DAE-BRNS SYMPOSIUM' CETHENP-2022

Heavy Quark Diffusion coefficients in the light of Gribov-Zwanziger(GZ) action

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November 16, 2022

Ultrarelativistic heavy ion collision

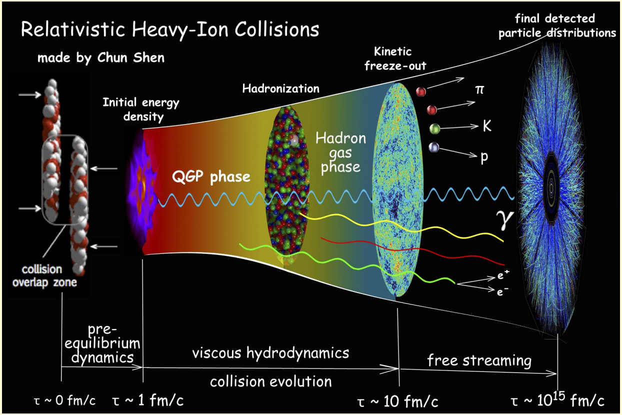


Figure (1) –Time evolution of an ultra-relativistic heavy-ion collision(Cartoon)

http://u.osu.edu/vishnu/2014/08/06/sketch-of-relativistic-heavy-ion-collisions/

Heavy Quark

1. "**Heavy Quark**": $m_Q \gg \Lambda_{QCD}$, $m_Q \gg T_{medium}$
2. thermal production is energetically forbidden in the QGP medium
3. produced in the primordial phase of the collision
4. $\tau_{therm}^{(HQ)} \sim \tau_{therm}^{(LQ)} \cdot (m_Q/T)$
   $\Rightarrow$ greater thermalization time as compared to their lighter counterpart (comparable to or even larger than the lifetime of the fireball)
5. Final spectral distribution disclose the interaction history in the medium(QGP) .

Diffusion coefficient

1. HQ dynamics $\xrightarrow{\text{QGP}}$ Brownian motion .
2. Momentum of HQ in QGP medium, evolves following Langevin equation (Phys. Rev. C 71 (2005) 064904).

$$\frac{dp_i}{dt} = \xi_i(t) - \eta_D p_i \ , \tag{1}$$

$$\langle \xi_i(t)\xi_j(t')\rangle \propto \delta_{ij}\delta(t-t') = \kappa \ \delta_{ij}\delta(t-t') \ . \tag{2}$$

3. $\kappa \Rightarrow$ momentum diffusion coefficient/ mean squared momentum transfer per unit time
4. $\mathcal{D}_s$, spatial diffusion coefficient, can be found as mean squared position of particle(starting at $x=0, t=0$)at any later time t

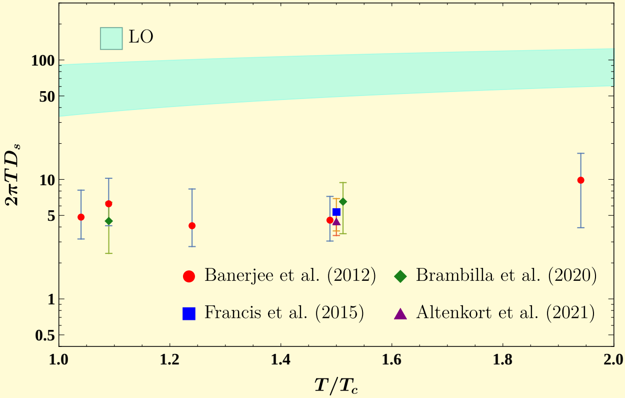
$$\langle x_i(t)x_j(t)\rangle = 2\delta_{ij} \ \mathcal{D}_s \ t \implies \langle x^2(t)\rangle = 6\mathcal{D}_s t \tag{3}$$

5. At vanishing momentum transfer, the spatial diffusion coefficient, $\mathcal{D}_s = (2T^2/\kappa)$

As of now

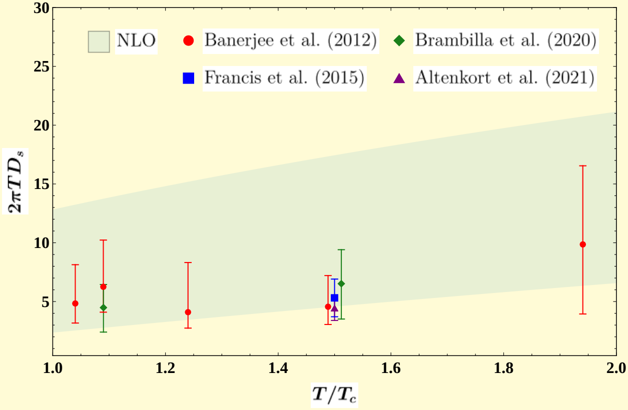
$$3\kappa = \frac{C_F g^4}{18\pi}\left[\left(N_c + \frac{N_f}{2}\right)\left[\ln\frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right] + \frac{N_f}{2}\ln 2\right]$$

PRC 71, 064904 (2005), Moore Teaney



$$3\kappa = \frac{C_F g^4}{18\pi}\left[\underbrace{\left(N_c + \frac{N_f}{2}\right)\left[\ln\frac{2T}{m_D} + \frac{1}{2} - \gamma_E + \frac{\zeta'(2)}{\zeta(2)}\right] + \frac{N_f}{2}\ln 2}_{\textbf{LO-HTLpt}((m_D/T)\ll 1)} + 2.3302\left(N_c\frac{m_D}{T}\right)\right]$$

PRL 100, 052301 (2008), Caron-Huot Moore



Going-over perturbative approach

1. At very high T, QCD thermodynamics is well established within the framework of resummed perturbation theory and lattice calculation.

2. Near $T \sim T_c$, pQCD calculations are stabbed by inconsistencies on being compared to lattice data. H.B. Meyer, Phys. Rev. D 76,101701(2007).

3. Relativistic heavy ion collision experiments signal that QGP behaves like a strongly interacting fluid. Gyulassy, McLerran: New form of QCD matter discovered at RHIC.

4. Strongly interacting system demands non-perturbative treatments(Asymptotic Freedom)

5. Free gas of Gribov quasiparticles captures the **non-perturbative** features of the lattice equation of state. PRL94, 182301 (2005)

6. Later in other references, **Gribov-Zwanziger(GZ)**(Phys. B 139 (1978) 1, Nucl. Phys. B 323 (1989) 513–544) action has been proven to be an efficient tool to encapsulate non-perturbative effects(which is effective near $T \sim T_c$).

Ambiguity in quantization of YM theory

1. One traditional way to quantize Yang-Mills theory is the Faddeev-Popov(FP) quantization, where the partition function is

$$Z_{FP} = V \int [\mathcal{D}\mathcal{A}] |\det \partial_\mu D_\mu^{ab}| \delta(\partial \mathcal{A}) e^{-S_{YM}} \quad (4)$$

2. Here, $(\partial_\mu D_\mu^{ab})$ is the FP operator, which could be negative as well as 0.

3. Ghost two point function is related to the inverse of the FP operator.

$$\langle k | (-\partial_\mu D_\mu^{ab})^{-1} | k \rangle = \frac{1}{k^2} \left[\frac{1}{1 - \sigma(k^2, \mathcal{A})} \right] \quad (5)$$

4. Gribov pointed out that multiple solutions of gauge fixing condition in non abelian gauge transformation cause problem in FP quantization → Gribov ambiguity

5. Gribov idea→ over counting(multiple solutions) of gauge fields need to be sorted.

Gribov proposal

1. Gribov suggested to restrict the path integral in a way that no gauge copies is considered. First Gribov region
2. To make this happen, he considered the ghost propagator, which is the vacuum expectation value of the inverse of the FP operator.
3. In usual perturbation theory, the propagator does have a pole, which implies the over counting of field configuration(or crossing the first Gribov region).
4. If this operator is always positive definite, the ghost propagator cannot have poles → No-pole condition, $\sigma(0, \mathcal{A}) \leq 1$
5. Restricting the integration to the first Gribov region is realized by inserting a function $V(\Omega)$ into the partition function, where

$$V(\Omega) = \theta[1 - \sigma(0)] = \int_{-i\infty+\epsilon}^{+i\infty+\epsilon} \frac{d\beta}{2\pi i\beta} e^{\beta[1-\sigma(0)]}$$

represent No-pole condition.

continue...

1. Mathematically:

$$\int \mathrm{DA} \int \mathrm{Dc} \int \mathrm{D\bar{c}} \longrightarrow \int_\Omega \mathrm{DA} \int \mathrm{Dc} \int \mathrm{D\bar{c}}\ \mathrm{V}(\Omega)\delta(\partial \cdot \mathrm{A})\ ,$$

$$\text{where } \Omega = \{\mathrm{A}^{\mathrm{a}}_\mu, \partial_\mu \mathrm{A}^{\mathrm{a}}_\mu = 0; -\partial_\mu \mathrm{D}^{\mathrm{ab}}_\mu > 0\}$$

2. Imposing this condition and solving one-loop gap equation, we get

$$\gamma_G^2 \approx \frac{\mu^2}{e^{1/g^2}} \tag{6}$$

3. Gribov parameter as a function of temperature, analytically, in the asymptotic limit, was later given by Zwanziger, as Phys. Rev. D 76, 125014

$$\gamma_G(T) = \begin{cases} \mu^2 \mathrm{Exp}\left(\frac{5}{12} - \frac{32\pi^2}{3\mathrm{N_c}g^2}\right) & T \to 0 \\ \frac{d-1}{d} \cdot \frac{\mathrm{N_c}}{4\sqrt{2}\pi} g^2(T) T & T \to \infty \end{cases} \tag{7}$$

where d is the space-time dimension.

As a consequence

1.) Faddeev-Popov quantization

$$D^{ab}_{\mu\nu}(P) = -i\delta^{ab}\left(\eta_{\mu\nu} - (1-\xi)\frac{P_\mu P_\nu}{P^2}\right)\frac{1}{P^2}$$

2.) Gribov quantization

$$D^{ab}_{\mu\nu}(P) = \delta^{ab}\left(\delta_{\mu\nu} - (1-\xi)\frac{P_\mu P_\nu}{P^2}\right)\frac{1}{P^2 + \left(\frac{\gamma_G^4}{P^2}\right)}$$

A new scale γ_G that leads to an IR-improved dispersion relation for gluons (Coulomb gauge)

[V. Gribov, NPB 139, 1 (1978); D. Zwanziger, NPB 323, 513 (1989);]

$$E(k) = k \quad \rightarrow \quad E(k) = \sqrt{k^2 + \frac{\gamma_G^4}{k^2}}$$

Dispersion relation

Solving for Gribov parameter, γ_G ($\mathrm{N_f} = 0$)

The one loop gap equation is:

$$\sum\!\!\!\!\!\!\!\int_P \frac{1}{P^4 + \gamma_G^4} = \frac{d}{(d-1)\mathrm{N_c}\, g^2}\ , \tag{8}$$

where, in $\overline{\mathrm{MS}}$ renormalization scheme,

$$\sum\!\!\!\!\!\!\!\int_P = \left(\frac{e^{\gamma_E}\Lambda^2}{4\pi}\right)^{\epsilon} \cdot T \cdot \sum_{p_4 = 2\pi n T} \cdot \int \frac{d^{d-1}p}{(2\pi)^{d-1}}\ . \tag{9}$$

Incorporating 9 in 8 and subtracting UV divergence part, we egt

$$1 = \frac{3\mathrm{N_c}\, g^2}{64\pi^2}\left[\frac{5}{6} - \ln\left(\frac{\gamma_G^2}{\Lambda^2}\right) + \frac{4}{i\gamma_G^2}\int_0^\infty dp\, p^2 \left(\frac{n_B(\omega_-)}{\omega_-} - \frac{n_B(\omega_+)}{\omega_+}\right)\right]. \tag{10}$$

Λ=1.69 GeV PhysRevD.88.076008, and

$$n_B(\omega_\pm) = \frac{1}{e^{\beta\omega_\pm} - 1}\ , \text{where } \omega_\pm = \sqrt{|\vec{p}|^2 \pm i\gamma_G^2}\ ,\quad (\beta = 1/T)\ . \tag{11}$$

Fixing γ_G perturbatively with $(\mathrm{N_f} = 3)$

Motive: Include quark contribution .

Analytic form of γ_G, in the limit $T \to \infty$, Phys. Rev. D 76, 125014

$$\gamma_G(T) = \frac{d-1}{d} \frac{\mathrm{N_c}}{4\sqrt{2}\pi} g^2(T) T \ , \tag{12}$$

d is dimension of space time(here, 4), $g(T)$ is the running coupling, in perturbative limit(one loop):

$$\alpha(T) = \frac{g^2(T)}{4\pi} = \frac{6\pi}{(11\mathrm{N_c} - 2\mathrm{N_f}) \ln\left(\frac{\mu}{\lambda_{\overline{\mathrm{MS}}}}\right)} \ , \tag{13}$$

where

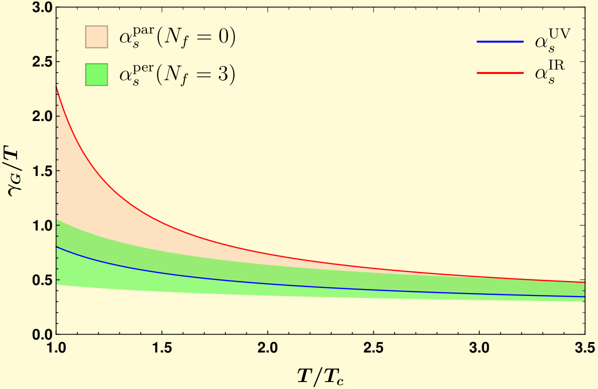
- $\pi T \leq \mu \leq 4\pi T$, and
- $\lambda_{\overline{\mathrm{MS}}} = 0.176$ GeV, (lattice result) .

γ_G/T vs (T/T_c) plot

Phys. Rev. D 88, 076008

$$\alpha_s(T/T_c) \equiv \frac{g^2(T/T_c)}{4\pi} = \frac{6\pi}{11\ \mathrm{N_c}\ \mathrm{Log}[c(T/T_c)]}\ , \tag{14}$$

$c = 1.43$ for IR and $c = 2.97$ for UV $\rightarrow \alpha^{IR}_{T=T_c} = 1.59$ and $\alpha^{UV}_{T=T_c} = 0.524$.



Calculation result

NOTE : Purely gluonic medium as a background medium for Heavy quark has been considered .
Secondly, calculations are done in non-relativistic limit such that for heavy quark, $P^\mu = (M, \vec{0})$.
In the end, quark contribution has been added perturbatively

Mean squared momentum transfer per unit time, which is 3κ, can be computed from the expression Phys. Rev. C 71 (2005) 064904

$$3\kappa = \frac{1}{2M}\int \frac{d^3\boldsymbol{k}d^3\boldsymbol{k}'d^3\boldsymbol{p}'}{(2\pi)^9(2k^0)(2k'^0)(2M)} \times (\boldsymbol{k}-\boldsymbol{k}')^2(2\pi)^4\delta^4(P'+K'-P-K) \times \left\{|\mathcal{M}|^2_{\text{quark}}\, n_F(k)[1-n_F(k')] + |\mathcal{M}|^2_{\text{gluon}}\, n_B(k)[1+n_B(k')]\right\}, \tag{15}$$

In the above equation, $\boldsymbol{k}' = \boldsymbol{k} - \boldsymbol{q}$.

$K \equiv (k^0, \boldsymbol{k})$ and $K' \equiv (k'^0, \boldsymbol{k}')$ are incoming and outgoing four momenta of light quarks and gluons, respectively.

t-channel heavy quark scattering

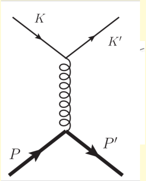


Figure (2) −**(qH → qH)**

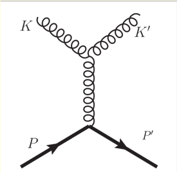


Figure (3) −**(gH → gH)**

$$|\mathcal{M}|^2_{\text{quark}} = 16\text{N}_\text{f} C_F g^4 M^2 k^2 \left(1 + \cos\theta_{kk'}\right) \frac{q^4}{(q^4 + \gamma_G^4)^2}$$

$$|\mathcal{M}|^2_{\text{gluon}} = 16\text{N}_\text{c} C_F g^4 M^2 k^2 \left(1 + \cos^2\theta_{kk'}\right) \frac{q^4}{(q^4 + \gamma_G^4)^2}$$

$$\cos\theta_{kk'} = 1 - \frac{(QM)^2}{2(K\cdot P)^2} = 1 - \frac{q^2}{2k^2} \;\; viz\; ((K\cdot P) \sim Mk, \text{Q} - \text{purely spatial})$$

3κ expression using GZ prescription

$$\frac{C_F g^4}{4\pi^3}\int_0^\infty k^2 dk \int_0^{2k} q dq \frac{q^6}{(q^4+\gamma_G^4)^2}$$
$$\times \mathrm{N_f}\, \mathrm{n_F}(\mathrm{k})[1-\mathrm{n_F}(\mathrm{k})]\left(2-\frac{\mathrm{q}^2}{2\mathrm{k}^2}\right)+$$
$$\mathrm{N_c}\, \mathrm{n_B}(\mathrm{k})[1+\mathrm{n_B}(\mathrm{k})]\left(2-\frac{\mathrm{q}^2}{\mathrm{k}^2}+\frac{\mathrm{q}^4}{4\mathrm{k}^4}\right)$$

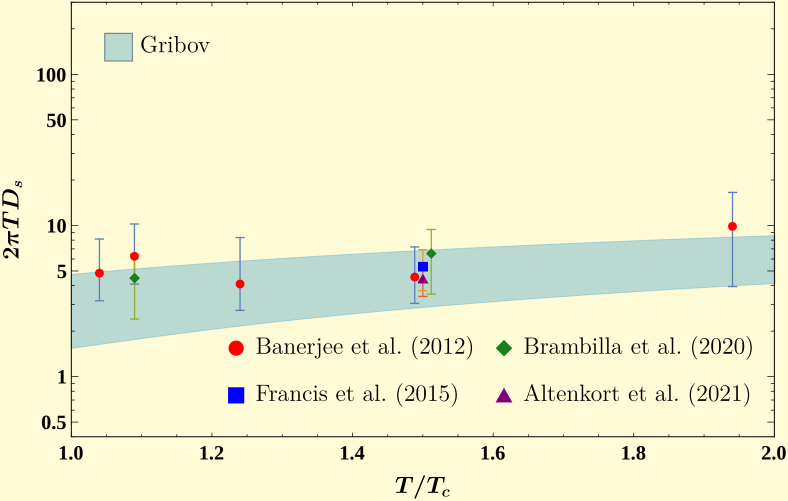
LO-HTLpt 3κ expression

$$\frac{C_F g^4}{4\pi^3}\int_0^\infty k^2 dk \int_0^{2k} q dq \frac{q^2}{(q^2+m_D^2)^2}$$
$$\times \mathrm{N_f}\, \mathrm{n_F}(\mathrm{k})[1-\mathrm{n_F}(\mathrm{k})]\left(2-\frac{\mathrm{q}^2}{2\mathrm{k}^2}\right)+$$
$$\mathrm{N_c}\, \mathrm{n_B}(\mathrm{k})[1+\mathrm{n_B}(\mathrm{k})]\left(2-\frac{\mathrm{q}^2}{\mathrm{k}^2}+\frac{\mathrm{q}^4}{4\mathrm{k}^4}\right)$$

- k',k are outgoing/incoming 3 momenta of thermal partiles.
- $q = k - k'$.
- $C_F = 4/3$; Color Casimir constant.
- n_F and n_B are the Fermi and Bose distribution, respectively.
- $m_D^2 = (1/3)g^2T^2(\mathrm{N_c} + \frac{\mathrm{N_f}}{2})$.
- $\mathrm{N_c} = \mathrm{N_f} = 3$.

Results

a.


Gribov
100
50
10
5
1
0.5
$2\pi T D_s$
1.0
1.2
1.4
1.6
1.8
2.0
T/T_c
Banerjee et al. (2012)
Brambilla et al. (2020)
Francis et al. (2015)
Altenkort et al. (2021)

b. $N_f = 0$

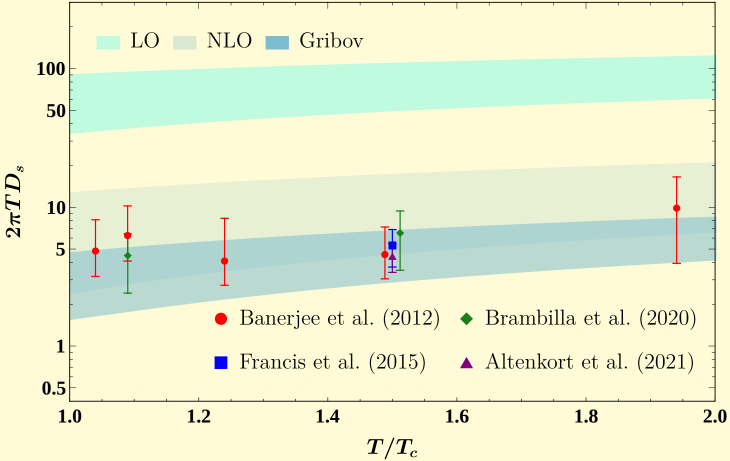


Figure (4) –Plot of $(2\pi T)D_S$ vs (T/T_c). For LO and NLO 2 loop coupling has been taken with $T_c/\Lambda_{\overline{MS}} = 1.15$.

c.

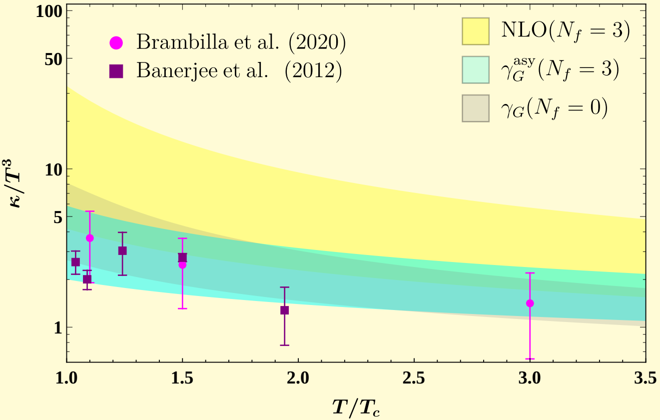


Figure (5) –Plot κ/T^3 vs (T/T_c). For NLO, one loop coupling has been taken with $\Lambda_{\overline{\rm MS}} = 0.176$ GeV.

Conclusions

1. We have discussed the motivation to include Gribov quantization to the estimation of heavy-quark diffusion coefficients.
2. We have discussed existing LO and NLO HTLpt results for heavy quark diffusion coefficient.
3. We have also discussed our recent results for heavy quark diffusion rate in Gribov plasma.
4. Result obtained is in good agreement with the lattice estimate.
5. Significant improvement over the LO estimation of the diffusion coefficient encourage us to investigate the other relevant transport properties of the medium following GZ prescription.

I rest my presentation here

!!! Thankyou All !!!

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