Quarkonia as probe of deconfinement

Theoretical overview

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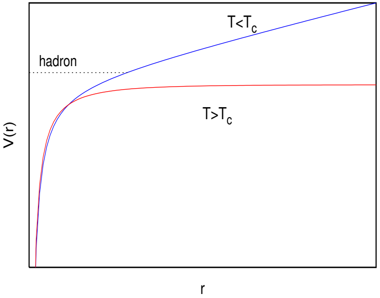
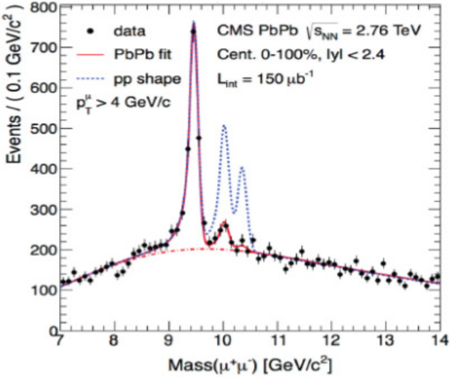
Introduction

Quarkonia: $Q\bar{Q}$ mesons where
$Q = b, c$: $m_Q \gg \Lambda_{QCD}$
Production in hard collisions.
Yield expected to reduce drastically in QGP.



Matsui & Satz 1987

Sharp peaks in dilepton channels from J/ψ, $\Upsilon(1S)$



CMS(2012)

For J/ψ in LHC, large regeneration also indicate deconfinement?

In-medium behavior

How to calculate interaction of quarkonia with medium?

- Dilepton peaks related to vector current spectral function.
- Calculate from lattice correlators?
- Various studies: gradual broadening of J/ψ peak.

  Asakawa & Hatsuda, 2004; Datta, Karsch, Petreczky, Wetzorke, 2004; Matsufuru et al., 2005

- Upsilon using NRQCD.

  Aarts et al., JHEP 11 (2011) 103. Kim, Petreczly & Rothkopf, JHEP 11 (2018) 088.

- Large systematics: quantitative predictions difficult.
- Extended operators give more control.

  Larsen, Meinel & Mukherjee, PLB 800 (2020) 135119.

  But connecting to physical observables difficult.

“Potential” at finite T

- Potential is a good leading order (in $\frac{1}{M_Q}$) approximation for nonrelativistic systems.
- Can we write down a “thermal potential” that can be used to study the properties of quarkonia, e.g., the dilepton peak for Υ?
- The problem has been studied in weak coupling.
- The actual form of the potential depends on the hierarchy of scales.
- Quarkonia scale hierarchy:

$$M \gg \frac{1}{r} \sim Mv \gg E_b \sim Mv^2$$

- For $T \gg \frac{1}{r}$ one would expect to see a screened ppotential.
- For $\frac{1}{r} \gg T$ one probably will not expect to see screening.

Effective field theory

- A systematic description of the thermal potential and other thermal effects has been carried out using effective field theory techniques.

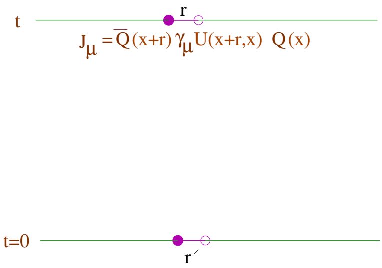
  M. Laine, O. Philipsen, P. Romatschke & M. Tassler, JHEP 03 (2007) 054
  N. Brambilla, J. Ghiglieri, A. Vairo & P. Petreczky, PRD 78 (2008) 014017.

- Theoretically most satisfactory way to study potentials is using pNRQCD, which writes an effective field theory at scales $< \frac{1}{r}$.
- The degrees of freedom in this effective theory are $\bar{Q}Q$ pairs in color singlet and color octet configurations, and the potentials appear as matching coefficients.

$$\begin{aligned} \mathcal{L}_{pNR} &= \mathrm{Tr} \left[S^\dagger \left(i\partial_0 - V_s(r) \right) S + O^\dagger \left(iD_0 - V_o(r) \right) O \right] \\ &+ \mathrm{Tr} \left[S^\dagger \vec{r}.g\vec{E} O + O^\dagger \vec{r}.g\vec{E} S + O^\dagger \vec{r}.g\vec{E} O \right] + \mathcal{O}(\frac{1}{M}) \end{aligned}$$

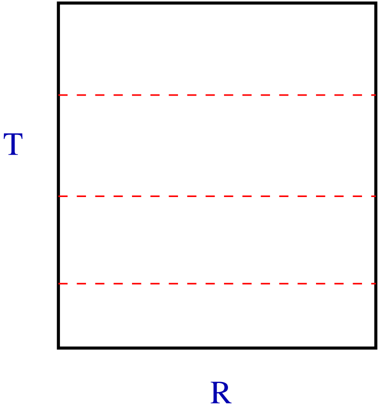
Pineda & Soto (1998); N. Brambilla, A. Pineda, J. Soto & A. Vairo, RMP 77 (2005) 1423

Calculation of $V(r)$ from QCD



Leads to a time-like Wilson loop at large M.
Define $V(r, T)$ through

$$\lim_{t \to \infty} i\partial_t W(r,t) = V(r)W(r,t)$$



In perturbation theory, potential obtained from the longitudinal gluon propagator.
Includes LO in $\frac{1}{M_Q}$ but all orders in α_s.

Thermal potential: Perturbative results

- $T \gg \frac{1}{r} \sim \alpha M, m_D \sim gT$

$$V(r) = -\frac{4}{3}\alpha_s \frac{e^{-m_D r}}{r} - i\,\frac{8}{3}\alpha_s T \int_0^\infty dz \frac{z}{(z^2+1)} \frac{\sin z\, m_D r}{z\, m_D r}$$

M. Laine, O. Philipsen, P. Romatschke and M. tassler, JHEP 0703 (2007) 054.

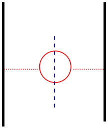
- For $\frac{1}{r} \gg m_D$:

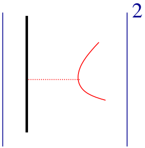
$$\begin{aligned} V_r(r) &= -\frac{4\alpha_S}{3r} - \frac{2\alpha_S}{3} r m_D^2 + \frac{2\alpha_S}{9} r^2 m_D^3 + \dots \\ V_i(r) &= \frac{2\alpha_S}{9} T\,(r m_D)^2 \left\{-2\gamma_E - \log (r m_D)^2 + \frac{8}{3}\right] + \dots \end{aligned}$$

N. Brambilla, J. Ghiglieri, A. Vairo & P. Petreczky, PRD 78 (2008) 014017.

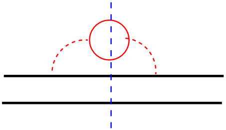
The physics of the imaginary part

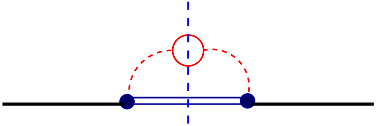
- The imaginary part is associated with decay width, and leads to the widening of the spectral function.
- The physics captured is that of Landau damping.





- For $\frac{1}{r} \gg T$ a different diagram dominates.





- In pNRQCD, the imaginary part comes from singlet to octet transitions.

Potential: perturbative results

- For bottomonia, $\frac{1}{r} > T$. For the scale separation

$$\frac{1}{r} \gg T \gg m_D \gg E_b \sim m_Q v^2$$

the form of the potential is quite different:

$$\begin{aligned} V^s_{\rm re}(\vec{r}; T) &= -\frac{4\alpha_s}{3r} + \frac{4\pi}{9}\alpha_s^2 r T^2 + \ldots \\ V^s_{\rm im}(\vec{r}; T) &= 2\alpha_s^3 T + \frac{2}{9}\alpha_s r^2 T m_D^2 (c - \log\frac{T^2}{m_D^2}) + \ldots \end{aligned}$$

Brambilla, Ghiglieri, Vairo & Petreczky, PR D 78 (2008) 014017.

Gludissociation

- If, on the other hand,

$$\frac{1}{r} \gg T \gg E_b \sim m_Q v^2 \gg m_D$$

  we also need to include the effect of transverse gluons.
- The contribution to the decay width in this case has been calculated recently; the leading term in an expansion in $\frac{m_D}{E_b}$ has been known for long.

  M. Peskin, NP B 156 (1979) 365.
  N. Brambilla, M. Escobedo, J. Ghiglieri & A. Vairo, JHEP 05 (2013) 130.
- In a perturbative estimate, the total thermal width is dominated by gluodissociation at small T. As temperature increases, Landau damping becomes more important.

How to calculate potential nonperturbatively

- The potential can be calculated non-perturbatively by analytical continuation of the Wilson loop calculated on lattice.

$$W(R,t) \;=\; \mathcal{N} \int d\omega \; e^{-\omega\tau} \; \rho(R,\omega)$$

A. Rothkopf T. Hatsuda & S. Sasaki, PRL 108 (2012) 162001

- A direct inversion, using Bayesian techniques, leads to large systematics.

  See, e.g., Y. Burnier, O. Kaczmarek & A. Rothkopf, PRL 114 (2015) 082001.

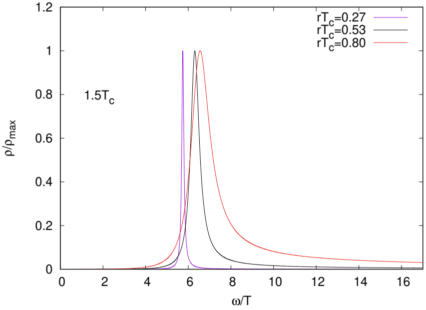
- Note that the spectral function needs a very specific structure for $W(R,t)$ to lead to a potential.
- Potential comes from long distance behavior, i.e., low-frequency part of $\rho(\omega)$.
  Assumption for the low-frequency structure.

low-ω peak

Demanding that the low ω structure leads to a potential in the long t limit, we are led to a structure which is Lorentzian-like near the maximum,

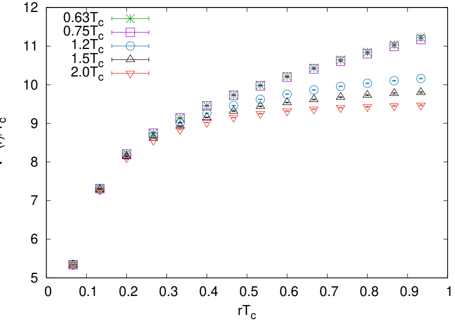
$$\rho(\omega)_{\rm low} \approx \sqrt{\frac{2}{\pi}} \frac{V_i}{(V_r - \omega)^2 + V_i^2}$$

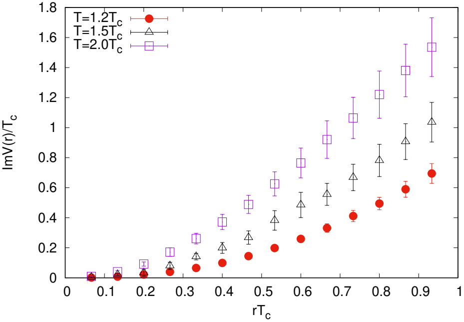
but has a powerlike and an exponential fall-off, respectively, in the high and the low ω sides.



Finite temperature potential for quarkonia

For gluonic plasma, we found that this peak structure gives an excellent description of the Wilson loop data.





D. Bala & S. Datta, PRD 101 (2020) 034507.

Other forms for $\rho(\omega)$)?

- Our form for $\rho(\omega)$ was guided by the requirement that it leads to a thermal potential.
- A recent study for ful QCD took a more agnostic approach: various low-ω peak structures were studied.

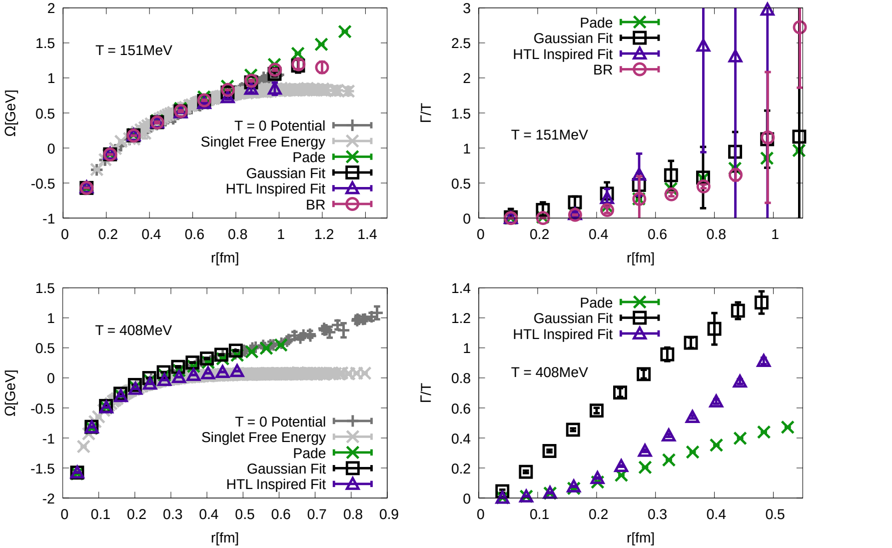
  D. Bala, O. Kaczmarek, R. Larsen, ... (HotQCD), PRD 105(2022) 054513

- A Gaussian peak, with an additional (small) very low-ω peak:

$$W(r,\tau) \sim A_{\mathrm{peak}}\, e^{-\Omega\tau + \frac{1}{2}\Gamma_G^2\tau^2} + A_{\mathrm{cut}}\, e^{-\omega_{\mathrm{cut}}\tau}$$

- A model-independent approach: calculate $\mathcal{F}(W)$ and estimate a Pade approximant $\Rightarrow \rho(\omega)$.
- Interpretations of the peaks are very different: only the first one leads to a thermal potential.

Full QCD: Different forms



D. Bala, O. Kaczmarek, R. Larsen, ... (HotQCD), PRD 105(2022)054513

Thermal potential from lattice: contd.

- We stress that the interpretations of the Ω, Γ are different for the different forms shown in previous slide.
- In particular, only the "HTL inspired fit" results can be interpreted as potential.
- However, the large spread in the position of the low ω peak is disconcerting.
- The full QCD results are from small ($N_\tau = 12$) lattices.
- Work with finer lattices in progress, both for quenched and for full QCD.

HotQCD: G. Parker, Nordic Lattice 2022; J. Weber, INT Workshop, 2022.

Form of $V^s_{\rm re}(\vec{r}; T)$

Let us look at the results from quenched QCD in more detail.
While $V^s_{\rm re}(\vec{r}; T)$ shows the expected screening behavior, its r dependence is very different from perturbation theory.
Adding a screened string term as in 1+1-D Schwinger model:

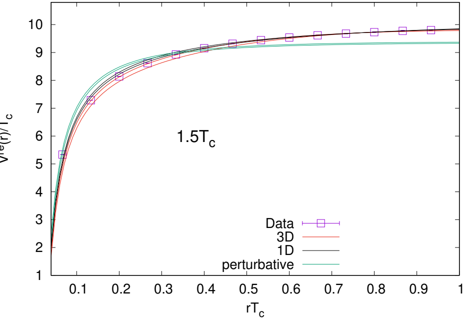
$$V^{\rm re}_{1D}((\vec{r}; T)) = -\frac{4\alpha}{3r} e^{-m_D r} \; + \; \frac{\sigma}{m_D} \left(1 - e^{-m_D r}\right)$$

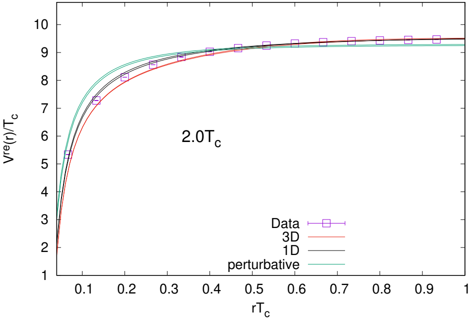
Karsch, Mehr, Satz, Z. Phys. C 37 (1988) 617

Similarly, a 3D screened form of the potential, $V^{\rm re}_{3D}((\vec{r}; T))$ has also been discussed.

Dixit, Mod. Phys. Lett. A 5 (1990) 227; Burnier & Rothkopf, PL B 753(2016) 332.

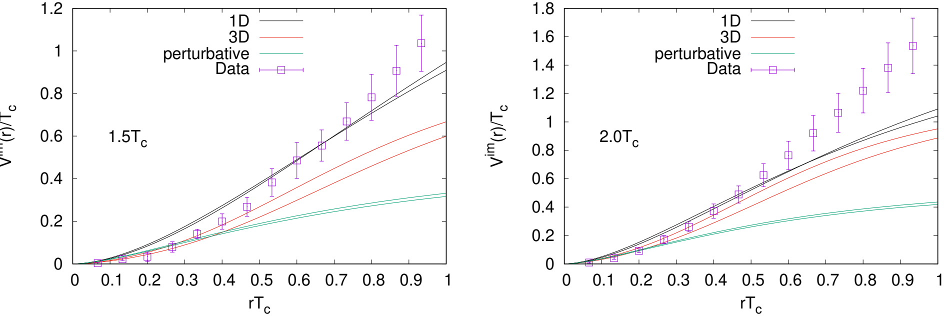
Model for potential





While the 1D screening form gives slightly better χ^2, our data is not accurate enough to distinguish between the 1D and 3D forms.

Form of $V^s_{\rm im}(\vec{r};T)$



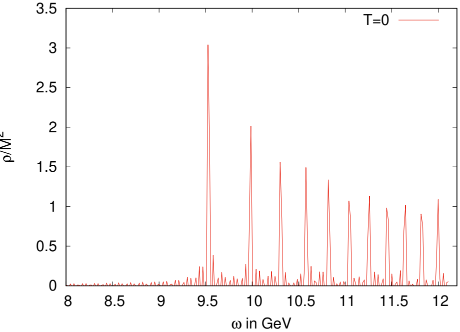
"Nonperturbative" forms of $V^s_{\rm im}(\vec{r};T)$ have been suggested by defining a perturbative "complex permittivity" and using it with the above screening forms.

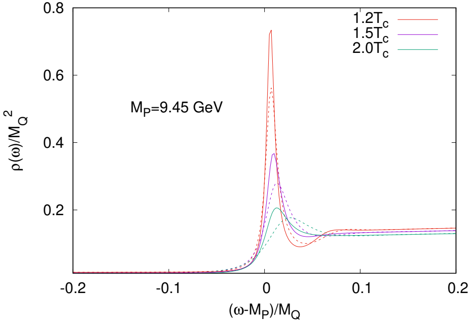
Thakur, Kakade, Patra, PR D 89 (2014) 094020. Burnier & Rothkopf 2016; Guo et al, PR D 100(2019) 036011.

Data steeper than $V^{\rm im}_{1D}((\vec{r};T))$ but slower than quadratic.

Spectral function for $\bar{Q}\gamma_i Q$ current

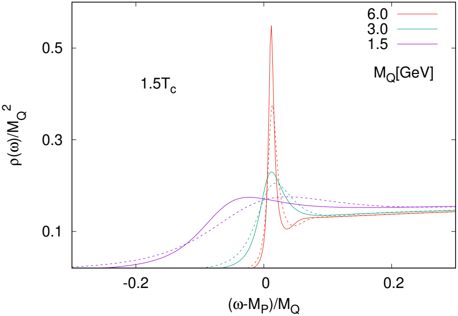
Following Laine, JHEP 05 (2007) 028 we can investigate the fate of the quarkonia peak in the vector current spectral function using the thermal potential.





Calculate $C(t, r)$ from potential with $C(0, r) = \delta^3(r)$
Peak structure in spectral function till 2 T_c, at 1.5 T_c narrow peak.
Shift in peak position small.

Spectral function for $\bar{Q}\gamma_i Q$ current



At 1.5 T_c charm peak very broad. Here 1.5 $T_c \sim$ 420 MeV.

D. Bala & S. Datta, PRD 101 (2020) 034507.

Interpreting thermal potential stochastically

- An useful interpretation of the thermal potential, that opens up a path to its use in an evolving medium, was forwarded by Akamatsu.

  Y. Akamatsu & A. Rothkopf, PRD 85 (2012) 105011; Akamatsu, 2013-2022; see Y. Akamatsu, arXiv:2009.10559 for a review.

- Introduce a noise term,

$$H(\vec{r},t) = -\frac{\nabla_r^2}{M} + V^s_{\rm re}(\vec{r};T) + \Theta(\vec{r},t), \qquad \Theta(\vec{r},t) = \theta(\vec{R}+\vec{r}/2,t) - \theta(\vec{R}-\vec{r}/2,t)$$

$$\langle \theta(\vec{r},t) \rangle = 0, \qquad \langle \theta(\vec{r},t)\,\theta(\vec{s},t') \rangle = D(\vec{r}-\vec{s})\,\delta(t-t')$$

- Noise averaging would give

$$i\frac{\partial}{\partial t}\langle \Psi_{\bar{Q}Q}(\vec{r},t)\rangle = \left[-\frac{\nabla_r^2}{M} + V^s_{\rm re}(\vec{r};T) - i\,(D(0)-D(r))\right] \langle \Psi_{\bar{Q}Q}(\vec{r},t)\rangle$$

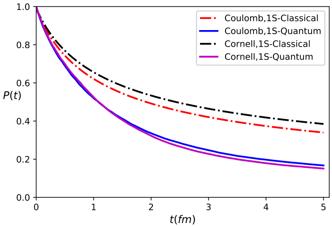
  This paves the way for study of quarkonia in evolving QGP.

- More formally, one can use the formalism of the open quantum systems: the evolution equation for the density matrix of the $\bar{Q}Q$ system,

$$i\frac{d\rho(x,y;t)}{dt} = H_{\rm eff}\,\rho - \rho\, H^\dagger_{\rm eff} + i\sum_n L_n\,\rho\, L_n^\dagger, \qquad H_{\rm eff} = H - \frac{i}{2}\sum_n L_n^\dagger L_n\,.$$

Colored and correlated noise

- In our case, LO form of the noise term is
  $gE^a(\vec{r}/2,t)\; t^a \otimes 1 - gE^a(-\vec{r}/2,t)\; 1 \otimes t^{a\star}$
- Such a term will cause transition between color singlet and octet states, as well as between $l=0$ and $l=1$ states.
- Also, one can move beyond the approximation of Markovian $\delta(t-t')$ correlation.
- Evolution with all these features has been studied.
  R. Sharma & A. Tiwari, PRD 101 (2020) 074004
- Large difference from a classical, "rate equation" evolution.



Open quantum system in pNRQCD

- The color transitions are naturally handled in pNRQCD.

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

- For $\frac{1}{r} \gg T \sim m_D \gg E_b$ one gets

$$H_{\text{eff}} = \begin{pmatrix} h_s + \frac{r^2}{2}(\gamma - i\kappa) & 0 \\ 0 & h_o + \frac{r^2}{2}(\gamma - i\kappa)\, \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

and the $L_i \propto \sqrt{\kappa}$ connect different color and l states.

Brambilla, Escobedo, Soto, Vairo, PRD 96(2017) 034201; PRD 97 (2018) 074005

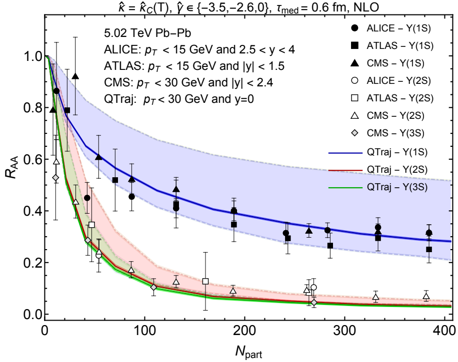
- κ and γ are real and imaginary parts of

$$\frac{g^2}{6N_c} \int_{-\infty}^{\infty} ds\, e^{i(V_o - V_s)t} \langle T\, E^{a,i}(s,\vec{0})\, W^{ab}(s,0)\, E^{b,i}(0,\vec{0})\rangle.$$

- $\mathcal{O}(E_b/T)$ corrections have also been worked out.

A calculation of R_{AA}

A recent, elaborate calculation of evolution of a narrow $\bar{b}\gamma^i b$ current, including proper system evolution and various nonperturbative ingredients:



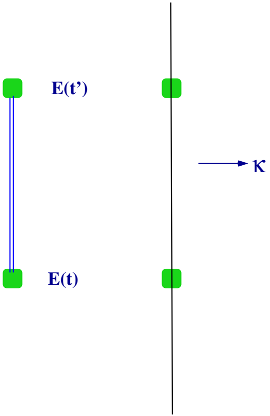
N. Brambilla, M. Escobedo, A. Islam, M. Strickland, A. Tiwari, A. Vairo, P. Griend, arXiv:2205.10289.

the κ term

- For κ, results from a momentum diffusion coefficient calculation have been used.
- The two are not the same.

  T. Binder, K. Mukaida, B. Scheihing-Hitschfeld, X. Yao, JHEP 01 (2022) 137
- Also the $V_o - V_s$ in the exponent have been ignored.
- The effect of this has been estimated in a perturbative calculation of κ.

  R. Sharma & B. Singh, INT Workshop, 2022



$\bar{Q}, Q$ in octet color configuration

- Another quantity that appeared repeatedly is the color octet potential, V_o.
- How to study color octet potential nonperturbatively?
- Color octet $\bar{Q}Q$ current

$$J^a(\vec{x}, \vec{y}; \vec{z}; t) = \bar{Q}(\vec{x}; t) U(\vec{x}, \vec{z}; t) T^a U(\vec{z}, \vec{y}; t) Q(\vec{y}; t)$$

- Wilson loop analogously to singlet case: not gauge invariant.
- No known way to get octet potential from gauge fixed Wilson loop.

O. Philipsen & M. Wagner, PR D89 (2014) 014509

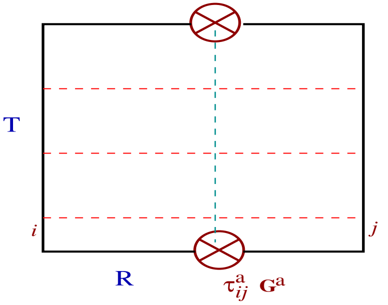
Potential for Q and $\bar{Q}$ in color octet configuration

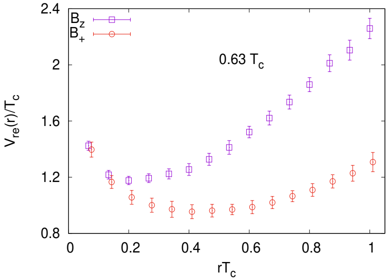
A gauge invariant current:

$$J_G(\vec{x},\vec{y};\vec{z};t) = \bar{Q}(\vec{x};t)U(\vec{x},\vec{z};t)T^a G^a U(\vec{z},\vec{y};t)Q(\vec{y};t)$$

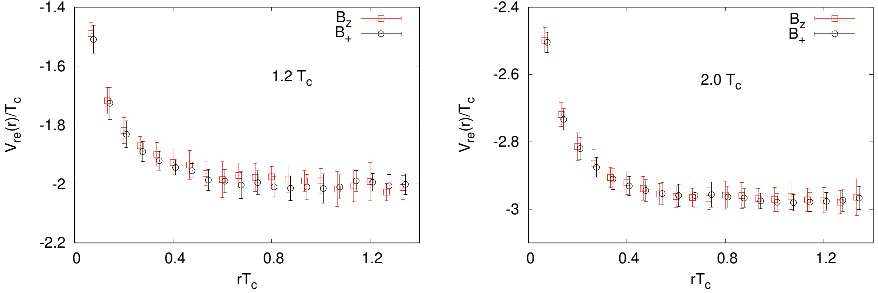
In perturbation theory: ladder sum independent of G^a and in lowest orders, gives octet potential

$$V_o(\vec{r}) = -\frac{1}{2N_c}V_s(\vec{r}) \; + \; \mathcal{O}(\alpha_s^3)$$



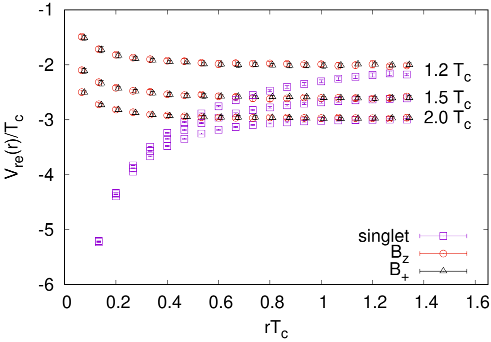


Octet potential above T_c



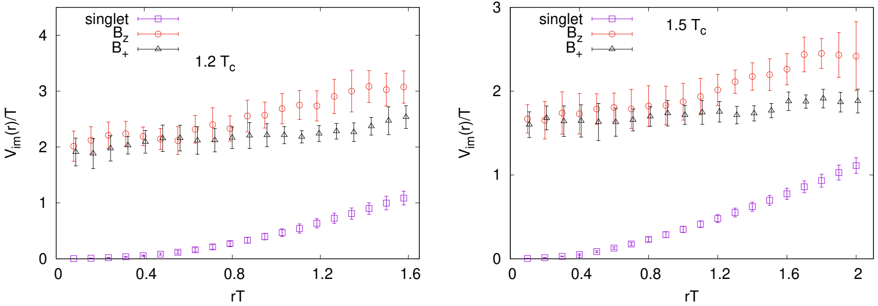
D. Bala & S. Datta, PRD 103 (2021) 014512

$V^o_{\rm re}(\vec{r}; T)$ above T_c



- $V^o_{\rm re}(\vec{r}; T)$ screened and repulsive everywhere.
- $V^o_{\rm re}(\vec{r}; T)$ for Σ_u^- and Π_u agree everywhere.
- Comes close to $V^s_{\rm re}(\vec{r}; T)$ for $rT \gtrsim 2$.

$V^o_{\rm im}(\vec{r}; T)$ above T_c



- $V^o_{\rm im}(\vec{r}; T)$ shows some channel dependence at long distances, though error bar too large.
- $V^o_{\rm im}(\vec{r}; T)$ nonzero at $r = 0$.
- $V^o_{\rm im}(\vec{r}; T) > V^s_{\rm im}(\vec{r}; T)$ in the distance range studied by us.

D. Bala & S. Datta, PRD 103 (2021) 014512

Summary

- Quarkonia remain a topic of active interest in the QGP community: from an experimental probe of deconfinement, it has also grown into a nice theoretical setup where controlled calculation is possible.
- A combination of effective field theory (NRQCD, pNRQCD) and lattice techniques has provided us a route to controlled nonperturbative computations.
- New development in last decade: an open quantum system treatment of quarkonia.
- A complete nonperturbative calculation in this framework seems very much in the cards in the near future.
- Hopefully these developments can be carried forward to quarkonia in other setups (e.g., in presence of magnetic field).

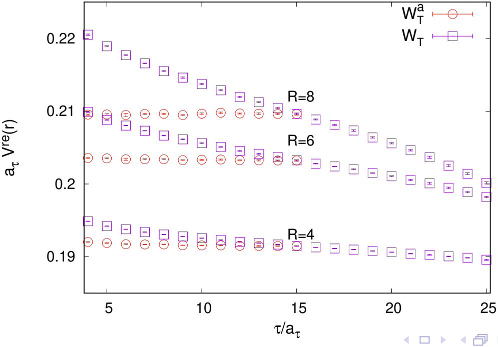
BACKUP SLIDES

Our method to get potential

T=0: $\bar{Q}Q$ potential obtained as $W(\tau, \vec{r}) \xrightarrow[\text{large}\tau]{} e^{-\tau\, V(\vec{r})}$

Write $W_T(\tau, \vec{r}) \;=\; e^{\omega(\tau,\vec{r})}\; W_T\left(\frac{\beta}{2}, \vec{r}\right)$

In analogy with T=0, we expect $\tilde{w}(\tau, \vec{r}) \sim -(\tau - \frac{\beta}{2}) V^s_{\rm re}(\vec{r}; T) + \ldots$
Splitting W_T into parts symmetric(W_s) and antisymmetric (W_a) around $\frac{\beta}{2}$, we find $\log W_a$ to show exactly the behavior we expect.



Our method to get potential

What about $A(\tau, \vec{r}) = \log W_s(\tau, \vec{r})$? Its behavior can be understood if we write a spectral decomposition:

$$A(\tau, \vec{r}) = \int d\omega \; \sigma(\omega; T) \frac{1}{2} \left(e^{-\omega\tau} + e^{-\omega(\beta-\tau)} \right) + \tau - \text{independent terms}$$

Going to real time $\tau \rightarrow it$,

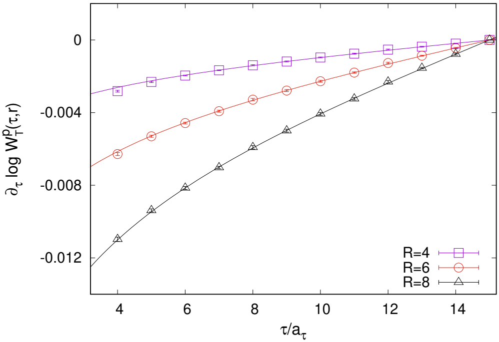
$$i\partial_t A(it) = \int d\omega \; \sigma(\omega, T) \frac{\omega}{2} \left(e^{-i\omega t} - e^{-\omega\beta} e^{i\omega t} \right)$$

in limit of large t, $A(it)$ leads to an imaginary potential.

How to extract $V^s_{\rm im}(\vec{r}; T)$ from W_a? Since $e^{-i\omega t} - e^{i\omega t - \omega\beta} \xrightarrow[\text{large} t]{} -2\pi i \omega \delta(\omega)$, in order to obtain a potential from $\lim_{\text{large} t} i\partial_t A(it)$ we need $\sigma(\omega; T) \underset{\omega \rightarrow 0}{\sim} \frac{1}{\omega^2}$

Our method to get potential

Just the leading $\frac{1}{\omega^2}$ behavior gives a good fit to the data near center.



Note that near $\omega \to 0$, a factor of ω comes from the Bose distribution (for low energy gluons). So we model $\sigma(\omega; T)$ as:

$$\sigma(\omega; T) \ = \ (1 + n_B(\omega)) \ \left(\frac{\beta \, V_i}{\pi \, \omega} + c_2 \omega + c_3 \omega^3 + \dots \right)$$

Low-ω peak

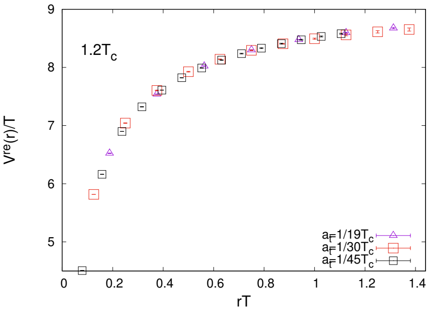
This indicates, near $\tau \sim 1/2T$,

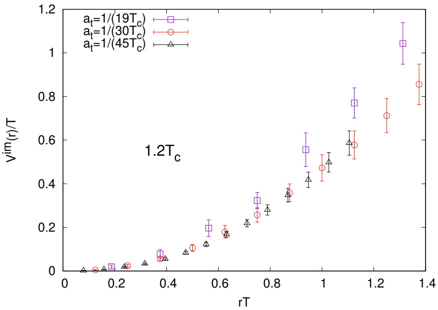
$$W_T(\tau, \vec{r}) \;=\; e^{-V_r\left(\tau - \frac{\beta}{2}\right) \,-\frac{\beta}{\pi} V_i \,\log\,\sin\left(\frac{\pi\tau}{\beta}\right) - \cdots} \; W_T\left(\frac{\beta}{2}, \vec{r}\right)$$

The low-ω peak can be inferred from this. It has the behavior

$$\begin{aligned} \rho(\omega)_{\rm low} &\approx \sqrt{\frac{2}{\pi}}\, \frac{V_i}{(V_r - \omega)^2 \;+\; V_i^2} && |V_r - \omega|,\; V_i \ll T \\ &\sim (\omega - V_r)^{-\left(1 - \frac{\beta V_i}{\pi}\right)} && \omega - V_r \gg T \\ &\sim e^{-\beta(V_r - \omega)}\; (V_r - \omega)^{-\left(1 - \frac{\beta V_i}{\pi}\right)} && \omega - V_r \ll -T \end{aligned} \tag{1}$$

Lattice spacing dependence





Cutoff effect seen to be small already for lattices with $a = \frac{1}{30\,T_c}$. So our finest lattices with $a = \frac{1}{45\,T_c}$ can be taken to give the continuum results.