Diffusion of HQ in magnetic field-perturbative vs non-perturbative

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Introduction

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Background

- Two heavy nuclei colliding at a relativistic energy create a system of deconfined quarks and gluons called Quark Gluon Plasma at finite temperature and finite baryon density. System is governed by Quantum Chromo Dynamics.
- In non-central HICs, due to the electric charge imbalance, a very strong magnetic field is created, $eB \sim m_\pi^2$ at RHIC and $eB \sim m_\pi^2$ at LHC energies.
- Though fast decaying, it is possible for the field to be present with a significant magnitude after the formation of the thermalised QCD medium.
- Electrical conductivity, σ of the medium can induce a current opposing the rate of decay of the field as per Lenz's law. relaxation time (time over which the field remains reasonably strong), $\tau = L^2\sigma/4$. For $L = 10fm$ and temperature of 200 MeV, $\tau \sim 1fm$.

The premise:magnetic field

- Important to investigate the salient features of the QCD medium in presence of a background magnetic field.
- HQs are good probe, due to higher mass w.r.t the characteristic temperature and w.r.t the d.o.f of the medium.
- Existence of more than one scales renders it necessary to propose a hierarchy. $M^2 \gg eB \gg T^2$.
- Consequently, HQ motion is not directly affected by the field, light quarks are quantised in different Landau Levels and gluon spectrum are affected by the quark loop.

Non-perturbative aspects

- pQCD calculations of the HQ transport coefficients are successful in explaining experimental realisation through R_{AA}, v_2 etc. at higher p_T of HQ and higher T of the medium.
- Low to intermediate p_T region as well as the low temperature region are regions of non-perturbative physics.
- Various ways to incorporate npQCD within the framework of pQCD in HQ transport in agreement with the inputs of the finite temperature lattice QCD results as the baseline.
- $Q\bar{Q}$ potential can be used as the effective gluon propagator to calculate the transport coefficients.
- Short range Yukawa potential and the long range string potential are attributed to the pQCD and npQCD respectively.

Formalism

HQ potentials

- HQ potential inside a thermal medium

$$V(r) = V_C(r) + V_S(r) = -\frac{4}{3}\frac{\alpha_s e^{-m_D r}}{r} - \frac{\sigma e^{-m_S r}}{m_S} \quad (1)$$

  V_C is the Coulombic part and V_S is the string part.

- More extensively, HQ potential in momentum representation from the respective differential equations of $V_C(r)$ and $V_S(r)$:

$$\begin{aligned} -\nabla^2 V_C(r) + m_D^2 V_C(r) &= 4\pi\alpha_s\delta(r) - i\alpha_s T m_{Dg}^2 g(m_D r) \\ -\frac{1}{r^2}\frac{d^2 V_S(r)}{dr^2} + m_s^4 V_S(r) &= 4\pi\sigma\delta(r) - i\sigma T m_{Dg}^2 g(m_D r) \end{aligned}$$

- Fourier transform of the differential equations gives HQ potential in momentum representation

$$V_C(q) = 4\pi\alpha_s\left[\frac{1}{(q^2+m_D^2)} - i\pi T m_{Dg}^2 \frac{1}{q(q^2+m_D^2)^2}\right]$$
$$V_S(q) = 4\pi\sigma\left[\frac{1}{(q^2+m_s^2)^2} - \frac{4i\pi^2\sigma T m_{Dg}^2}{q(q^2+m_D^2)(q^2+m_s^2)^3}\right]$$

- $m_{Dg}^2 = \frac{4}{3}\alpha_s T^2 N_c$ is the Debye mass from the gluon loop contributon and m_D is Debye mass from both the gluon and quark loop of the gluon self energy and in presence of the magnetic field, is given by $\mathrm{m}_D^2 = m_{Dg}^2 + \sum_f \frac{q_f B\alpha_s}{2\pi}$
- HQ potentials are used as the effective gluon propagator. V_C is responsible for the perturbative part whereas V_S is attributed to the non-perturbative regime accommodated inside the framework of pQCD.

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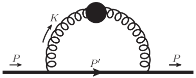
Elastic scattering of HQ with the partons

- Interaction rate of HQ is connected to the imaginary part of its retarded self energy in the medium



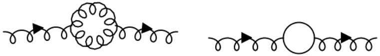
$$\Gamma(E) = -\frac{1}{2E}\,(1 - f(E))\,Tr[(\not{P} + M)\Im(\Sigma(P))], \tag{2}$$

in which Σ is the self energy of HQ with potential as

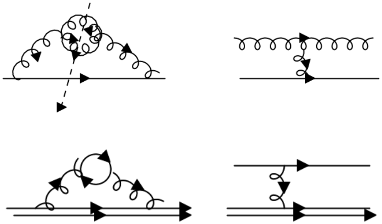
$$\Sigma(P) = ig^2 \int \frac{d^4K}{(2\pi)^4} D^{\mu\nu}\gamma_\mu S(P')\gamma_\nu \tag{3}$$

- The gluon with a black solid blob represents the effective medium gluon propagator in presence of a strong magnetic field with light quarks in the LLLs.

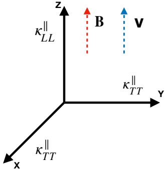
- Two scattering processes are considered: 1) HQ with gluons, i.e. QCD Compton scattering and 2) HQ with light quarks, i.e. Coulomb scattering.
- So, the following gluon self energy is calculated:



- One gets the two scattering processes by the cutting the HQ self energy diagram with gluon propagator calculated from the above 2 diagrams.



Diffusion coefficients: $\vec{v} \parallel \vec{B}$

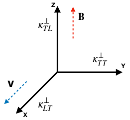


- The broadening of the variance of the HQ momentum distribution are described by $\frac{1}{2}\frac{d}{dt}\langle(\Delta p_T)^2\rangle = \kappa_{TT}^{\parallel}(p)$ and $\frac{d}{dt}\langle(\Delta p_L)^2\rangle = \kappa_{LL}^{\parallel}(p)$.
- These two components can be calculated from the scattering rate

$$\kappa_{TT}^{\parallel}(v) = \frac{1}{2}\int \frac{d^3\vec{q}}{(2\pi)^3} q_T^2 \frac{d\Gamma(E)}{d^3\vec{q}}$$

$$\kappa_{LL}^{\parallel}(v) = \int \frac{d^3\vec{q}}{(2\pi)^3} q_z^2 \frac{d\Gamma(E)}{d^3\vec{q}}$$

$\vec{v} \perp \vec{B}$



- The Langevin Equation $\frac{d}{dt}\langle (p_x^2)^2\rangle = \kappa^{\perp}_{LT}(p)$, $\frac{d}{dt}\langle (p_y^2)^2\rangle = \kappa^{\perp}_{TT}(p)$ and $\frac{d}{dt}\langle (p_z^2)^2\rangle = \kappa^{\perp}_{TL}(p)$.
-

$$\kappa^{\perp}_{LT}(v) = \int \frac{d^3\vec{q}}{(2\pi)^3} q_x^2 \frac{d\Gamma(E)}{d^3\vec{q}}$$
$$\kappa^{\perp}_{TL}(v) = \int \frac{d^3\vec{q}}{(2\pi)^3} q_z^2 \frac{d\Gamma(E)}{d^3\vec{q}}$$
$$\kappa^{\perp}_{LT}(v) = \int \frac{d^3\vec{q}}{(2\pi)^3} q_x^2 \frac{d\Gamma(E)}{d^3\vec{q}}$$

Yukawa scattering rate

- Scattering rate for Yukawa or pQCD

$$\Sigma_Y(P) = i \int \frac{d^4Q}{(2\pi)^4} \gamma_\mu \frac{1}{\not{P}' - M} \gamma^\mu V_Y(\vec{q})$$

- Using imaginary time formalism,

$$\Gamma_Y(E) = \frac{2\pi M^2}{E^2} \int \frac{d^3q}{(2\pi)^3} \rho_Y(q) \int d\omega [1 + n_B(\omega)] \delta(\omega - \vec{v} \cdot \vec{q})$$

- Yukawa spectral function

$$\rho_Y(q) = \frac{T m_{Dg}^2}{q(4\pi\alpha_s)} |V_Y(q)|^2$$

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String scattering rate

- Scattering rate due to the string part of potential

$$\Sigma_S(P) = i \int \frac{d^4Q}{(2\pi)^4} \frac{(\not{P}' + M)}{P'^2 - M^2} V_S(q)$$

- Using imaginary time formalism

$$\Gamma_S(E) = \frac{2\pi M^2}{E^2} \int \frac{d^3q}{(2\pi)^3} \int d\omega \rho_S(q)[1 + n_B(\omega)]\delta(\omega - \vec{v} \cdot \vec{q})$$

- where string spectral function

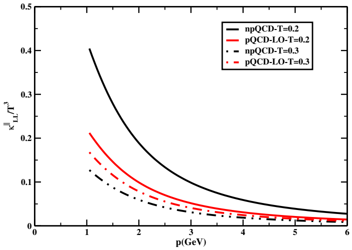
$$\rho_S(q) = \frac{T m_{Dg}^2 (q^2 + m_s^2)}{q(q^2 + m_D^2)} |V_S(q)|^2$$

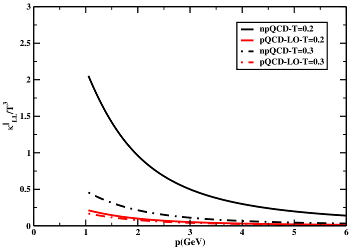
Results

Diffusion coefficients: $\vec{v} \parallel \vec{B}$-momentum dependence

$$\alpha_s(T) = \frac{6\pi}{(33-2N_f)\log(2\pi T/\Lambda)}$$

$$\sigma(T) = \sigma_0\sqrt{1-\frac{\pi T^2}{3\sigma_0}}$$





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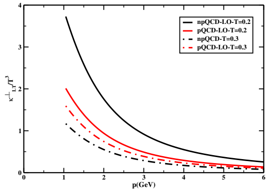
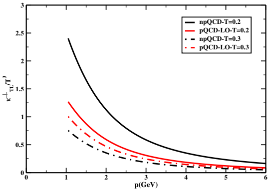
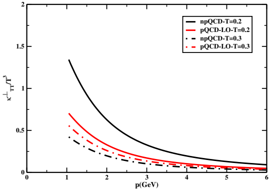
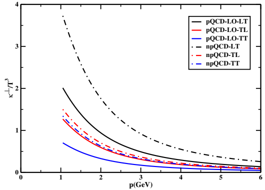
Diffusion coefficients: $\vec{v} \perp \vec{B}$: Momentum dependence

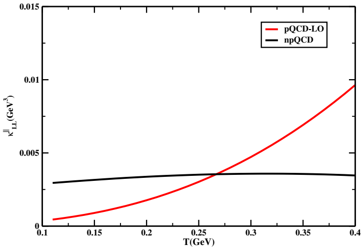


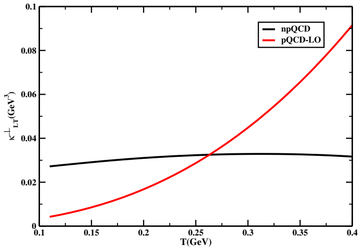






Diffusion coefficients: T dependence





Epilogue

Summary

- Methodology to calculate various diffusion coefficients of HQ moving with a velocity inside a thermal and magnetic medium in the npQCD domain within the scope of pQCD treatment.
- HQ potential is used as an effective gluon propagator. Yukawa potential is responsible for the pQCD and string part gives rise to the npQCD effect.
- Two features of finite T QCD, 1) confining T dependent string tension and 2) asymptotic freedom attributed to the T dependent running coupling.
- npQCD dominates over pQCD at low momenta of HQ and low T of medium. As T and p increase pQCD starts to increase and crosses npQCD at a certain T and p.
- Important in explaining low p_T experimental observations in R_{AA}, v_2 etc.