

Quantum Field Theoretical structure of conductivity and dilepton

Sabyasachi Ghosh (Physics Dept, IIT Bhilai)

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One-loop Kubo estimations of the shear and bulk viscous coefficients for hot and magnetized bosonic and fermionic systems

Snigdha Ghosh and Sabyasachi Ghosh
Phys. Rev. D **103**, 096015 – Published 18 May 2021

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Kubo estimation of the electrical conductivity for a hot relativistic fluid in the presence of a magnetic field

Sarthak Satapathy, Snigdha Ghosh, and Sabyasachi Ghosh
Phys. Rev. D **104**, 056030 – Published 28 September 2021

Quantum field theoretical structure of electrical conductivity of cold and dense fermionic matter in the presence of a magnetic field

Sarthak Satapathy, Snigdha Ghosh, and Sabyasachi Ghosh
Phys. Rev. D **106**, 036006 – Published 9 August 2022



Time line of Magnetic field in QGP topic

2004

2009

2012

2017

2022

SQGP →

Strong B →

IMC →

Vorticity →

CME test

Estimate of the magnetic field strength in heavy-ion collisions

V. Skokov (Darmstadt, GSI and Frankfurt U., FIAS and Dubna, JINR), A.Yu. Illarionov (Trento U.), V. Toneev (Darmstadt, GSI and Dubna, JINR)

Jul, 2009

8 pages

Published in: *Int.J.Mod.Phys.A* 24 (2009) 5925-5932

QCD quark condensate in external magnetic fields

G.S. Bali (Regensburg U. and Tata Inst.), F. Bruckmann (Regensburg U.), G. Endrodi (Regensburg U.), Z. Fodor (Wuppertal U. and Eotvos U. and Julich, Forschungszentrum), S.D. Katz (Julich, Forschungszentrum) et al.

Phys.Rev.D 86 (2012) 071502 • e-Print: [1206.4205](https://arxiv.org/abs/1206.4205) • DOI: [10.1103/PhysRevD.86.071502](https://doi.org/10.1103/PhysRevD.86.071502)

STAR Collaboration (Sep 7, 2022)

e-Print: [2209.03467 \[nucl-ex\]](https://arxiv.org/abs/2209.03467)



NJL model at finite B

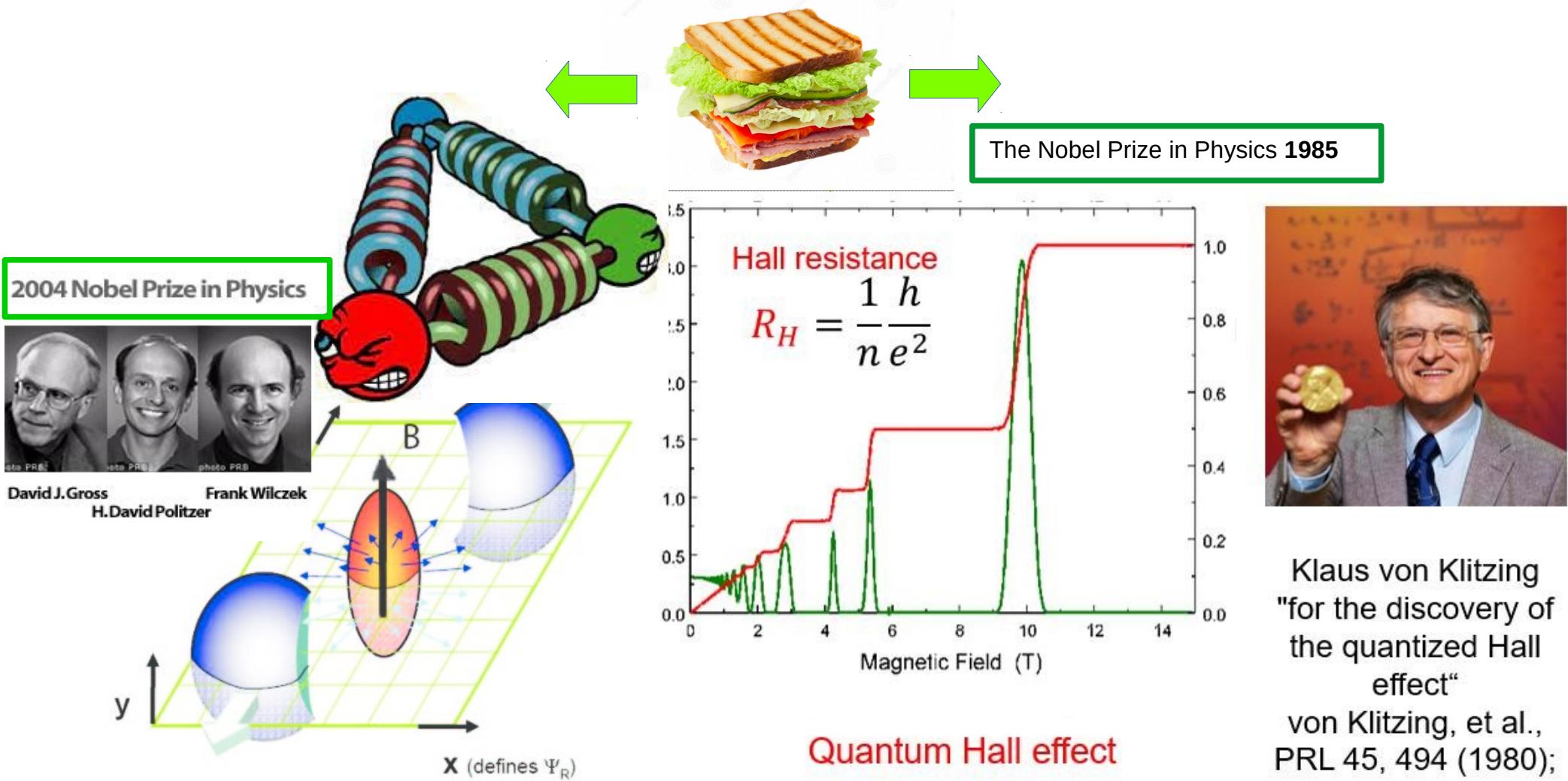
QCD Thermodynamics at finite B

Transport Coefficients at finite B

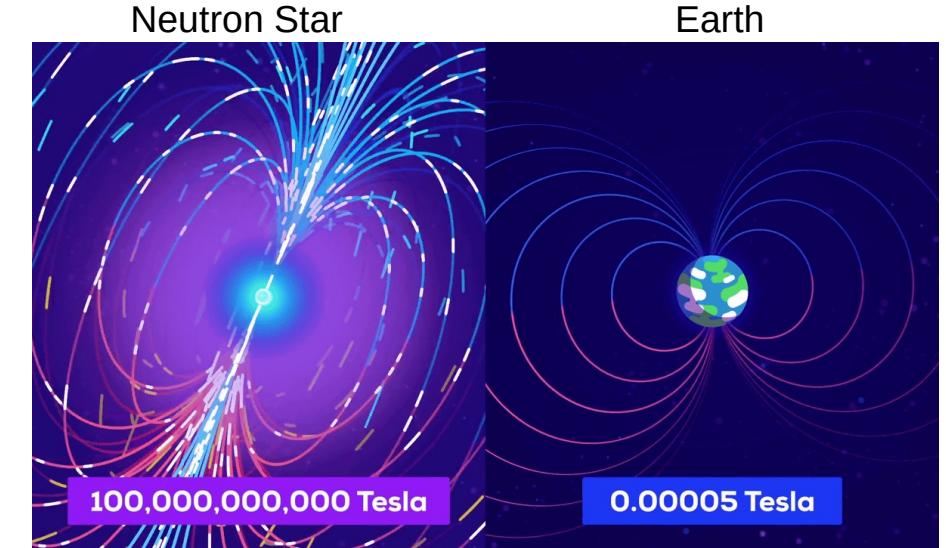
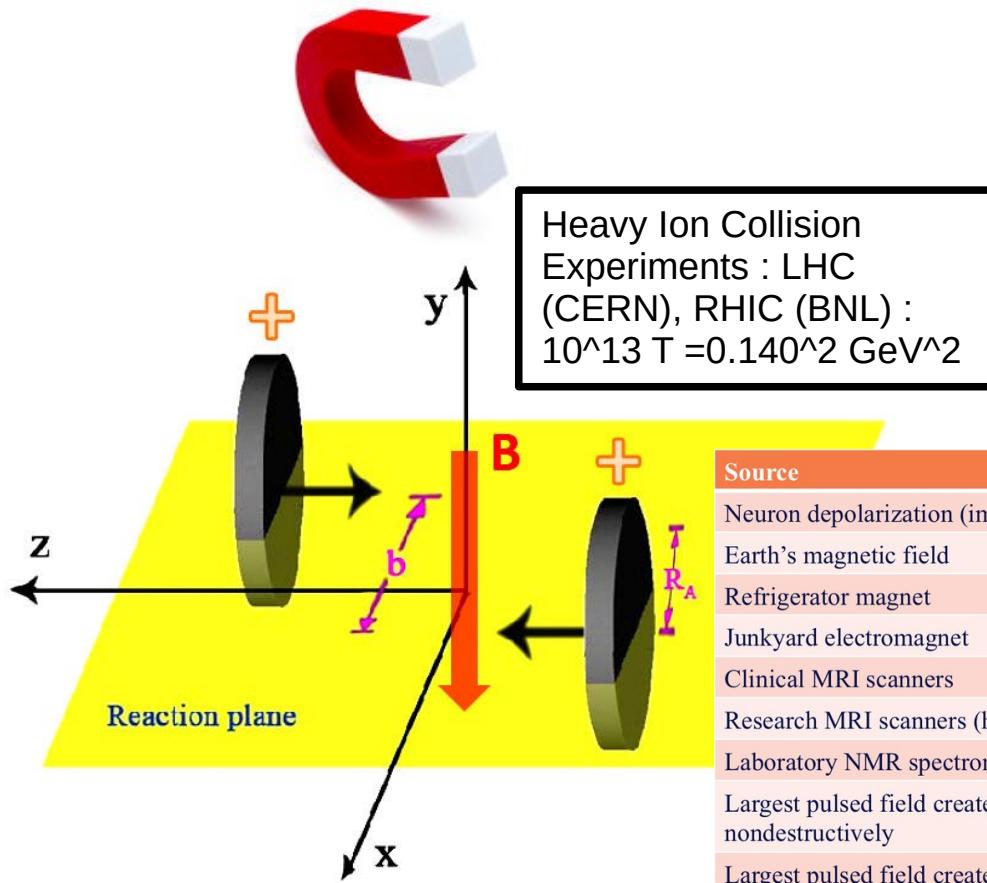


.....& more Institutes

Our Query: Quantum Hall effect in Nuclear Matter?



Magnetic Field:



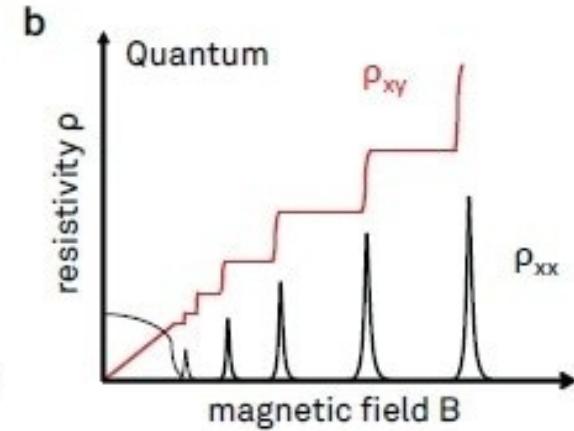
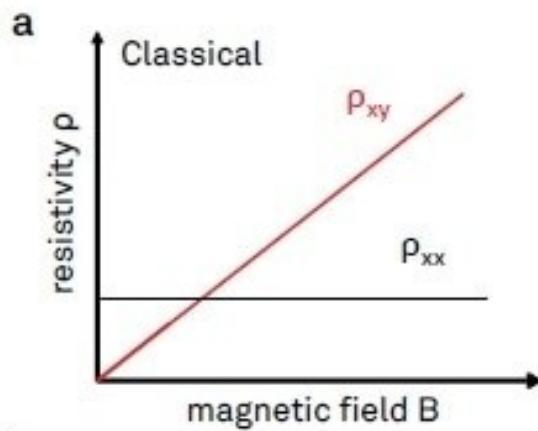
Source	Approximate Magnetic Field
Neuron depolarization (imaged by MEG)	$0.5 \text{ pT} (5 \times 10^{-13} \text{ T})$
Earth's magnetic field	$0.5 \text{ G} (50 \mu\text{T})$
Refrigerator magnet	$50 \text{ G} (5 \text{ mT})$
Junkyard electromagnet	1 T
Clinical MRI scanners	$0.5 - 3.0 \text{ T} (\text{typical})$
Research MRI scanners (human)	$7.0 \text{ T} - 11.7 \text{ T}$
Laboratory NMR spectrometers	$6 - 23 \text{ T}$
Largest pulsed field created in lab nondestructively	97 T
Largest pulsed field created in lab (destroying equipment but not the lab)	730 T



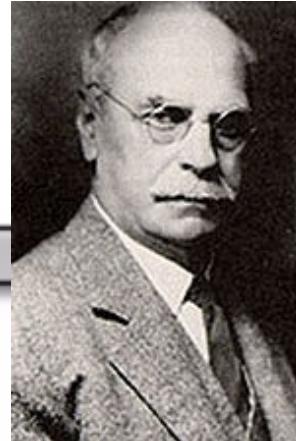
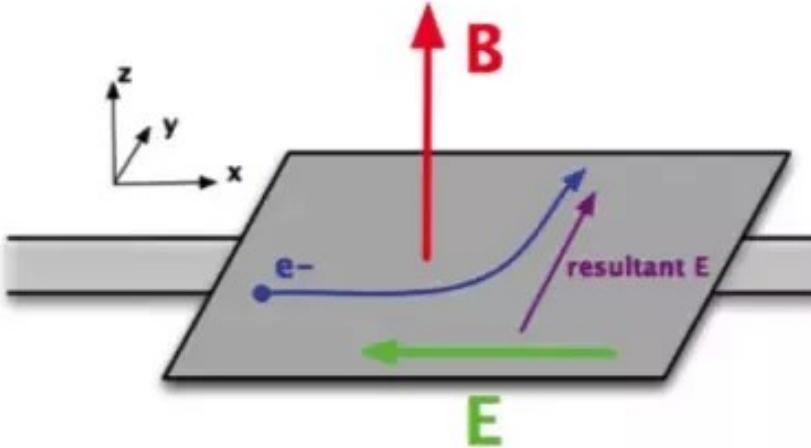
Classical and Quantum Hall effect:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Electric force Magnetic force



(Discovered by
Edwin Hall in 1879)



(Discovered by
Klaus von Klitzing in 1980)

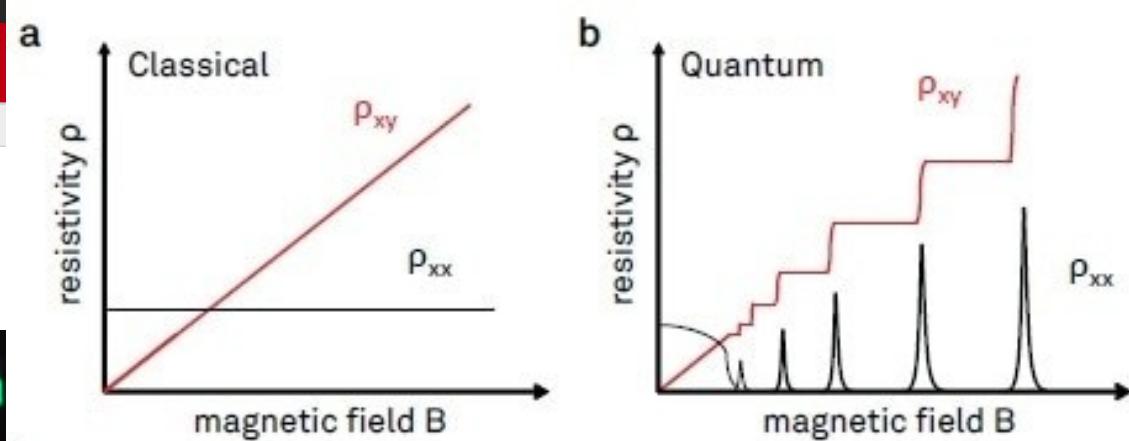




Lectures on the Quantum Hall Effect

David Tong

(Submitted on 21 Jun 2016 (v1), last revised 20 Sep 2016 (this version, v2))



Acknowledgements

These lectures were given in TIFR, Mumbai. I'm grateful to the students, postdocs, faculty and director for their excellent questions and comments which helped me a lot in understanding what I was saying.

Resistivity Matrix:

The *resistivity* is defined as the inverse of the conductivity matrix, both are matrices,

$$\rho = \sigma^{-1} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ -\rho_{xy} & \rho_{yy} \end{pmatrix}$$

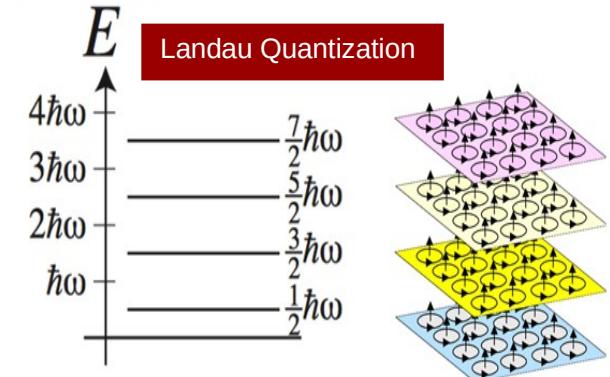
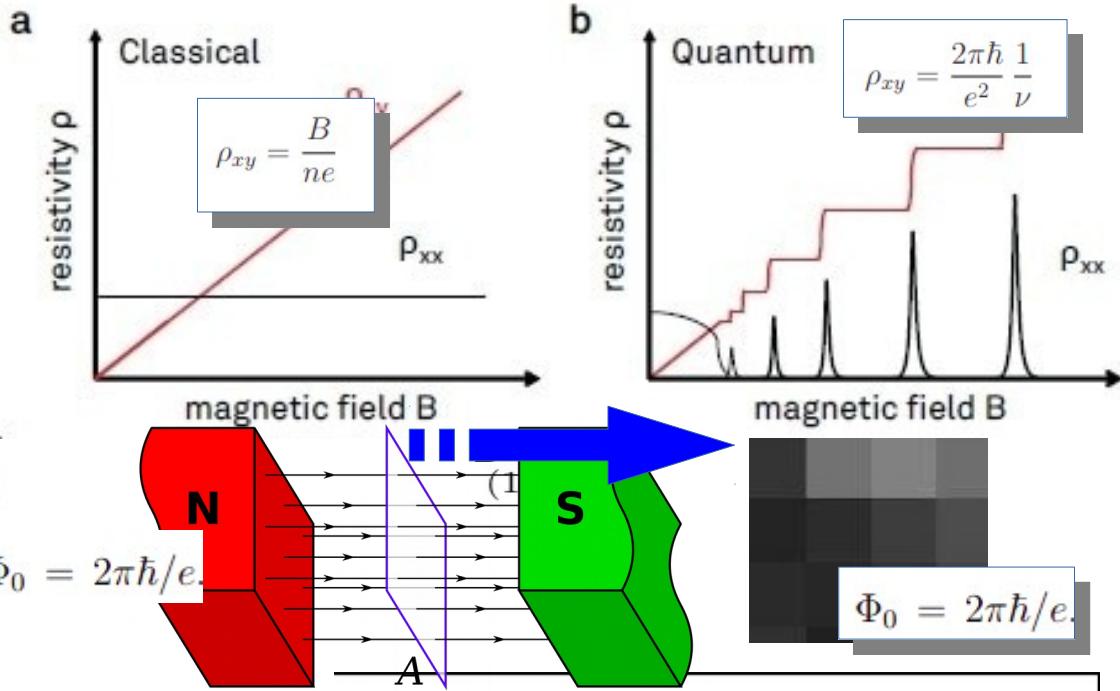
From the Drude model, we have

$$\rho = \frac{1}{\sigma_{DC}} \begin{pmatrix} 1 & \omega_B \tau \\ -\omega_B \tau & 1 \end{pmatrix}$$

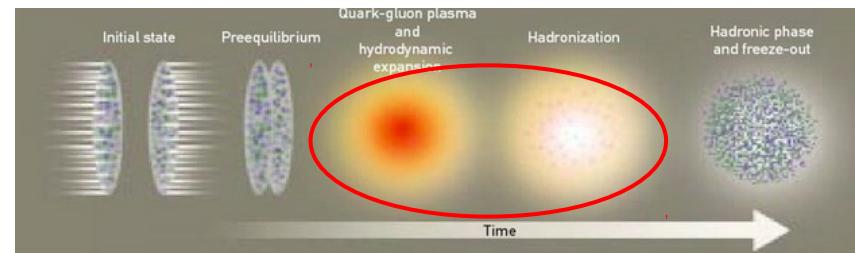
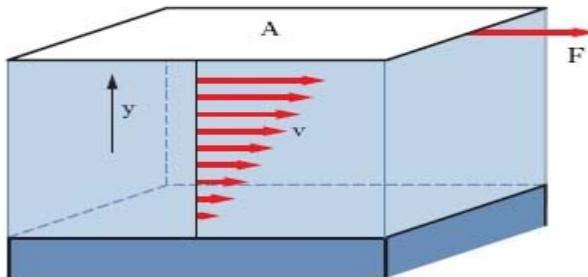
$$\Phi_0 = 2\pi\hbar/e$$

Electrical Conductivity Matrix:

$$\sigma = \frac{\sigma_{DC}}{1 + \omega_B^2 \tau^2} \begin{pmatrix} 1 & -\omega_B \tau \\ \omega_B \tau & 1 \end{pmatrix} \quad \text{with} \quad \sigma_{DC} = \frac{ne^2 \tau}{m}$$



Kinetic Theory (Relaxation Time Approximation) for $B=0$



Macro

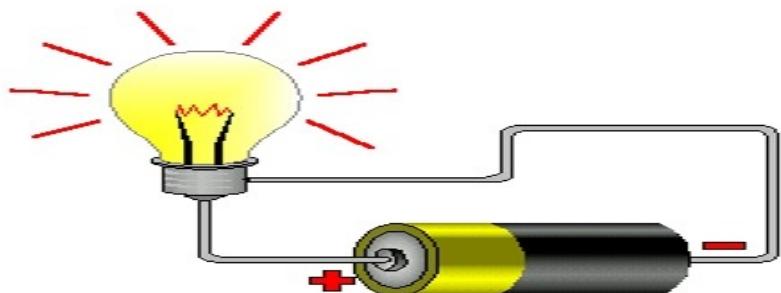
$$\begin{aligned} \eta U_{\eta}^{ij} &= T^{ij} = g \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu p^\nu}{E} \delta f \\ \sigma^{ij} E_j &= J^i = g_e e \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{E} \delta f , \end{aligned}$$

Micro

$$\delta f = (A_{ij} U_{\eta}^{ij} + C_i E^i) f (1 \pm f) ,$$

$$\frac{p^\mu}{E} \partial_\mu f^\pm + F^\mu \frac{\partial f^\pm}{\partial p^\mu} = - \left(\frac{p^\mu u_\mu}{E} \right) \frac{\delta f^\pm}{\tau_c}$$

Relativistic Boltzmann Equation



$$\begin{aligned} \eta_{g,Q} &= \frac{g_{g,Q}}{15T} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left(\frac{\vec{p}^2}{\omega} \right)^2 \tau f (1 \pm f) \\ \sigma_Q &= \frac{e_Q^2 g_e}{3T} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left(\frac{\vec{p}}{\omega} \right)^2 \tau f (1 - f) . \end{aligned}$$

Kinetic Theory (Relaxation Time Approximation) for finite B

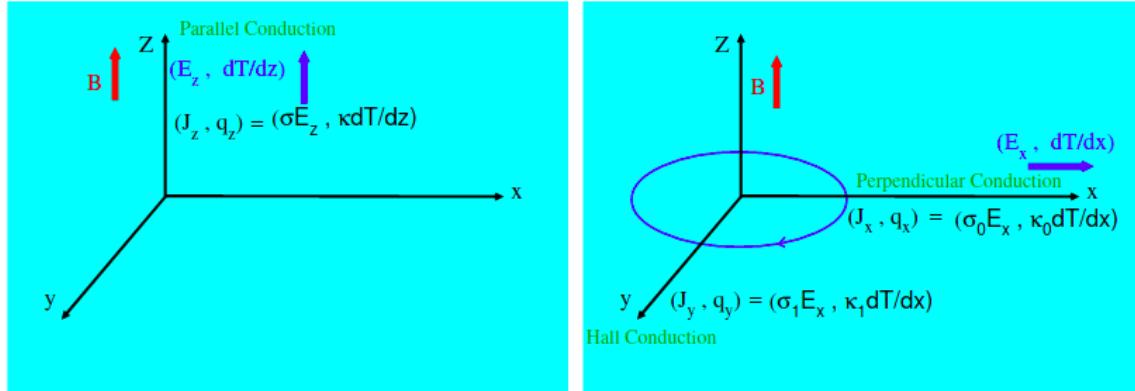
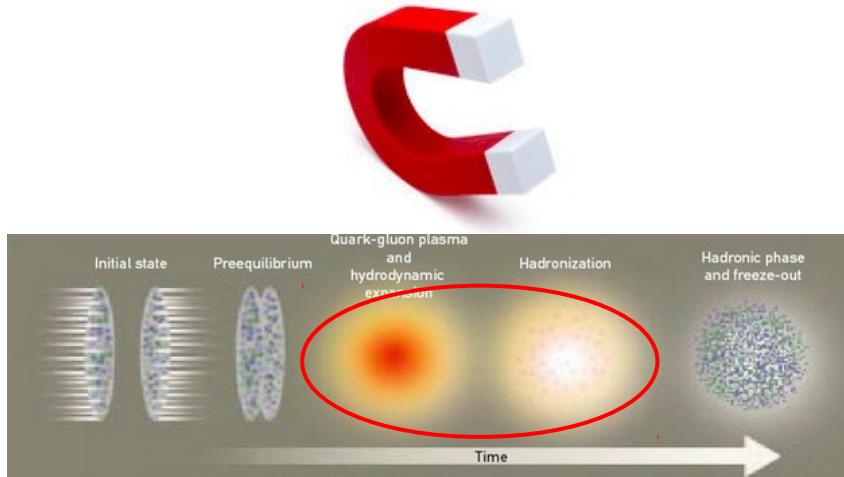


Fig. 5 Schematic diagram of parallel (Left), perpendicular and Hall (Right) components of electrical and thermal conductivity.

Macro

Micro

$$\sigma^{ij} E_j = J^i = g_e e \int \frac{d^3 p}{(2\pi)^3} \frac{p^\mu}{E} \delta f,$$

$$\sigma_Q = \frac{e_Q^2 g_e}{3T} \int \frac{d^3 \vec{p}}{(2\pi)^3} \left(\frac{\vec{p}}{\omega}\right)^2 \tau f(1-f).$$

τ_c

$$\begin{aligned} & \tau_c/[1 + (\tau_c/\tau_B)^2] \\ & \tau_c(\tau_c/\tau_B)/[1 + (\tau_c/\tau_B)^2] \end{aligned}$$

Relativistic Boltzmann Equation

$$\frac{p^i}{E} \partial_i f_0^\pm + e_{Q,\bar{Q}} \mathcal{E}^i \frac{\partial f_0^\pm}{\partial p^i} + e_{Q,\bar{Q}} B b^{ij} v_j \frac{\partial}{\partial p^i} (\delta f^\pm) = -\frac{\delta f^\pm}{\tau_c}$$

Green-Kubo Relation of Transport coefficients



Operators

$$\begin{aligned}\pi^{ij} &\equiv T^{ij} - g^{ij}T_k^k/3, \\ \mathcal{P} &\equiv -T_k^k/3 - c_s^2 T^{00} \text{ (for vanishing chemical potential } \mu = 0\text{)}, \\ \mathcal{K}^i &\equiv T^{0i} - hN^i,\end{aligned}$$



Static Limit

Shear Viscosity

$$\eta = \frac{1}{20} \lim_{q_0, \vec{q} \rightarrow 0} \frac{A_\eta(q_0, \vec{q})}{q_0}, \quad A_\eta = \int d^4x e^{iq \cdot x} \langle [\pi^{ij}(x), \pi_{ij}(0)] \rangle_\beta;$$

Bulk Viscosity

$$\zeta = \frac{1}{2} \lim_{q_0, \vec{q} \rightarrow 0} \frac{A_\zeta(q_0, \vec{q})}{q_0}, \quad A_\zeta = \int d^4x e^{iq \cdot x} \langle [\mathcal{P}(x), \mathcal{P}(0)] \rangle_\beta;$$

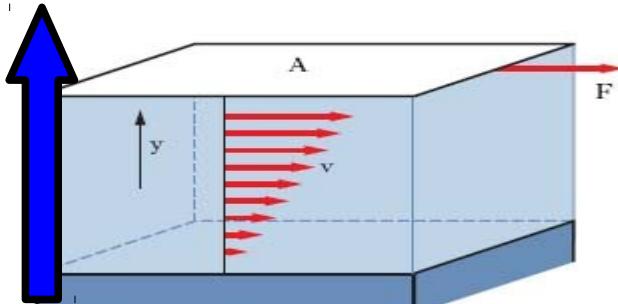
Thermal Conductivity

$$\kappa = \frac{\beta}{6} \lim_{q_0, \vec{q} \rightarrow 0} \frac{A_\kappa(q_0, \vec{q})}{q_0}, \quad A_\kappa = \int d^4x e^{iq \cdot x} \langle [\mathcal{K}^i(x), \mathcal{K}_i(0)] \rangle_\beta$$

Transport Coefficients

Energy-momentum tensor & conserved current

R. Kubo, Soc. Jpn. Phys., 12, 570 (1957).



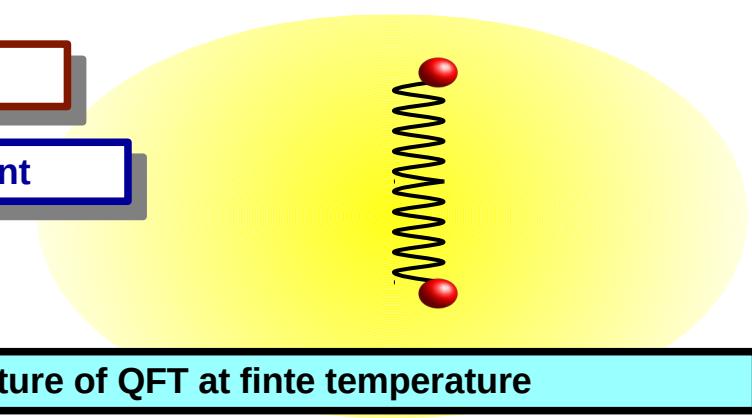
Heat Energy Transport

Momentum Transport

Classical Picture

Velocity gradient

Temperature gradient



Picture of QFT at finite temperature



Dynamical structure

Thermal Correlators

Kubo Formulism: QFT \rightarrow TFT \rightarrow TFT for finite B

Fermion Self-energy in Medium

$$\langle \bar{\psi}_C J^\mu(x) J^\nu(0) \rangle_{11}^B$$

$$\sigma_{\parallel} = e^2 \left(\frac{eB}{2\pi^2} \right) \frac{1}{\Gamma\mu} \sum_{l=0}^{l_{\max}} (2 - \delta_l^0) \sqrt{\mu^2 - m_l^2} \Theta(\mu - m_l),$$

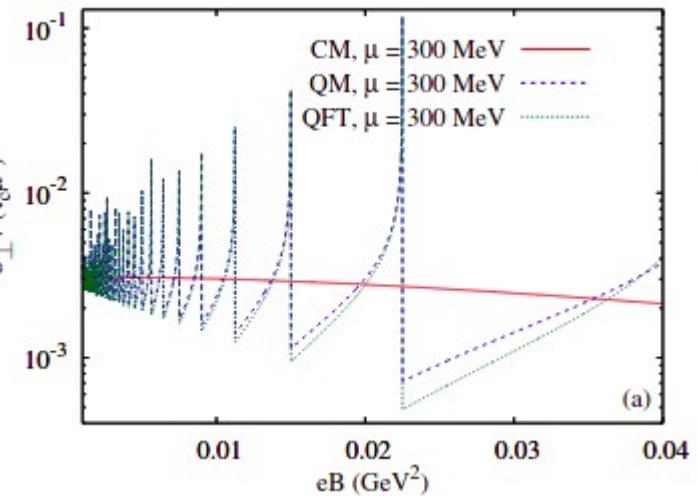
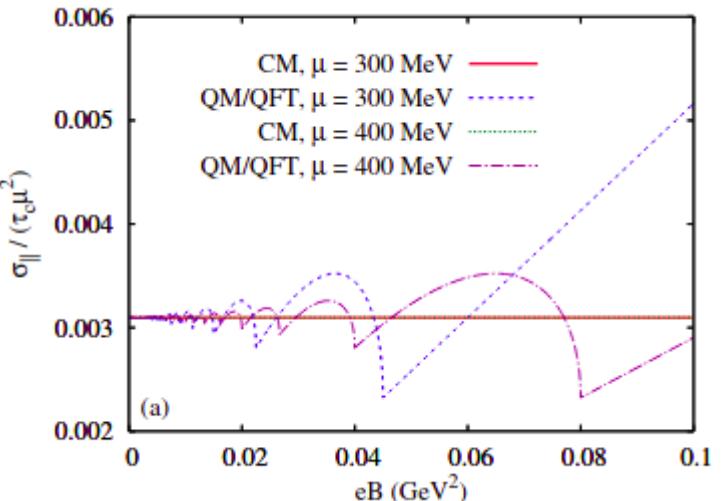
$$\sigma_{\perp} = e^2 \left(\frac{eB}{2\pi^2} \right) \frac{\Gamma}{\Gamma^2 + (\mu - \sqrt{\mu^2 - 2eB})^2} \frac{1}{\sqrt{\mu^2 - 2eB}} \sum_{l=1}^{l_{\max}} \frac{(2l-1)eB}{\sqrt{\mu^2 - m_l^2}} \Theta(\mu - m_l),$$

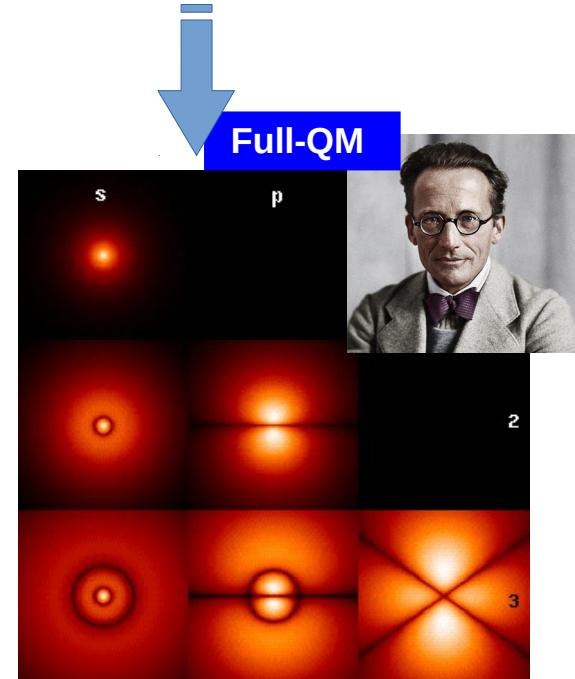
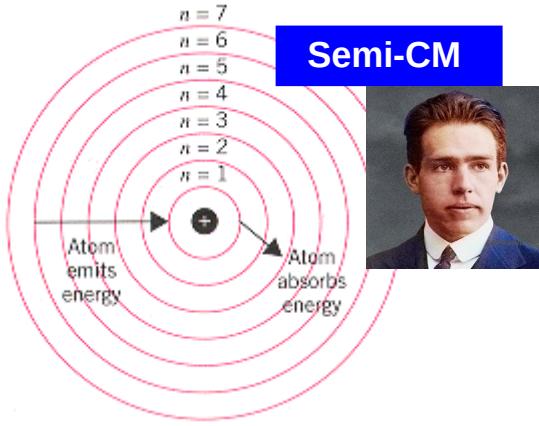
QFT

$$l_{\max} = \left\lfloor \frac{\mu^2 - m^2}{2eB} \right\rfloor$$

CM

$$\sigma_{\parallel, \perp}^{\text{RTA}} = 2e^2 \int \frac{d^3 k}{(2\pi)^3} \tau_c^{\parallel, \perp} \frac{\vec{k}^2}{3\omega_k^2} \delta(\omega_k - \mu) = \frac{e^2}{3\pi^2} \frac{(\mu^2 - m^2)^{3/2}}{\mu} \tau_c^{\parallel, \perp}$$





Concluding Remarks and future plans

Cyclotron time in QFT

$$\tilde{\tau}_c^\perp = \frac{\Gamma}{\Gamma^2 + (\mu - \sqrt{\mu^2 - 2eB})^2} = \tau_c / \left(1 + \frac{\tau_c^2}{\tilde{\tau}_B^2} \right),$$

where,

$$\frac{1}{\tilde{\tau}_B} = (\mu - \sqrt{\mu^2 - 2eB}) = \frac{eB}{\mu} \left\{ 1 + \frac{eB}{2\mu^2} + \frac{(eB)^2}{2\mu^4} + \frac{5(eB)^3}{8\mu^6} + \dots \right\} = \frac{1}{\tau_B} \left\{ 1 + \frac{1}{2\mu\tau_B} + \frac{1}{2(\mu\tau_B)^2} + \frac{5}{8(\mu\tau_B)^3} + \dots \right\}$$

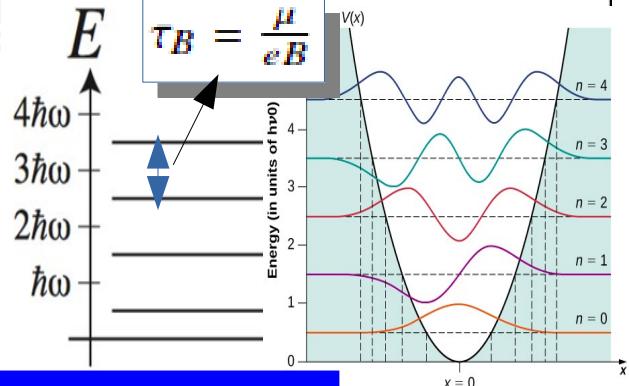
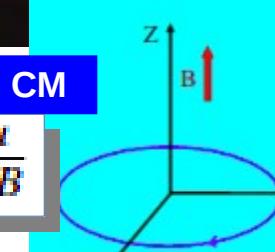
SHM in CM



Cyclotron motion in QM

$$\tau_B = \frac{\mu}{eB}$$

Cyclotron motion in CM



Future plans: impact of this QFT version of cyclotron motion in EM probes