Spatial diffusion of heavy quarks in background magnetic field

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Spatial Diffusion of heavy quarks at $B = 0$

- Heavy quarks (charm & bottom quarks) are one of the fine probes of Quark Gluon Plasma (QGP).
- Mass of heavy quarks is significantly higher than the QCD energy scale ($\Lambda_{\text{QCD}} = 200$ MeV) and the temperature at which QGP is created.
- Do not thermalize quickly and witness the entire evolution of the fireball.

Einstein's relation for diffusion in condensed matter physics (Romatschke [4]):

$$\boxed{D = \frac{\sigma}{\chi}}$$

where σ is the electrical conductivity & χ is the susceptibility.
For a relativistic fluid (QGP) (Mikkio Laine [6]) :

$$\boxed{D = \frac{1}{3\chi} \lim_{q_0 \to 0^+} \frac{\rho^{ii}(q_0, \vec{0})}{q_0} = \frac{\sigma}{\chi}}$$

where

$$\rho^{ii}(q_0, \vec{q}) = \text{Im}\ i \int d^4x\ e^{iq\cdot x} \langle J^i(x) J^i(0) \rangle_\beta \quad .$$

One loop Kubo formula expression of σ is the same as Relaxation Time Approximation expression in the absence of magnetic field. Motivation for studying heavy quark diffusion using this formula.

Relaxation times for D^0 mesons and comparing results at zero magnetic field

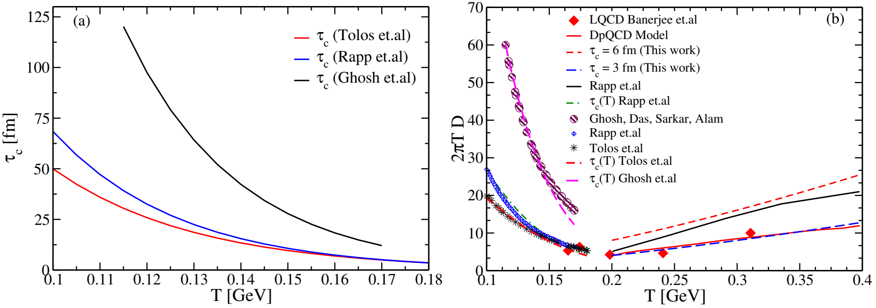

Figure: (a)Relaxation time parameterized as a function of T shown by $\tau_c(T)$ vs T. Black solid line : results from Das. [1], blue solid line : results of He et.al. [2], red solid line : results of Tolos et.al. [3] and (b) Spatial diffusion coefficient (D) for D^0 mesons and charm quarks multiplied by $2\pi T$ plotted as $2\pi TD$ vs T.

- In Hadrons has been plotted between $T = 0.1$ - 0.18 GeV and charm quarks have been plotted for $T = 0.2$ - 0.4 GeV.
- Results for charm quarks calibrated between $\tau_c = 3$ and 6 fm.
- Existing results for charm quarks obtained via LQCD(Banerjee et.al) [7], DpQCD model(Berrehrah et.al) [8] and T-matrix approach(Riek & Rapp) [9] span our results.

Diffusion at $B \neq 0$

Magnetic field renders transport coefficients anisotropic :

- Electrical conductivity splits into three components : longitudinal ($\sigma^{\|}$) and transverse ($\sigma^{\perp}$)
- Spatial Diffusion coefficient : longitudinal ($D^{\|}$) and transverse ($D^{\perp}$)

Anisotropic factors in electrical conductivities ($\sigma^{\|}, \sigma^{\perp}$) is from the relaxation times ($\tau_c^{\|}, \tau_c^{\perp}$) :

$$\boxed{\text{longitudinal : } \tau_c^{\|} = \tau_c \quad \& \quad \text{transverse : } \tau_c^{\perp} = \tau_c \Big/ \left(1 + \frac{\tau_c^2 e^2 B^2}{\omega^2}\right)}$$

Similar anisotropic factors in spatial diffusion calculated in AdS/CFT (D. Dudal et.al [5])[1]

$$\boxed{D_{\|} = \frac{T}{\gamma m} \quad \& \quad D_{\perp} = \frac{D_{\|}}{1 + \frac{q^2 B^2}{m^2 \gamma^2}}}, \quad \text{where } \gamma \equiv \tau_c^{-1}.$$

[1] D. Dudal and T. G. Mertens, "Holographic estimate of heavy quark diffusion in a magnetic field," Phys. Rev. D **97**, no.5, 054035 (2018).

Electrical conductivities and susceptibility at $B \neq 0$

At $B \neq 0$

$\tau_c^{\parallel} = \tau_c, \quad \tau_c^{\perp} = \frac{\tau_c}{1+\frac{\tau_c^2 e^2 B^2}{\omega^2}}$

$$\sigma_{\parallel}^{\text{RTA}} = \frac{gq^2\beta}{3}\int \frac{d^3k}{(2\pi)^3}\frac{(k_z)^2}{\omega_k^2}\tau_c f\left[1-f\right] \quad (1)$$

$$\sigma_{\perp}^{\text{RTA}} = \frac{gq^2\beta}{3}\int \frac{d^3k}{(2\pi)^3}\frac{(k_{x,y})^2}{\omega_k^2}\frac{\tau_c}{1+\frac{\tau_c^2}{\tau_B^2}} f\left[1-f\right] \quad (2)$$

$$\sigma_{\perp}^{\text{QM}} = \frac{2e^2}{T}\sum_{l=0}^{\infty} g_l \frac{qB}{2\pi}\int_{-\infty}^{+\infty}\frac{dk_z}{2\pi}\frac{lqB}{\omega_l^2}\tau^{\perp} f(1-f) \quad (3)$$

$$\sigma_{\parallel}^{\text{QM}} = \frac{2e^2}{T}\sum_{l=0}^{\infty} g_l \frac{qB}{2\pi}\int_{-\infty}^{+\infty}\frac{dk_z}{2\pi}\frac{k_z^2}{\omega_l^2}\tau^{\parallel} f(1-f) \quad (4)$$

$$\chi_{\text{RTA}} = \beta\int \frac{d^3k}{(2\pi)^3}\, f(1-f), \qquad \chi_{\text{QM}} = \beta\sum_{l=0}^{\infty}\frac{g_l qB}{2\pi}\int_{-\infty}^{+\infty}\frac{dk_z}{2\pi}\, f(1-f) \quad (5)$$

$$\tau_c^{\parallel} = \tau_c, \quad \tau^{\parallel}_{c,\ D^0\text{Meson}} = 2794.15 \ \ e^{-37T}\text{fm}, \quad \tau^{\parallel}_{c,\ \text{charm}} = 6 \text{ fm}$$

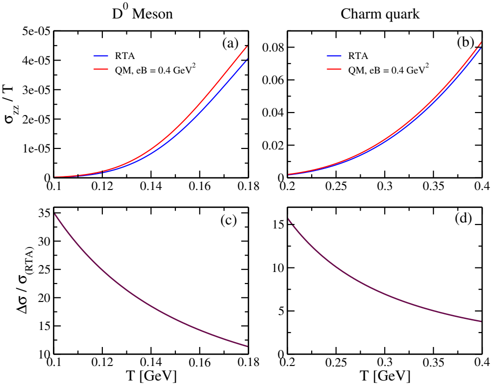

Figure: Parallel component of conductivity (scaled) σ_{zz}/T vs T.

- Here $\Delta\sigma = (\sigma_{\text{QM}} - \sigma_{\text{RTA}}) \times 100$.
- Increasing temperature contributes to kinetic energy and hence σ increases.

Longitudinal electrical conductivity as a function of magnetic field

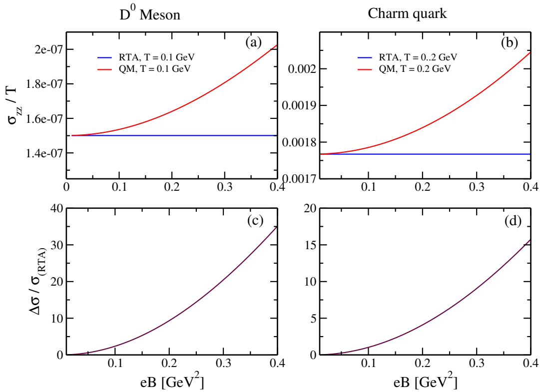

Figure: Parallel component of conductivity (scaled) σ_{zz}/T vs eB.

- RTA results of longitudinal electrical conductivity remains unaffected by magnetic field as Lorentz force does not act along the direction of B.
- Landau quantization induces magnetic field dependence in QM results.
- In strong magnetic field regime RTA and QM results differ.

Longitudinal spatial diffusion as a function of temperature

$$\tau_c^{||} = \tau_c, \quad \tau^{||}_{c,\ D^0\text{Meson}} = 2794.15 \ \ e^{-37T}\text{fm}, \quad \tau^{||}_{c,\ \text{charm}} = 6 \text{ fm}$$

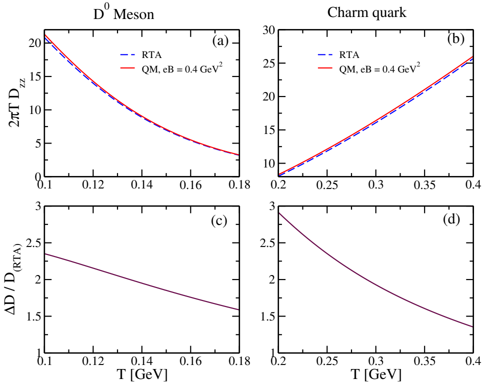

Figure: Parallel component of diffusion (D) (scaled) by $2\pi T$ vs T.

- Here $\Delta D = \left(D_{\text{QM}} - D_{\text{RTA}}\right) \times 100$.
- For the D^0 meson case results are consistent with Das. [1], He et.al. [2] and Tolos et.al. [3].

Longitudinal spatial diffusion as a function of magnetic field

$$\tau_c^{\parallel} = \tau_c, \quad \tau^{\parallel}_{c,\ D^0\text{Meson}} = 2794.15 \ \ e^{-37T}\text{fm}, \quad \tau^{\parallel}_{c,\ \text{charm}} = 6 \text{ fm}$$

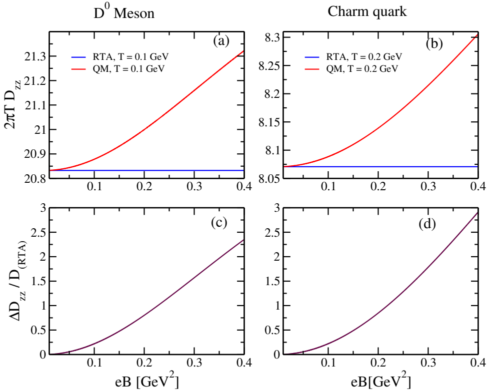

Figure: Parallel component of diffusion (D) (scaled) by $2\pi T$ vs eB.

- RTA results of longitudinal diffusion remains unaffected as Lorentz force does not act along B.
- For the QM case the contribution of all Landau levels (via Landau quantization) renders magnetic field dependent longitudinal diffusion.

Transverse electrical conductivity as a function of temperature

$$\tau_c^{\perp} = \frac{\tau_c}{\left(1 + \frac{\tau_c^2 e^2 B^2}{\omega^2}\right)}, \quad \tau_{c,\text{ Hadrons}} = 2794.15 \quad e^{-37T}\text{fm}, \quad \tau_{c,\text{ charm}} = 6 \text{ fm}$$

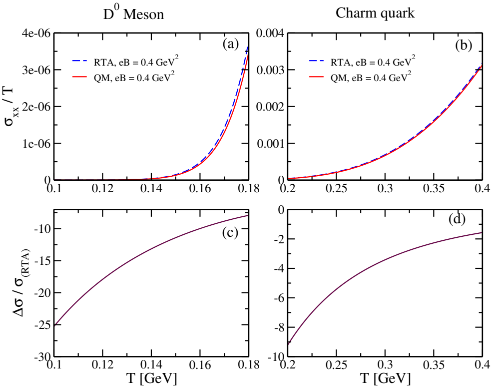

Figure: Perpendicular component of conductivity (scaled) σ_{zz}/T vs T.

Transverse electrical conductivity as a function of magnetic field

$$\tau_c^{\perp} = \frac{\tau_c}{\left(1 + \frac{\tau_c^2 e^2 B^2}{\omega^2}\right)}, \quad \tau_{c,\text{ Hadrons}} = 2794.15 \;\; e^{-37T}\text{fm}, \quad \tau_{c,\text{ charm}} = 6 \text{ fm}$$

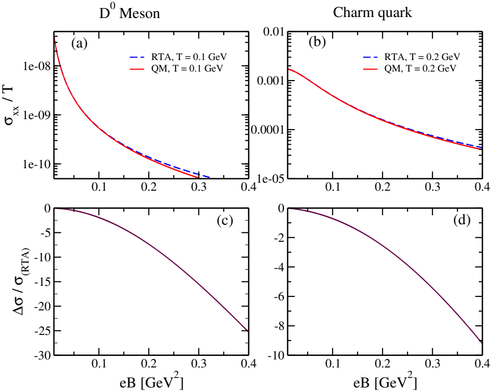

Figure: Perpendicular component of conductivity (scaled) σ_{zz}/T vs eB.

- Opposite effect of B compared to T for transverse case. Temperature increases randomness whereas magnetic field aligns the system.
- In strong magnetic field regime RTA and QM results differ.

$$\tau_c^{\perp} = \frac{\tau_c}{\left(1 + \frac{\tau_c^2 e^2 B^2}{\omega^2}\right)}, \quad \tau_{c,\text{ Hadrons}} = 2794.15 \;\; e^{-37T}\text{fm}, \quad \tau_{c,\text{ charm}} = 6 \text{ fm}$$

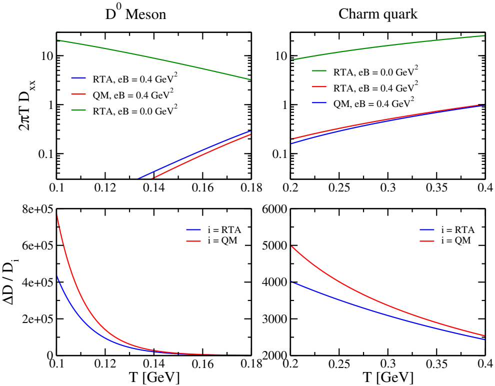

Figure: Perpendicular component of Diffusion (D) (scaled by $2\pi T$) vs T.

- For the hadronic case $2\pi T D_{xx}$ increases.
- Previously in η at $B \neq 0$: $\eta^{\parallel}$ follows KSS bound ($\eta/s \leq 1/4\pi$), $\eta^{\perp}$ violates KSS bound.
- Similar behaviour of transverse diffusion is observed.

Transverse spatial diffusion as a function of magnetic field

$$\tau_c^{\perp} = \frac{\tau_c}{\left(1 + \frac{\tau_c^2 e^2 B^2}{\omega^2}\right)}, \quad \tau_{c,\ \text{Hadrons}} = 2794.15 \quad e^{-37T}\text{fm}, \quad \tau_{c,\ \text{charm}} = 6 \text{ fm}$$

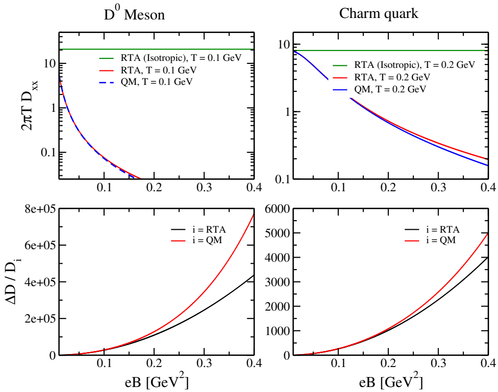

Figure: Perpendicular component of Diffusion (D) (scaled by $2\pi T$) vs eB.

Susceptibilities in magnetic field : χ/T^2 vs $\{T, eB\}$

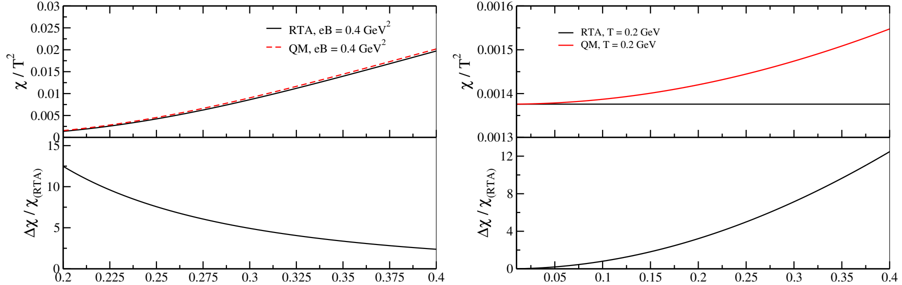

Figure: susceptibilities as a function of T, eB.

- RTA expression of susceptibility remains unchanged as it is independent of magnetic field.
- QM expression induces magnetic field dependence through Landau quantization.

Summary

- Spatial diffusion is computed as a ratio of electrical conductivity and susceptibility.
- In the presence of magnetic field spatial diffusion splits into two components : longitudinal and transverse relative to the direction of magnetic field.
- Electrical conductivities and susceptibilities is calculated via RTA and QM method (Landau quantization).
- Spatial diffusion has been studied as a function of temperature and magnetic field.
- Anisotropy comes in through the relaxation time of longitudinal and transverse components of diffusion.

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