\nu - \left(P + \frac{B^2}{2}\right) \Delta^{\mu\nu} - B^2\, b^\mu\, b^\nu$$

- In this case, energy momentum tensor of fluid and magnetic field are separately conserved [Denicol et. al. PRD 98 (2018) 076009].
- Resistive dissipative formulation has also been attempted [Denicol et. al. PRD 99 (2019) 056017]
- Several groups actively working on MHD and effect of magnetic field.
- There are some theoretical support from astrophysics literature.
- More phenomenological work required within this framework.

Relativistic hydrodynamics: fast developing field.

Active involvement of bright young minds needed!